61A Lecture 19

Wednesday, October 10
Generic Functions, Continued
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A function might want to operate on multiple data types
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Last time:
Generic Functions, Continued

A function might want to operate on multiple data types

Last time:
• Polymorphic functions using message passing
Generic Functions, Continued

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• Polymorphic functions using message passing
• Interfaces: collections of messages with a meaning for each
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- An arithmetic system over related types
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- Type coercion

What's different? Today's generic functions apply to multiple arguments that don't share a common interface
Rational Numbers
Rational Numbers

Rational numbers represented as a numerator and denominator
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Rational numbers represented as a numerator and denominator

```python
class Rational(object):
```

Rational Numbers

Rational numbers represented as a numerator and denominator

class Rational(object):
    def __init__(self, numer, denom):
        g = gcd(numer, denom)
        self.numer = numer // g
        self.denom = denom // g
Rational Numbers

Rational numbers represented as a numerator and denominator

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Greatest common divisor
Rational Numbers

Rational numbers represented as a numerator and denominator

class Rational(object):

def __init__(self, numer, denom):
    g = gcd(numer, denom);
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def __repr__(self):
    return 'Rational({0}, {1})'.format(self.numer, self.denom)
Rational Numbers

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    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)

    def add_rational(self, y):
        nx, dx = x.numer, x.denom
        ny, dy = y.numer, y.denom
        return Rational(nx * dy + ny * dx, dx * dy)
```

Greatest common divisor
Rational Numbers

Rational numbers represented as a numerator and denominator

class Rational(object):
    def __init__(self, numer, denom):
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        self.denom = denom // g

    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)

    def add_rational(x, y):
        nx, dx = x.numer, x.denom
        ny, dy = y.numer, y.denom
        return Rational(nx * dy + ny * dx, dx * dy)

    def mul_rational(x, y):
        return Rational(x.numer * y.numer, x.denom * y.denom)
Complex Numbers: the Rectangular Representation
class ComplexRI(object):
    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

@property
def magnitude(self):
    return (self.real ** 2 + self.imag ** 2) ** 0.5

@property
def angle(self):
    return atan2(self.imag, self.real)

def __repr__(self):
    return 'ComplexRI({0}, {1})'.format(self.real, self.imag)
Complex Numbers: the Rectangular Representation

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def magnitude(self):
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@property
def angle(self):
    return atan2(self.imag, self.real)

__repr__ = lambda self: 'ComplexRI({0}, {1})'.format(self.real, self.imag)

def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real, z1.imag + z2.imag)
Complex Numbers: the Rectangular Representation

class ComplexRI(object):
    def __init__(self, real, imag):
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def magnitude(self):
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def angle(self):
    return atan2(self.imag, self.real)

def __repr__(self):
    return 'ComplexRI({0}, {1})'.format(self.real, self.imag)

def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real, z1.imag + z2.imag)

Might be either ComplexMA or ComplexRI instances
Special Methods
Special Methods

Adding instances of user-defined classes with \texttt{\_\_add\_\_}.
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Adding instances of user-defined classes with \_\_add\_.

Demo
Special Methods

Adding instances of user-defined classes with `__add__`.

Demo

```python
>>> ComplexRI(1, 2) + ComplexMA(2, 0)
ComplexRI(3.0, 2.0)
```
Special Methods

Adding instances of user-defined classes with `__add__`.

Demo

```python
>>> ComplexRI(1, 2) + ComplexMA(2, 0)
ComplexRI(3.0, 2.0)
```

```python
>>> ComplexRI(0, 1) * ComplexRI(0, 1)
ComplexMA(1.0, 3.141592653589793)
```
Special Methods

Adding instances of user-defined classes with `__add__`.

Demo

```python
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ComplexRI(3.0, 2.0)
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```


http://docs.python.org/py3k/reference/datamodel.html#special-method-names
The Independence of Data Types
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Data abstraction and class definitions keep types separate
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Some operations need to cross type boundaries
The Independence of Data Types

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Some operations need to cross type boundaries.

\begin{flushleft}
\begin{tabular}{ll}
add\textunderscore rational & mul\textunderscore rational \\
\text{Rational numbers as} &\text{numerators \& denominators}
\end{tabular}
\end{flushleft}
The Independence of Data Types

Data abstraction and class definitions keep types separate

Some operations need to cross type boundaries

```
add_rational  mul_rational
```

Rational numbers as numerators & denominators

```
add_complex  mul_complex
```

Complex numbers as two-dimensional vectors
The Independence of Data Types

Data abstraction and class definitions keep types separate. Some operations need to cross type boundaries.

How do we add a complex number and a rational number together?

Rational numbers as numerators & denominators

Complex numbers as two-dimensional vectors
The Independence of Data Types

Data abstraction and class definitions keep types separate.

Some operations need to cross type boundaries.

How do we add a complex number and a rational number together?

Rational numbers as numerators & denominators

Complex numbers as two-dimensional vectors

There are many different techniques for doing this!
Type Dispatching
Type Dispatching

Define a different function for each possible combination of types for which an operation (e.g., addition) is valid.
Type Dispatching

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```python
def iscomplex(z):
    return type(z) in (ComplexRI, ComplexMA)
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Type Dispatching

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    def iscomplex(z):
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```python
def iscomplex(z):
    return type(z) in (ComplexRI, ComplexMA)

def isrational(z):
    return type(z) is Rational

def add_complex_and_rational(z, r):
    return ComplexRI(z.real + r.numer/r.denom, z.imag)
```
Type Dispatching

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Converted to a real number (float)
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def add_by_type_dispatching(z1, z2):
    """Add z1 and z2, which may be complex or rational.""
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def add_complex_and_rational(z, r):
    return ComplexRI(z.real + \( \frac{r.\text{numer}}{r.\text{denom}} \), z.imag)

def add_by_type_dispatching(z1, z2):
    """Add z1 and z2, which may be complex or rational."""
    if iscomplex(z1) and iscomplex(z2):
        return add_complex(z1, z2)
```

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Define a different function for each possible combination of types for which an operation (e.g., addition) is valid.

```python
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    return type(z) in (ComplexRI, ComplexMA)

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    return ComplexRI(z.real + (r.numer/r.denom), z.imag)

def add_by_type_dispatching(z1, z2):
    """Add z1 and z2, which may be complex or rational.""
    if iscomplex(z1) and iscomplex(z2):
        return add_complex(z1, z2)
    elif iscomplex(z1) and isrational(z2):
        return add_complex_and_rational(z1, z2)
```

Converted to a real number (float)
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    """Add z1 and z2, which may be complex or rational.""
    if iscomplex(z1) and iscomplex(z2):
        return add_complex(z1, z2)
    elif iscomplex(z1) and isrational(z2):
        return add_complex_and_rational(z1, z2)
    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
```

(Converted to a real number (float))
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    if iscomplex(z1) and iscomplex(z2):
        return add_complex(z1, z2)
    elif iscomplex(z1) and isrational(z2):
        return add_complex_and_rational(z1, z2)
    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
    else:
        add_rational(z1, z2)
```

Converted to a real number (float)
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    elif isrational(z1) and iscomplex(z2):
        return add_complex_and_rational(z2, z1)
    else:
        add_rational(z1, z2)
```

Demo
Tag-Based Type Dispatching
Tag-Based Type Dispatching

**Idea:** Use dictionaries to dispatch on type
Tag-Based Type Dispatching

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```python
def type_tag(x):
    return type_tag.tags[type(x)]
```
Tag-Based Type Dispatching

Idea: Use dictionaries to dispatch on type

```python
def type_tag(x):
    return type_tag.tags[type(x)]

type_tag.tags = {ComplexRI: 'com',
                 ComplexMA: 'com',
                 Rational: 'rat'}
```
Tag-Based Type Dispatching

**Idea:** Use dictionaries to dispatch on type

```python
def type_tag(x):
    return type_tag.tags[type(x)]
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```python
type_tag.tags = {
    ComplexRI: 'com',
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}
```

Declares that ComplexRI and ComplexMA should be treated uniformly.
Tag-Based Type Dispatching

**Idea**: Use dictionaries to dispatch on type

```python
def type_tag(x):
    return type_tag.tags[type(x)]

type_tag.tags = {'ComplexRI': 'com',
                 'ComplexMA': 'com',
                 'Rational': 'rat'}

def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add.implementations[types](z1, z2)
```

Declares that ComplexRI and ComplexMA should be treated uniformly
Tag-Based Type Dispatching

**Idea:** Use dictionaries to dispatch on type

```python
def type_tag(x):
    return type_tag.tags[type(x)]


def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add.implementations[types](z1, z2)

add.implementations = {}
add.implementations[('com', 'com')] = add_complex
add.implementations[('rat', 'rat')] = add_rational
add.implementations[('com', 'rat')] = add_complex_and_rational
add.implementations[('rat', 'com')] = add_rational_and_complex
```

Declares that ComplexRI and ComplexMA should be treated uniformly
Tag-Based Type Dispatching

**Idea:** Use dictionaries to dispatch on type

```python
def type_tag(x):
    return type_tag.tags[type(x)]
type_tag.tags = {ComplexRI: 'com',
                 ComplexMA: 'com',
                 Rational: 'rat'}

def add(z1, z2):
    types = (type_tag(z1), type_tag(z2))
    return add.implementations[types](z1, z2)

add.implementations = {}
add.implementations[(('com', 'com'))] = add_complex
add.implementations[(('rat', 'rat'))] = add_rational
add.implementations[(('com', 'rat'))] = add_complex_and_rational
add.implementations[(('rat', 'com'))] = add_rational_and_complex
```

- Declares that ComplexRI and ComplexMA should be treated uniformly
- Lambda function: `lambda r, z: add_complex_and_rational(z, r)`
Type Dispatching Analysis
Type Dispatching Analysis

Minimal violation of abstraction barriers: we define cross-type functions as necessary, but use abstract data types.
Type Dispatching Analysis

Minimal violation of abstraction barriers: we define cross-type functions as necessary, but use abstract data types.

Extensible: Any new numeric type can "install" itself into the existing system by adding new entries to various dictionaries.
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Extensible: Any new numeric type can "install" itself into the existing system by adding new entries to various dictionaries

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**Question:** How many cross-type implementations are required to support $m$ types and $n$ operations?
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$$m \cdot (m - 1) \cdot n$$
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**Question:** How many cross-type implementations are required to support \( m \) types and \( n \) operations?

\[
m \cdot (m - 1) \cdot n
\]

\[
4 \cdot (4 - 1) \cdot 4 = 48
\]
Type Dispatching Analysis

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integer, rational, real, complex \[ m \cdot (m - 1) \cdot n \]

\[ 4 \cdot (4 - 1) \cdot 4 = 48 \]
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```

**Question:** How many cross-type implementations are required to support $m$ types and $n$ operations?

integer, rational, real, complex

$m \cdot (m - 1) \cdot n$

add, subtract, multiply, divide

$4 \cdot (4 - 1) \cdot 4 = 48$
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Minimal violation of abstraction barriers: we define cross-type functions as necessary, but use abstract data types.

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<tr>
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<tbody>
<tr>
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Message Passing

Type Dispatching
Data-Directed Programming
Data-Directed Programming

There's nothing addition-specific about add_by_type
Data-Directed Programming

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**Idea:** One dispatch function for (operator, types) pairs
Data-Directed Programming

There's nothing addition-specific about add_by_type

**Idea:** One dispatch function for (operator, types) pairs

```python
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
    return apply.implementations[key](x, y)
```
There's nothing addition-specific about `add_by_type`

**Idea:** One dispatch function for (operator, types) pairs

```python
def apply(operator_name, x, y):
    tags = (type_tag(x), type_tag(y))
    key = (operator_name, tags)
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Demo
Coercion
Coercion

**Idea:** Some types can be converted into other types
Coercion

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Takes advantage of structure in the type system
Coercion

Idea: Some types can be converted into other types

Takes advantage of structure in the type system

```python
>>> def rational_to_complex(x):
    return ComplexRI(x.numer/x.denom, 0)
```
Coercion

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Takes advantage of structure in the type system

```python
>>> def rational_to_complex(x):
    return ComplexRI(x.numer/x.denom, 0)

>>> coercions = {('rat', 'com'): rational_to_complex}
```
Coercion

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Takes advantage of structure in the type system

```python
>>> def rational_to_complex(x):
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**Question:** Can any numeric type be coerced into any other?
Coercion

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>>> def rational_to_complex(x):
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```

**Question:** Can any numeric type be coerced into any other?

**Question:** Have we been repeating ourselves with data-directed programming?
Applying Operators with Coercion
Applying Operators with Coercion

1. Attempt to coerce arguments into values of the same type
Applying Operators with Coercion

1. Attempt to coerce arguments into values of the same type
2. Apply type-specific (not cross-type) operations
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Demo
Coercion Analysis
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Minimal violation of abstraction barriers: we define cross-type coercion as necessary, but use abstract data types
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Requires that all types can be coerced into a common type
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More sharing: All operators use the same coercion scheme
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<tbody>
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