

# 61A Lecture 22

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Wednesday, October 17

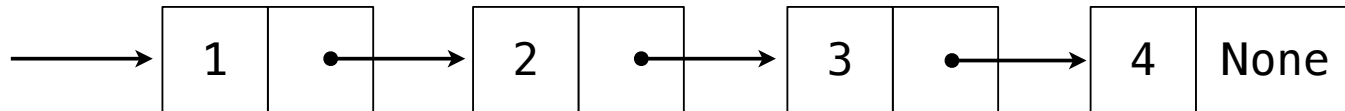
## Closure Property of Data

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A tuple can contain another tuple as an element.

Pairs are sufficient to represent sequences.

Recursive list representation of the sequence 1, 2, 3, 4:



Recursive lists are recursive: the rest of the list is a list.

Nested Tuples (old): `(1, (2, (3, (4, None))))` Demo

Rlist class (new): `Rlist(1, Rlist(2, Rlist(3, Rlist(4))))`

## Recursive List Class

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Methods can be recursive as well!

This part was all  
in Homework 6

```
class Rlist(object):  
  
    class EmptyList(object):  
        def __len__(self):  
            return 0  
  
    empty = EmptyList()  
  
    def __init__(self, first, rest=empty):  
        self.first = first  
        self.rest = rest
```

There's the  
base case!

```
    def __len__(self):  
        return 1 + len(self.rest)
```

Yes, this call  
is recursive

```
    def __getitem__(self, i):  
        if i == 0:  
            return self.first  
        return self.rest[i-1]
```

Demo

## Recursive Operations on Recursive Lists

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Recursive list processing almost always involves a recursive call on the rest of the list.

```
>>> s = Rlist(1, Rlist(2, Rlist(3)))
```

```
>>> s.rest  
Rlist(2, Rlist(3))
```

```
>>> extend_rlist(s.rest, s)  
Rlist(2, Rlist(3, Rlist(1, Rlist(2, Rlist(3))))))
```

```
def extend_rlist(s1, s2):  
    if s1 is Rlist.empty:  
        return s2  
    return Rlist(s1.first, extend_rlist(s1.rest, s2))
```

Demo

## Map and Filter on Recursive Lists

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We want operations on a whole list, not an element at a time.

```
>>> def map_rlist(s, fn):
    if s is Rlist.empty:
        return s
    return Rlist(fn(s.first), map_rlist(s.rest, fn))
```

```
>>> def filter_rlist(s, fn):
    if s is Rlist.empty:
        return s
    rest = filter_rlist(s.rest, fn)
    if fn(s.first):
        return Rlist(s.first, rest)
    return rest
```

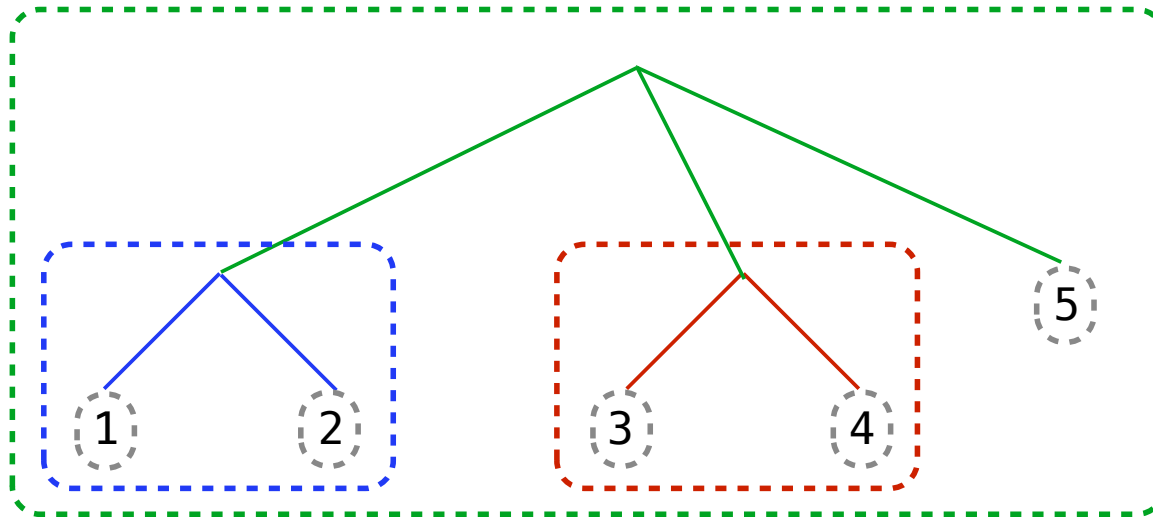
Demo

## Tree Structured Data

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Nested Sequences are Hierarchical Structures.

$((1, 2), (3, 4), 5)$



*In every tree, a vast forest*

## Recursive Tree Processing

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Tree operations typically make recursive calls on branches

```
def count_leaves(tree):  
    if type(tree) != tuple:  
        return 1  
    return sum(map(count_leaves, tree))
```

```
def map_tree(tree, fn):  
    if type(tree) != tuple:  
        return fn(tree)  
    return tuple(map_tree(branch, fn) for branch in tree)
```

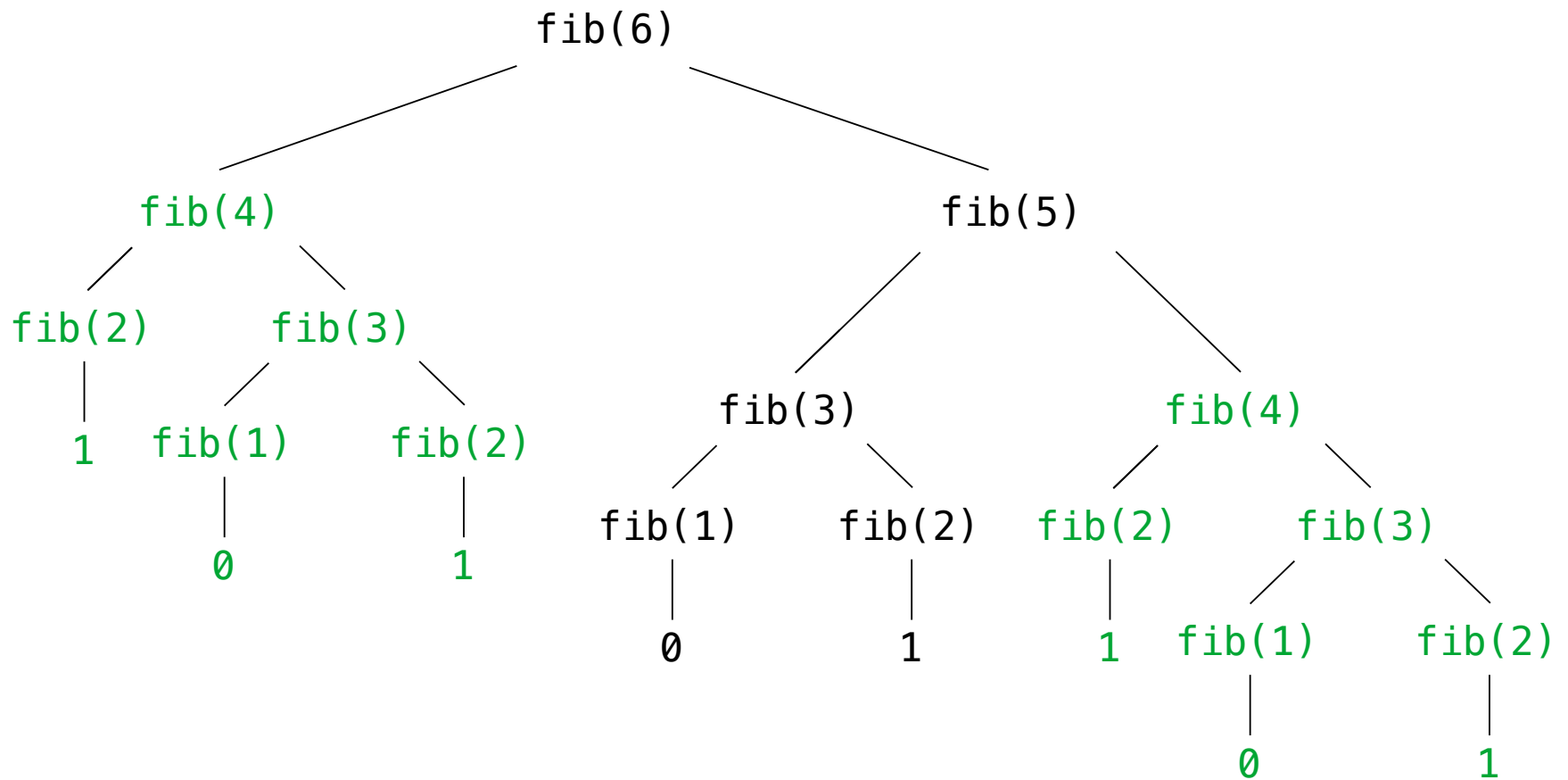
Demo

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## Trees with Internal Node Values

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Trees can have values at their roots as well as their leaves.





## Trees with Internal Node Values (Entries)

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Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n-2)
    right = fib_tree(n-1)
    return Tree(left.entry + right.entry, left, right)
```

Demo

## Sets

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One more built-in Python container type

- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}

>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Demo