Closure Property of Data

A tuple can contain another tuple as an element.
Pairs are sufficient to represent sequences.

Recursive list representation of the sequence 1, 2, 3, 4:

```
1  2  3  4  None
```

Recursive lists are recursive: the rest of the list is a list.

Nested Tuples (old):  
(1, (2, (3, (4, None))))  
Rlist class (new): 
Rlist(1, Rlist(2, Rlist(3, Rlist(4))))

Recursive List Class

Methods can be recursive as well!

```python
class Rlist(object):
    def __init__(self, first, rest=EmptyList()):
        self.first = first
        self.rest = rest

def extend_rlist(s1, s2):
    if s1 is Rlist().empty:
        return s2
    return Rlist(s1.first, extend_rlist(s1.rest, s2))
```

Recursive Operations on Recursive Lists

Recursive list processing almost always involves a recursive call on the rest of the list.

```python
>>> s = Rlist(1, Rlist(2, Rlist(3)))
>>> s.rest
Rlist(2, Rlist(3))
>>> extend_rlist(s.rest, s)
Rlist(2, Rlist(3, Rlist(1, Rlist(2, Rlist(3)))))
```

```python
def extend_rlist(s1, s2):
    if s1 is Rlist().empty:
        return s2
    return Rlist(s1.first, extend_rlist(s1.rest, s2))
```

Map and Filter on Recursive Lists

We want operations on a whole list, not an element at a time.

```python
>>> def map_rlist(s, fn):
...     if s is Rlist().empty:
...         return s
...     return Rlist(fn(s.first), map_rlist(s.rest, fn))
```

```python
>>> def filter_rlist(s, fn):
...     if s is Rlist().empty:
...         return s
...     rest = filter_rlist(s.rest, fn)
...     if fn(s.first):
...         return Rlist(s.first, rest)
...     return rest
```

Tree Structured Data

Nested Sequences are Hierarchical Structures.

```
((1, 2), (3, 4), 5)
```

```
In every tree, a vast forest
```
Recursive Tree Processing

Tree operations typically make recursive calls on branches.

def count_leaves(tree):
    if type(tree) != tuple:
        return 1
    return sum(map(count_leaves, tree))

def map_tree(tree, fn):
    if type(tree) != tuple:
        return fn(tree)
    return tuple(map_tree(branch, fn) for branch in tree)

Trees with Internal Node Values

Trees can have values at their roots as well as their leaves.

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

    def fib_tree(n):
        if n == 1:
            return Tree(0)
        if n == 2:
            return Tree(1)
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        return Tree(left.entry + right.entry, left, right)

Sets

One more built-in Python container type
- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}

>>> 3 in s
True

>>> len(s)
4

>>> s.union({1, 5})
{1, 2, 3, 4, 5}

>>> s.intersection({6, 5, 4, 3})
{3, 4}