61A Lecture 30

Wednesday, November 7
Functional Programming
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All functions are pure functions
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No assignment and no mutable data types (except for re-define)
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• The value of an expression is independent of the order in which sub–expressions are evaluated

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• Referential transparency: The value of an expression does not change when we substitute one of its subexpression with the value of that subexpression.
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But... Can we make basic loops efficient?

Yes!
Iteration Versus Recursion in Python

In Python, recursive calls always create new active frames.
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def exp(b, n):
    if n == 0:
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    return b * exp(b, n-1)
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\[
\begin{array}{c|c}
\text{Time} & \text{Space} \\
\hline
\Theta(n) & \\
\end{array}
\]

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<table>
<thead>
<tr>
<th></th>
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<td>Θ(n)</td>
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Tail Recursion

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"Implementations of Scheme are required to be \textit{properly tail-recursive}. This allows the execution of an iterative computation in constant space, even if the iterative computation is described by a syntactically recursive procedure."


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```
(define (factorial n k)
  (if (= n 0) k
    (factorial (- n 1) (factorial (- n 1) (* k n))))))
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\begin{verbatim}
(define (factorial n k)
  (if (= n 0) k
      (factorial (- n 1)
                  (* k n))))

def factorial(n, k):
  while n > 0:
    n, k = n-1, k*n
  return k
\end{verbatim}
Tail Calls
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A procedure call that has not yet returned is active. Some procedure calls are tail calls. A Scheme interpreter should support an unbounded number of active tail calls.
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A tail call is a call expression in a tail context:

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- Sub–expressions 2 & 3 in a tail context if expression
A procedure call that has not yet returned is *active*. Some procedure calls are *tail calls*. A Scheme interpreter should support an unbounded number of active tail calls.

A tail call is a call expression in a *tail context*:  
- The last expression in a lambda expression  
- Sub-expressions 2 & 3 in a tail context *if* expression  
- All non-predicate sub-expressions in a tail context *cond*
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  (if (null? s) 0
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Linear recursions can often be re-written to use tail calls.
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(define (length-tail s)
Example: Length of a List

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Eval with Tail Call Optimization
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The return value of the tail call is the return value of the current procedure call.
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In the interpreter, recursive calls to scheme_eval for tail calls must instead be expressed iteratively.

Demo
Logical Special Forms, Revisited

Logical forms may only evaluate some sub-expressions.

- **If** expression:  \((\text{if } \langle\text{predicate}\rangle \ \langle\text{consequent}\rangle \ \langle\text{alternative}\rangle)\)
- **And** and **or**:  \((\text{and } \langle\text{e}_1\rangle \ ... \ \langle\text{e}_n\rangle), \ (\text{or } \langle\text{e}_1\rangle \ ... \ \langle\text{e}_n\rangle)\)
- **Cond expr'n**:  \((\text{cond } (\langle\text{p}_1\rangle \ \langle\text{e}_1\rangle) \ ... \ (\langle\text{p}_n\rangle \ \langle\text{e}_n\rangle) \ (\text{else } \langle\text{e}\rangle))\)
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The value of an **if** expression is the value of a sub-expression.
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- **Cond expr’n**: \((\text{cond } (<p_1> <e_1>) \ldots (<p_n> <e_n>) \text{ (else } <e>))\)

The value of an **if** expression is the value of a sub-expression.
- Evaluate the predicate.
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- Evaluate the predicate.
- Choose a sub-expression: \(<\text{consequent}> or <\text{alternative}>\).
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Logical forms may only evaluate some sub-expressions.

- **If** expression: \( (\text{if } \text{<predicate>} \ \text{<consequent>} \ \text{<alternative>}) \)
- **And** and **or**: \( (\text{and } \text{<e}_1 \ \ldots \ \text{<e}_n) \), \( (\text{or } \text{<e}_1 \ \ldots \ \text{<e}_n) \)
- **Cond expr'n**: \( (\text{cond}(\text{<p}_1 \ \text{<e}_1) \ \ldots \ (\text{<p}_n \ \text{<e}_n)(\text{else } \text{<e>})) \)

The value of an **if** expression is the value of a sub-expression.

- Evaluate the predicate.
- Choose a sub-expression: **<consequent>** or **<alternative>**.
- Evaluate that sub-expression in place of the whole expression.
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- **Cond** expr'n: `(cond (<p₁> <e₁>) ... (<pₙ> <eₙ>) (else <e>))`

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Evaluation of the tail context does not require a recursive call.
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- **And** and **or**: \((\text{and } \text{<e}1\text{> ... } \text{<e}n\text{>}), \text{(or } \text{<e}1\text{> ... } \text{<e}n\text{>})\)
- **Cond** expr'n: \((\text{cond } \text{(<p}1\text{> <e}1\text{>)} ... \text{(<p}n\text{> <e}n\text{>) (else } <e>))\)

The value of an **if** expression is the value of a sub–expression.

- Evaluate the predicate.
- Choose a sub–expression: \text{<consequent>} or \text{<alternative>}.
- Evaluate that sub–expression in place of the whole expression.

Evaluation of the tail context does not require a recursive call. E.g., replace \((\text{if false 1 (+ 2 3)})\) with \((+ 2 3)\) and repeat.
Example: Reduce
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\((\text{define (reduce \textit{fn} \textit{s} \textit{start})})\)
Example: Reduce

```
(define (reduce fn s start)
  (reduce * '(3 4 5) 2)
```
Example: Reduce

\[(\text{define } (\text{reduce} \ \text{fn} \ s \ \text{start}))\]

\[(\text{reduce} * \ '(3 \ 4 \ 5) \ 2) \quad 120\]
Example: Reduce

\[(\text{define (reduce \textbf{fn} \textbf{s} \textbf{start})})\]

\[(\text{reduce \* '(3 4 5) 2}) \quad 120\]

\[(\text{reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))}\]
Example: Reduce

\[(\text{define} \ (\text{reduce} \ \text{fn} \ s \ \text{start}))\]

\[(\text{reduce} \ * \ '(3 \ 4 \ 5) \ 2) \quad 120\]

\[(\text{reduce} \ (\lambda (x \ y) \ (\text{cons} \ y \ x)) \ '(3 \ 4 \ 5) \ '(2)) \quad (5 \ 4 \ 3 \ 2)\]
Example: Reduce

\[
\text{(define (reduce fn s start)}
\]

\[
\text{(if (null? s) start}
\]

\[
(\text{reduce } * \text{ '(3 4 5) 2}) \quad 120
\]

\[
(\text{reduce (lambda (x y) (cons y x)) '}(3 4 5) '}(2)) \quad (5 4 3 2)
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Example: Reduce

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(define (reduce fn s start)
  (if (null? s) start
    (reduce fn
        (reduce fn
            '(3 4 5) 2)
    (reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))
```
Example: Reduce

(define (reduce fn s start)
  (if (null? s) start
      (reduce fn
        (cdr s)
      ))

(reduce * '(3 4 5) 2) 120
(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2)) (5 4 3 2)
Example: Reduce

\[
(\text{define (reduce } \text{fn } s \text{ start)} \\
\quad (\text{if (null? } s) \text{ start} \\
\quad \quad (\text{reduce } \text{fn} \\
\quad \quad \quad (\text{cdr } s) \\
\quad \quad \quad (\text{fn start } (\text{car } s))) ))) \\
\]

(\text{reduce } \ast ' (3 4 5) 2) \quad \quad 120

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(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))  (5 4 3 2)
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(\(\text{reduce }* \ (3 \ 4 \ 5) \ 2\)) 120

(\(\text{reduce } (\lambda (x \ y) \ (\text{cons } y \ x)) \ (3 \ 4 \ 5) \ '(2))\) \(5 \ 4 \ 3 \ 2\)
Example: Reduce

\[
\text{(define (reduce \textit{fn} \textit{s} \textit{start})}
\]

\[
\text{(if (null? \textit{s}) \textit{start}}
\]

\[
\text{(reduce \textit{fn}}
\]

\[
\text{(cdr \textit{s})}
\]

\[
\text{(\textit{fn} \textit{start} (car \textit{s}))}}
\]

\[
\)(})
\]

Recursive call is a tail call.

\[
\text{(reduce \* '}(3 \ 4 \ 5) \ 2)
\]

120

\[
\text{(reduce (lambda (x y) (cons y x)) '}(3 \ 4 \ 5) '(2))}
\]

(5 4 3 2)
Example: Reduce

```
(define (reduce fn s start)
  (if (null? s) start
      (reduce fn
        (cdr s)
        (fn start (car s)))))
```

Recursive call is a tail call.
Other calls are not; constant space depends on `fn`.

```
(reduce * '(3 4 5) 2) 120
(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2)) (5 4 3 2)
```
Example: Map
Example: Map

(define (map fn s)
Example: Map

(define (map fn s)
  (define (map-iter fn s m))
Example: Map

(define (map fn s)
  (define (map-iter fn s m)
    (if (null? s) m

Example: Map

```
(define (map fn s)
  (define (map-iter fn s m)
    (if (null? s) m
       (map-iter fn
```
Example: Map

```
(define (map fn s)
  (define (map-iter fn s m)
    (if (null? s) m
        (map-iter fn
                     (cdr s))
```
Example: Map

```
(define (map fn s)
  (define (map-iter fn s m)
    (if (null? s) m
      (map-iter fn
        (map-iter fn
          (cdr s)
        (cons (fn (car s)) m))))
```
Example: Map

(define (map fn s)
  (define (map-iter fn s m)
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      (map-iter fn
        (map-iter fn
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(define (reverse s)
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  (reverse (map-iter fn s nil)))

(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s) r
      (reverse-iter (car s) (cons (cdr s) r)))))
Example: Map

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(define (map fn s)
  (define (map-iter fn s m)
    (if (null? s) m
    (map-iter fn
      (cdr s)
      (cons (fn (car s)) m))))
  (reverse (map-iter fn s nil)))

(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s) r
      (reverse-iter s (cons (fn (car s)) r))))
  (reverse-iter s nil))
```
Example: Map

(define (map fn s)
  (define (map-iter fn s m)
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Example: Map

(define (map fn s)
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```
An Analogy: Programs Define Machines
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Programs specify the logic of a computational device
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Programs specify the logic of a computational device

factorial
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

\[ \text{factorial} = \text{factorial} \times 1 \]
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

factorial

5

1

1

1

*
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

5 \rightarrow factorial \rightarrow 1 \rightarrow 1 \rightarrow * \rightarrow 120

factorial

= 1

\downarrow

- \downarrow 1

* \downarrow
Interpreters are General Computing Machine
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

```
(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
```

5 ➔ \textbf{Scheme Interpreter} ➔ 120
An interpreter can be parameterized to simulate any machine

\[
\text{(define (factorial n)} \\
\text{ (if (zero? n) 1 (* n (factorial (- n 1)))))}
\]

Our Scheme interpreter is a universal machine
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

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(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1))))
```

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

\[
\begin{align*}
5 & \quad \xrightarrow{\text{Scheme Interpreter}} \quad 120 \\
(\text{define (factorial n)} & \quad (\text{if (zero? n) 1 (* n (factorial (- n 1))))})
\end{align*}
\]

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Internally, it is just a set of manipulation rules
Interpretation in Python
Interpretation in Python

*eval*: Evaluates an expression in the current environment and returns the result. Doing so may affect the environment.
**Interpretation in Python**

*eval*: Evaluates an expression in the current environment and returns the result. Doing so may affect the environment.

*exec*: Executes a statement in the current environment. Doing so may affect the environment.
Interpretation in Python

**eval**: Evaluates an expression in the current environment and returns the result. Doing so may affect the environment.

**exec**: Executes a statement in the current environment. Doing so may affect the environment.

```python
eval('2 + 2')
```
Interpretation in Python

eval: Evaluates an expression in the current environment and returns the result. Doing so may affect the environment.

exec: Executes a statement in the current environment. Doing so may affect the environment.

```
eval('2 + 2')

exec('def square(x): return x * x')
```
**Interpretation in Python**

`eval`: Evaluates an expression in the current environment and returns the result. Doing so may affect the environment.

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```python
exec('def square(x): return x * x')
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`os.system('python <file>')`: Directs the operating system to invoke a new instance of the Python interpreter.
**Interpretation in Python**

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*os.system('python <file> '): Directs the operating system to invoke a new instance of the Python interpreter.

Demo