Logic Language Review

Expressions begin with query or fact followed by relations.

Expressions and their relations are Scheme lists.

(fact (append-to-form () ?x ?x))  \textbf{Simple Fact}

(fact (append-to-form (?a . ?r) ?y (?a . ?z))
  (append-to-form ?r ?y ?z))  \textbf{Conclusion}

(query (append-to-form ?left (c d) (e b c d)))
Success!
\texttt{left: (e b)}

If a query has more than one relation, all must be satisfied.
The interpreter lists all bindings of variables to values that it can find to satisfy the query.

Logic Example: Anagrams

A permutation (i.e., anagram) of a list is:
- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

<table>
<thead>
<tr>
<th>Element</th>
<th>List</th>
<th>List with element</th>
</tr>
</thead>
<tbody>
<tr>
<td>(fact (insert ?a ?r (?a . ?r)))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| (fact (insert ?a (?b . ?r) (?b . ?s))
  (insert ?a ?r ?s)) |
| (fact (anagram () ())) |
| (fact (anagram (?a . ?r) ?b)
  (insert ?a ?s ?b)
  (anagram ?r ?s)) |

Pattern Matching

The basic operation of the Logic interpreter is to attempt to unify two relations.

Unification is finding an assignment to variables that makes two relations the same.

\begin{align*}
( (a b) c (a b) ) & \rightarrow \text{True, } \{x: (a b)\} \\
( ?x c ?x ) & \rightarrow \text{True, } \{y: b, z: c\} \\
( (a b) c (a b) ) & \rightarrow \text{False}
\end{align*}

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\begin{align*}
( (a b) c (a b) ) & \rightarrow (a b)
( ?x c ?x ) & \rightarrow ?x ?x \rightarrow \text{Success!}
\end{align*}

Unification with Two Variables

Two relations that contain variables can be unified as well.

\begin{align*}
( ?x ) & \rightarrow (a ?y c)
( ?x ) & \rightarrow (a b ?z)
\end{align*}

True, \(\{x: (a ?y c), y: b, z: c\}\)

Substituting values for variables may require multiple steps.

lookup('?x') \(\rightarrow (a ?y c)\)
lookup('?y') \(\rightarrow b\)
Implementing Unification

```python
def unify(e, f, env):
e = lookup(e, env)
f = lookup(f, env)
if e == f:
    return True
elif isvar(e):
    env.define(e, f)
    return True
elif isvar(f):
    env.define(f, e)
    return True
elif scheme_atomp(e) or scheme_atomp(f):
    return False
else:
    return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment
2. Establish new bindings to unify elements.

Symbols/relations without variables only unify if they are the same

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z)) (app (app (?r (c d) (e b c d)))
(conclusion <- hypothesis
(app (?r (c d) (c d))
{r2: (), x: (c d)})
(left: (e b . ())) \(\rightarrow\) (e b))
```

Depth-First Search

The space of facts is searched exhaustively, starting from the query and following a depth-first exploration order.

Depth-first search: A possible proof approach is explored exhaustively before another one is considered.

```
def search(clauses, env, depth):
    if clauses is nil:
        yield env
    elif DEPTH_LIMIT is None or depth <= DEPTH_LIMIT:
        for fact in facts:
            fact = rename_variables(fact, get_unique_id())
            env_head = Frame(env)
            if unify(fact.first, clauses.first, env_head):
                for env_rule in search(fact.second, env_head, depth+1):
                    for result in search(clauses.second, env_rule, depth+1):
                        yield result
        yield result
```

Limiting depth of the search avoids infinite loops.
Each time a fact is used, its variables are renamed.
Bindings are stored in separate frames to allow backtracking.

Implementing Depth-First Search

```
def search(clauses, env, depth):
    if clauses is nil:
        yield env
    elif DEPTH_LIMIT is None or depth <= DEPTH_LIMIT:
        for fact in facts:
            fact = rename_variables(fact, get_unique_id())
            env_head = Frame(env)
            if unify(fact.first, clauses.first, env_head):
                for env_rule in search(fact.second, env_head, depth+1):
                    for result in search(clauses.second, env_rule, depth+1):
                        yield result
        yield result
```

Whatever calls search can access all yielded results