Announcements
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• Project 4 due Thursday 11/21 @ 11:59pm, and it's a Scheme interpreter!
  • Also, the project is very long. Get started today.
Dynamic Scope
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**Lexical scope:** The parent for f's frame is the global frame.
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Tail Recursion
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**But...** no for/while statements! Can we make basic iteration efficient? Yes!
Recursion and Iteration in Python

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How? Eliminate the middleman!

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Should use resources like

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```

(Demo)

http://goo.gl/tu9s3W
Tail Calls
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A procedure call that has not yet returned is *active*. Some procedure calls are *tail calls*. A Scheme interpreter should support an *unbounded number* of active tail calls using only a *constant* amount of space.
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Linear recursive procedures can often be re-written to use tail calls.
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(define (length s)
  (if (null? s) 0  ; Not a tail context
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The return value of the tail call is the return value of the current procedure call.
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(Demo)
Tail Recursion Examples
Which Procedures are Tail Recursive?

Which of the following procedures run in constant space? \( \Theta(1) \)

\[\text{;; Compute the length of } s.\]
\[
\text{(define (length s)}
\] + 1 (\text{if (null? s)}
\] -1
\] (length (cdr s)))
\)

\[\text{;; Return the } n\text{th Fibonacci number.}\]
\[
\text{(define (fib n)}
\] (define (fib-iter current k)
\] (if (= k n)
\] current
\] (fib-iter (+ current
\] (fib (- k 1)))
\] (+ k 1))
\)
\] (if (= 1 n) 0 (fib-iter 1 2)))

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Which of the following procedures run in constant space? \( \Theta(1) \)

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\text{(define (length s)}\\
\text{ (+ 1 (if (null? s)\\
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\]

\[
\text{;;; Return the } n\text{th Fibonacci number.}\\
\text{(define (fib n)}\\
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\text{ (if (= k n)\\
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\]
Map and Reduce
Example: Reduce
Example: Reduce

(define (reduce procedure s start)
Example: Reduce

(define (reduce procedure s start)
  (reduce * '(3 4 5) 2))
Example: Reduce

```
(define (reduce procedure s start)
  (reduce * '(3 4 5) 2) 120)
```
Example: Reduce

\[
(\text{define } (\text{reduce} \ \text{procedure} \ s \ \text{start}))
\]

\[
(\text{reduce } * \ '(3 \ 4 \ 5) \ 2)
\]

\[
(\text{reduce } (\text{lambda} \ (x \ y) \ (\text{cons} \ y \ x)) \ '(3 \ 4 \ 5) \ '(2))
\]
Example: Reduce

\[\text{(define (reduce procedure s start)}\]

\[
\text{(reduce } * \text{ '(3 4 5) 2)} \quad 120
\]

\[
\text{(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))} \quad (5 4 3 2)
\]
Example: Reduce

(define (reduce procedure s start)
  (if (null? s) start

  (reduce * '(3 4 5) 2)

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  120

  (5 4 3 2)
Example: Reduce

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  (if (null? s) start
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      120
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Example: Reduce

\[
\text{(define (reduce procedure s start)}
\]
\[
\text{(if (null? s) start)}
\]
\[
\text{(reduce procedure)}
\]
\[
\text{(cdr s)}
\]

\[
\text{(reduce * '(3 4 5) 2)}
\]
\[
120
\]

\[
\text{(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))}
\]
\[
(5 4 3 2)
\]
Example: Reduce

\[
\begin{align*}
\text{(define (reduce procedure s start)} & \\
\text{  (if (null? s) start)} & \\
\text{  (reduce procedure} & \\
\text{    (reduce procedure} & \\
\text{      (cdr s)} & \\
\text{    (procedure start (car s))) ) ) ) } & \\
\text{(reduce * '(3 4 5) 2)} & \quad 120 \\
\text{(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))} & \quad (5 4 3 2)
\end{align*}
\]
Example: Reduce

```
(define (reduce procedure s start)
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        (cdr s)
        (procedure start (car s)))))
```

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(reduce * '(3 4 5) 2) 120
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(define (reduce procedure s start)
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```
Example: Reduce

\[
\text{(define (reduce procedure s start)} \\
\hspace{1cm} \text{(if (null? s) start)} \\
\hspace{2cm} \text{(reduce procedure)} \\
\hspace{3cm} \text{(cdr s)} \\
\hspace{4cm} \text{(procedure start (car s))}} \}}))
\]

(reduce * '(3 4 5) 2) \quad 120

(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2)) \quad (5 4 3 2)
Example: Reduce

\[
\text{(define (reduce procedure s start)}
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\[
\begin{align*}
& \quad \text{(if (null? s) start} \\
& \qquad \text{(reduce procedure)} \\
& \qquad \quad \text{(cdr s)} \\
& \qquad \quad \quad \text{(procedure start (car s))})
\end{align*}
\]

Recursive call is a tail call.

\[
\begin{align*}
\text{(reduce * '(3 4 5) 2)} & \quad 120 \\
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\end{align*}
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Example: Reduce

(define (reduce procedure s start)
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Recursive call is a tail call.
Other calls are not; constant space depends on whether procedure requires constant space.

(reduce * '(3 4 5) 2) 120
(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2)) (5 4 3 2)
Example: Map with Only a Constant Number of Frames
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(define (map procedure s))
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(define (map procedure s)
  (if (null? s)
     ))
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      ....)
```

Example: Map with Only a Constant Number of Frames

```scheme
(define (map procedure s)
  (if (null? s)
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Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
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(map (lambda (x) (- 5 x)) (list 1 2))
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\[
\text{(define (map procedure s)}
\begin{align*}
\text{(if (null? s) } \\
\text{ \hspace{1em} nil} \\
\text{ \hspace{1em} (cons (procedure (car s))} \\
\text{ \hspace{2.5em} (map procedure (cdr s)))))}
\end{align*}
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(define (map procedure s)
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```
Example: Map with Only a Constant Number of Frames

(\texttt{define} (\texttt{map} \texttt{procedure} \texttt{s})
  (if (null? \texttt{s})
    \texttt{nil}
    (cons (\texttt{procedure} (\texttt{car} \texttt{s}))
      (\texttt{map} \texttt{procedure} (\texttt{cdr} \texttt{s}))))))

(map (\texttt{lambda} (\texttt{x}) (- 5 \texttt{x})) (\texttt{list} 1 2))
Example: Map with Only a Constant Number of Frames

\[
\begin{align*}
(\text{define} & \ (\text{map} \ \text{procedure} \ s) \\
& \ (\text{if} \ (\text{null?} \ s) \\
& \hspace{1cm} \text{nil} \\
& \hspace{1cm} (\text{cons} \ (\text{procedure} \ (\text{car} \ s)) \\
& \hspace{1.5cm} (\text{map} \ \text{procedure} \ (\text{cdr} \ s)))))
\end{align*}
\]

\[
(\text{map} \ (\text{lambda} \ (x) \ (-5 \ x)) \ (\text{list} \ 1 \ 2))
\]
Example: Map with Only a Constant Number of Frames

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```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s)))))
```
Example: Map with Only a Constant Number of Frames

```
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(map (lambda (x) (- 5 x)) (list 1 2))
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Example: Map with Only a Constant Number of Frames

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General Computing Machines
An Analogy: Programs Define Machines
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Programs specify the logic of a computational device
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Programs specify the logic of a computational device

factorial
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

```
factorial
= 1
- 1

1

1

1

* factorial
```
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

\[
5 = \text{factorial} * 1
\]
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

$$\text{factorial}$$

$$5$$

$$=$$

$$1$$

$$1$$

$$*$$

$$1$$

$$120$$

$$\text{factorial}$$
Interpreters are General Computing Machine
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An interpreter can be parameterized to simulate any machine.
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(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
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Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

\[
\begin{align*}
5 & \quad \rightarrow \quad \text{Scheme Interpreter} \quad \rightarrow \quad 120 \\
\text{(define (factorial n)} & \quad \text{(if (zero? n) 1 (* n (factorial (- n 1)))))}
\end{align*}
\]

Our Scheme interpreter is a universal machine
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

\[
\begin{align*}
5 \rightarrow & \quad \text{Scheme Interpreter} \\
& \quad \downarrow \\
& \quad (\text{define} (\text{factorial} \ n) \\
& \quad \quad (\text{if} (\text{zero?} \ n) 1 (* \ n (\text{factorial} \ (- n 1)))))) \\
& \quad \uparrow \\
& \quad 120
\end{align*}
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Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself
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5 → Scheme Interpreter → 120

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Internally, it is just a set of evaluation rules