Announcements

- Homework 8 due Tuesday 11/12 @ 11:59pm, and it's in Scheme!
- Project 4 due Thursday 11/21 @ 11:59pm, and it's a Scheme interpreter!
- Also, the project is very long. Get started today.

Dynamic Scope

The way in which names are looked up in Scheme and Python is called lexical scope (or static scope).

Lexical scope: The parent of a frame is the environment in which a procedure was defined.

Dynamic scope: The parent of a frame is the environment in which a procedure was called.

```
(define f (lambda (x) (+ x y))
(define g (lambda (x y) (f (+ x x)))
(g 3 7)
```

Lexical scope: The parent for f's frame is the global frame.
Dynamic scope: The parent for f's frame is g's frame.

Error: unknown identifier: y

Special form to create dynamically scoped procedures

Functional Programming

All functions are pure functions.
No re-assignment and no mutable data types.
Name-value bindings are permanent.

Advantages of functional programming:
- The value of an expression is independent of the order in which sub-expressions are evaluated.
- Sub-expressions can safely be evaluated in parallel or on demand (lazily).
- Referential transparency: The value of an expression does not change when we substitute one of its subexpression with the value of that subexpression.

But... no for/while statements! Can we make basic iteration efficient? Yes!

Tail Recursion

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Special form to create dynamically scoped procedures
Recursion and Iteration in Python

In Python, recursive calls always create new active frames.

```python
def factorial(n, k):
    if n == 0:
        return k
    else:
        return factorial(n-1, k*n)
```

- **Time**: $O(n)$
- **Space**: $O(n)$

```python
def factorial(n, k):
    while n > 0:
        n, k = n-1, k*n
    return k
```

- **Time**: $O(n)$
- **Space**: $O(1)$

Tail Recursion

From the Revised Report on the Algorithmic Language Scheme:

"Implementations of Scheme are required to be properly tail-recursive. This allows the execution of an iterative computation in constant space, even if the iterative computation is described by a syntactically recursive procedure."

```scheme
(define (factorial n k)
  (if (zero? n) k
      (factorial (- n 1) (+ k n)))))
```

Tail Calls

A procedure call that has not yet returned is active. Some procedure calls are tail calls. A Scheme interpreter should support an unbounded number of active tail calls using only a constant amount of space.

A tail call is a call expression in a tail context:

- The last body sub-expression in a lambda expression
- Sub-expressions 2 & 3 in a tail context if expression
- All non-predicate sub-expressions in a tail context cond
- The last sub-expression in a tail context and or or
- The last sub-expression in a tail context begin

```scheme
(define (factorial n k)
  (if (= n 0) k
      (factorial (- n 1) (+ k n)))))
```

Tail Calls

Example: Length of a List

```scheme
(define (length s)
  (if (null? s) 0
      (+ 1 (length (cdr s)))))
```

A call expression is not a tail call if more computation is still required in the calling procedure.

Linear recursive procedures can often be re-written to use tail calls.

```scheme
(define (length-tail s)
  (if (null? s) 0
      (length-tail (cdr s) (+ 1 n)))))
```

Eval with Tail Call Optimization

The return value of the tail call is the return value of the current procedure call. Therefore, tail calls shouldn't increase the environment size.
Tail Recursion Examples

Which Procedures are Tail Recursive?

Which of the following procedures run in constant space? \( \Theta(1) \)

\[
\text{\textbullet Compute the length of } s. \\
(\text{define} \ length \ s) \\
\begin{array}{l}
(\text{if} \ (\text{null?} \ s) \\
\quad \text{1}) \\
(\text{else}) \\
\quad \text{length} \ (\text{cdr} \ s)) \\
\end{array}
\]

\[
\text{\textbullet Return the } n\text{-th Fibonacci number.} \\
(\text{define} \ fib \ n) \\
\begin{array}{l}
(\text{define} \ \text{fib-iter} \ current \ k) \\
(\text{if} \ (\text{eq?} \ k \ n) \\
\quad current) \\
(\text{else}) \\
\quad (\text{fib-iter} \ (+ \ current \ (\text{fib} \ (- \ k \ 1))) \\
\quad (+ \ k \ 1)) \\
\end{array}
\]

\[
\text{\textbullet Return whether } s \text{ contains } v. \\
(\text{define} \ \text{contains} \ s \ v) \\
\begin{array}{l}
(\text{if} \ (\text{null?} \ s) \\
\quad \text{false}) \\
(\text{else}) \\
\quad (\text{if} \ (\text{eq?} \ v \ (\text{car} \ s)) \\
\quad \text{true}) \\
(\text{else}) \\
\quad \text{contains} \ (\text{cdr} \ s) \ v \\
\end{array}
\]

\[
\text{\textbullet Return whether } s \text{ has any repeated elements.} \\
(\text{define} \ \text{has-repeat} \ s) \\
\begin{array}{l}
(\text{if} \ (\text{null?} \ s) \\
\quad \text{false}) \\
(\text{else}) \\
\quad (\text{if} \ (\text{contains?} \ (\text{cdr} \ s) \ (\text{car} \ s)) \\
\quad \text{true}) \\
(\text{else}) \\
\quad \text{has-repeat} \ (\text{cdr} \ s) \\
\end{array}
\]

Map and Reduce

Example: Reduce

\[
(\text{define} \ \text{reduce} \ \text{procedure} \ s \ \text{start}) \\
\begin{array}{l}
(\text{if} \ (\text{null?} \ s) \ \text{start}) \\
(\text{reduce} \ \text{procedure} \ (\text{cdr} \ s) \ \text{procedure} \ \text{start} \ (\text{car} \ s))
\end{array}
\]

Recursive call is a tail call.
Other calls are not; constant space depends on whether \text{procedure} requires constant space.

\[
\begin{array}{l}
(\text{reduce} \ \ast \ ' (3 \ 4 \ 5) \ 2) \\
120 \\
(\text{reduce} \ (\lambda (x \ y) \ (\text{cons} \ y \ x)) \ ' (3 \ 4 \ 5) \ ' (2)) \\
(5 \ 4 \ 3 \ 2)
\end{array}
\]

Example: Map with Only a Constant Number of Frames

\[
\begin{array}{l}
(\text{define} \ \text{map} \ \text{procedure} \ s) \\
(\text{if} \ (\text{null?} \ s) \ \text{nil}) \\
(\text{else}) \\
(\text{cons} \ (\text{procedure} \ (\text{car} \ s))) \\
(\text{map} \ \text{procedure} \ (\text{cdr} \ s)))
\end{array}
\]

\[
\begin{array}{l}
(\text{define} \ \text{map-reverse} \ s \ \text{m}) \\
(\text{if} \ (\text{null?} \ s) \ \text{m}) \\
(\text{else}) \\
(\text{map-reverse} \ (\text{cdr} \ s)) \\
(\text{cons} \ (\text{procedure} \ (\text{car} \ s))) \\
\text{m}
\end{array}
\]

\[
(\text{reverse} \ (\text{map-reverse} \ \text{s} \ \text{m}))
\]

\[
\begin{array}{l}
(\text{define} \ \text{reverse} \ \text{s}) \\
(\text{define} \ \text{reverse-iter} \ \text{s} \ \text{r}) \\
(\text{if} \ (\text{null?} \ \text{s}) \ \text{r}) \\
(\text{else}) \\
(\text{reverse-iter} \ \text{cdr} \ \text{s}) \\
(\text{cons} \ \text{car} \ \text{s}) \ \text{r})
\end{array}
\]

\[
(\text{reverse-iter} \ \text{cdr} \ \text{s}) \\
(\text{cons} \ \text{car} \ \text{s}) \ \text{r})
\]

General Computing Machines
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

\[
\text{factorial} \quad 5 \quad 120
\]

Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

\[
\text{Scheme Interpreter} \quad 5 \quad 120
\]

(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Internally, it is just a set of evaluation rules