61A Lecture 32

Friday, November 22
Announcements
Announcements

• Homework 10 due Tuesday 11/26 @ 11:59pm
Announcements

• Homework 10 due Tuesday 11/26 @ 11:59pm
• No lecture on Wednesday 11/27 or Friday 11/29
Announcements

• Homework 10 due Tuesday 11/26 @ 11:59pm
• No lecture on Wednesday 11/27 or Friday 11/29
• No discussion section Wednesday 11/27 through Friday 11/29
Announcements

• Homework 10 due Tuesday 11/26 @ 11:59pm
• No lecture on Wednesday 11/27 or Friday 11/29
• No discussion section Wednesday 11/27 through Friday 11/29
  • Lab will be held on Wednesday 11/27
Announcements

• Homework 10 due Tuesday 11/26 @ 11:59pm
• No lecture on Wednesday 11/27 or Friday 11/29
• No discussion section Wednesday 11/27 through Friday 11/29
  • Lab will be held on Wednesday 11/27
• Recursive art contest entries due Monday 12/2 @ 11:59pm
Appending Lists

(Demo)
Lists in Logic
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

(fact (append-to-form () ?x ?x))
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))  Simple fact: Conclusion
```
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

(fact (append-to-form () ?x ?x))

(fact (append-to-form (?a . ?r) ?y (?a . ?z))
   (append-to-form ?r ?y ?z))
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```scheme
(fact (append-to-form () ?x ?x))<Simple fact: Conclusion
(fact (append-to-form (?a . ?r) ?y (?a . ?z))<Conclusion
 (append-to-form       ?r  ?y       ?z ))
```
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\text{(fact (append-to-form () ?x ?x))} \quad \text{Simple fact: Conclusion}
\]

\[
\text{(fact (append-to-form (?a . ?r) ?y (?a . ?z))} \quad \text{Conclusion}
\]

\[
\text{(append-to-form ?r ?y ?z )} \quad \text{Hypothesis}
\]
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))  \Simple fact: Conclusion\n
(fact (append-to-form (?a . ?r) ?y (?a . ?z))  \Conclusion\n  (append-to-form  ?r  ?y  ?z )) \Hypothesis\n
(query (append-to-form ?left (c d) (e b c d)))

Success!
left: (e b)
```
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\text{(fact (append-to-form () ?x ?x))} \quad \text{Simple fact: Conclusion}
\]

\[
\text{(fact (append-to-form (?a . ?r) ?y (?a . ?z)) (append-to-form ?r ?y ?z))} \quad \text{Conclusion and Hypothesis}
\]

\[
\text{(query (append-to-form ?left (c d) (e b c d)))}
\]
\text{Success!}
\text{left: (e b)}

In a *fact*, the first relation is the conclusion and the rest are hypotheses.
Lists in Logic

Expressions begin with `query` or `fact` followed by relations.

Expressions and their relations are Scheme lists.

(fact (append-to-form () ?x ?x))

(fact (append-to-form (?a . ?r) ?y (?a . ?z))
  (append-to-form ?r ?y ?z ))

(query (append-to-form ?left (c d) (e b c d)))

Success!
left: (e b)

In a `fact`, the first relation is the conclusion and the rest are hypotheses.

In a `query`, all relations must be satisfied.
Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\text{(fact (append-to-form () \(\text{x}\) \(\text{x}\)) \text{Simple fact: Conclusion}}
\]

\[
\text{(fact (append-to-form (\(\text{a}\) . \(\text{r}\)) \(\text{y}\) (\(\text{a}\) . \(\text{z}\))) Conclusion}
\]

\[
\text{(append-to-form \(\text{r}\) \(\text{y}\) \(\text{z}\)) Hypothesis}
\]

\[
\text{(query (append-to-form \(\text{left}\) (\(\text{c}\) \(\text{d}\)) (\(\text{e}\) \(\text{b}\) \(\text{c}\) \(\text{d}\)))) Success!}
\]

\[
\text{left: (e b)}
\]

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with query or fact followed by relations.

Expressions and their relations are Scheme lists.

(fact (append-to-form () ?x ?x))  Simple fact: Conclusion
(fact (append-to-form (?a . ?r) ?y (?a . ?z))  Conclusion
(append-to-form       ?r  ?y       ?z ))  Hypothesis

(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)  What ?left can append with (c d) to create (e b c d)

In a fact, the first relation is the conclusion and the rest are hypotheses.

In a query, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

- \[(\text{fact (append-to-form () ?x ?x)})\]
  - Simple fact: Conclusion
- \[(\text{fact (append-to-form (?a . ?r) ?y (?a . ?z))})\]
  - Conclusion
  - (append-to-form \(?r\) \(?y\) \(?z\))
  - Hypothesis

\[(\text{query (append-to-form ?left (c d) (e b c d))})\]
Success!
left: (e b)

What \(?left\) can append with (c d) to create (e b c d)

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

(\textcolor{red}{\textbf{fact}} (append-to-form (\textcolor{red}{\textbf{()}}) \textcolor{red}{\textbf{?x}} \textcolor{red}{\textbf{?x}})) \rightarrow \textcolor{red}{\textbf{Simple fact: Conclusion}} \\
\textcolor{red}{\textbf{(fact}}} (append-to-form (?a \textcolor{blue}{.} ?r) \textcolor{blue}{?y} (?a \textcolor{blue}{.} ?z)) \rightarrow \textcolor{red}{\textbf{Conclusion}} \\
\textcolor{red}{\textbf{(append-to-form}}} \textcolor{blue}{?r} \textcolor{blue}{?y} \textcolor{blue}{?z} \rightarrow \textcolor{red}{\textbf{Hypothesis}}

\textcolor{red}{\textbf{(query}}} (append-to-form \textcolor{red}{\textbf{?left}} (c \textcolor{blue}{d}) (e \textcolor{blue}{b} \textcolor{blue}{c} \textcolor{blue}{d})) \rightarrow \textcolor{red}{\textbf{Success!}} \\
\textcolor{red}{\textbf{left: (e b)}} \rightarrow \textcolor{red}{\textbf{What ?left can append with (c d) to create (e b c d)}}

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

- \( \text{(fact (append-to-form () ?x ?x))} \)

\( \text{(query (append-to-form ?left (c d) (e b c d)))} \)

Success!

**left**: \( (e b) \)

In a **fact**, the first relation is the conclusion and the rest are hypotheses.

In a **query**, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

- \((\text{fact } \text{(append-to-form () ?x ?x)})\)
- \((\text{fact } \text{(append-to-form (?a . ?r) ?y (?a . ?z))})\)
- \((\text{query } \text{(append-to-form ?left (c d) (e b c d))})\)

Success!
- \((\text{left: (e b)})\)
- \((\text{What ?left can append with (c d) to create (e b c d)})\)

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\begin{align*}
\text{(fact (append-to-form () ?x ?x))} & \quad \text{Simple fact: Conclusion} \\
\text{(fact (append-to-form (?a . ?r) ?y (?a . ?z))} & \quad \text{Conclusion} \\
& \quad \text{(append-to-form ?r ?y ?z )} \quad \text{Hypothesis}
\end{align*}
\]

\[
\begin{align*}
\text{(query (append-to-form ?left (c d) (e b c d)))} & \\
& \quad \text{Success!}
\end{align*}
\]

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\begin{align*}
\text{Simple fact: Conclusion} & \quad \text{Hypothesis} \\
\text{(fact (append-to-form () ?x ?x))} & \quad \text{(fact (append-to-form (?a . ?r) ?y (?a . ?z)) (append-to-form ?r ?y ?z))}
\end{align*}
\]

\[
\begin{align*}
& (\text{query (append-to-form ?left (c d) (e b c d))}) \\
& \text{Success!} \\
& \text{left: (e b)}
\end{align*}
\]

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\begin{align*}
\text{(fact (append-to-form () ?x ?x))} & \quad \text{Simple fact: Conclusion} \\
\text{(fact (append-to-form (?a . ?r) ?y (?a . ?z))} & \quad \text{Conclusion} \\
\text{(append-to-form ?r ?y ?z)} & \quad \text{Hypothesis} \\
\text{(query (append-to-form ?left (c d) (e b c d)))} & \quad \text{Success!} \\
\text{left: (e b)} & \quad \text{What ?left can append with (c d) to create (e b c d)}
\end{align*}
\]

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

- (fact (append-to-form () ?x ?x))
  - Simple fact: Conclusion
  - Conclusion
    - (b) (c d) => (b c d)
    - (e b) (c d) => (e b c d)
  - Hypothesis
    - (e . (b)) (c d) => (e . (b c d))

(query (append-to-form ?left (c d) (e b c d)))

Success!

- left: (e b)
  - What ?left can append with (c d) to create (e b c d)

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

*(Simple fact: Conclusion)*

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
    (append-to-form ?r ?y ?z ))
```

*(Conclusion)*

```
(query (append-to-form ?left (c d) (e b c d)))
```

*Success!*

**left**: *(e b)*

*What ?left can append with (c d) to create (e b c d)*

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\text{fact (append-to-form (\(\_\) ?x ?x))}
\]

Simple fact: Conclusion

\[
\text{fact (append-to-form (?a . ?r) ?y (?a . ?z))}
\]

Conclusion

\[
\text{(append-to-form ?r ?y ?z )}
\]

Hypothesis

\[
\text{(query (append-to-form ?left (c d) (e b c d)))}
\]

Success!

left: (e b)

What ?left can append with (c d) to create (e b c d)

In a **fact**, the first relation is the conclusion and the rest are hypotheses.

In a **query**, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\text{Simple fact: Conclusion}
\]
\[
\text{Conclusion}
\]
\[
\text{Hypothesis}
\]

\[
\text{Success!}
\]
\[
\text{What ?left can append with (c d) to create (e b c d)}
\]

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with query or fact followed by relations.

Expressions and their relations are Scheme lists.

\[
\text{fact (append-to-form () ?x ?x))} \quad \text{Simple fact: Conclusion} \quad \text{(c d)} \Rightarrow \text{(c d)}
\]

\[
\text{fact (append-to-form (?a . ?r) ?y (?a . ?z))} \quad \text{Conclusion} \quad \text{(b) (c d) => (b c d)}
\]

\[
\text{(append-to-form ?r ?y ?z)} \quad \text{Hypothesis} \quad \text{(e b) (c d) => (e b c d)}
\]

\[
\text{(query (append-to-form ?left (c d) (e b c d))} \quad \text{Success!}
\]

\[
\text{left: (e b)} \quad \text{What ?left can append with (c d) to create (e b c d)}
\]

In a fact, the first relation is the conclusion and the rest are hypotheses.

In a query, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[
\begin{align*}
\text{fact} & \ (\text{append-to-form} \ () \ ?x \ ?x) & \rightarrow & \ (c \ d) \\
\text{fact} & \ (\text{append-to-form} \ (?a \ . \ ?r) \ ?y \ (?a \ . \ ?z)) \ (\text{append-to-form} \ ?r \ ?y \ ?z) & \rightarrow & \ (b \ c \ d) \\
\text{query} & \ (\text{append-to-form} \ ?\text{left} \ (c \ d) \ (e \ b \ c \ d)) & \rightarrow & \ (e \ b \ c \ d)
\end{align*}
\]

Success!

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

- (fact (append-to-form () ?x ?x))
- (fact (append-to-form (?a . ?r) ?y (?a . ?z))
  (append-to-form ?r ?y ?z))
- (query (append-to-form ?left (c d) (e b c d)))

**Success!**

- left: (e b)  
  What ?left can append with (c d) to create (e b c d)

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

- `(fact (append-to-form () ?x ?x))` (Simple fact: Conclusion)
- `(fact (append-to-form (?a . ?r) ?y (?a . ?z)) (append-to-form ?r ?y ?z))` (Conclusion Hypothesis)
- `(query (append-to-form ?left (c d) (e b c d)))` (Success! left: (e b))

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[(\text{fact}\ (\text{append-to-form}\ (\text{?x}\ .\ \text{?x})))]\]

\[(\text{fact}\ (\text{append-to-form}\ (\text{?a}\ .\ \text{?r})\ \text{?y}\ (\text{?a}\ .\ \text{?z}))\ (\text{append-to-form}\ \text{?r}\ \text{?y}\ \text{?z})))\]

\[(\text{query}\ (\text{append-to-form}\ \text{?left}\ (\text{c}\ \text{d})\ (\text{e}\ \text{b}\ \text{c}\ \text{d})))\]

Success!

*left*: (e b)

What ?left can append with (c d) to create (e b c d)

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.
Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

\[(\text{fact (append-to-form } ?x ?x))\]  
Simple fact: Conclusion

\[(\text{fact (append-to-form } \text{?a . ?r} \text{?y } \text{(?a . ?z)})\]  
Conclusion

\[(\text{append-to-form } \text{?r } \text{?y } \text{?z })\]  
Hypothesis

\[(\text{query (append-to-form } \text{?left (c d) (e b c d)})\]

Success!

left: (e b)  
What ?left can append with (c d) to create (e b c d)

In a *fact*, the first relation is the conclusion and the rest are hypotheses.

In a *query*, all relations must be satisfied.

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

(Demo)
Permuting Lists
Anagrams in Logic
Anagrams in Logic

A permutation (i.e., anagram) of a list is:
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.
Anagrams in Logic

A permutation (i.e., anagram) of a list is:
• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:
• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
    (insert ?a       ?r        ?s))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.

• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
 (insert ?a ?r ?s))

List with ?a in front

List with ?a somewhere
A permutation (i.e., anagram) of a list is:
• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
 (insert ?a ?r ?s))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

\[
\text{(fact (insert ?a ?r (?a . ?r))) \quad \text{Bigger list with ?a somewhere}}
\]

\[
\text{(fact (insert ?a (?b . ?r) (?b . ?s))}
\quad \text{(insert ?a \quad ?r \quad ?s))}
\]

\[
\text{(fact (anagram () ()))}
\]

List with ?a somewhere

List with ?a somewhere

Element List List with ?a in front

6
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
(fact (insert ?a (?b . ?r) (?b . ?s)) (insert ?a ?r ?s))
(fact (anagram () () ))
(fact (anagram (?a . ?r) ?b))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

\[
\begin{align*}
\text{Element} & \quad \text{List} \quad \text{List with ?a in front} \\
\text{(fact (insert ?a ?r (?a . ?r)))} & \quad \text{Bigger list with ?a somewhere} \\
\text{(fact (insert ?a (?b . ?r) (?b . ?s))} & \quad \text{(insert ?a ?r ?s))} \\
\text{(fact (anagram () ()}) & \quad \text{List with ?a somewhere} \\
\text{(fact (anagram (?a . ?r) ?b) & \quad \text{(insert ?a ?s ?b)}
\end{align*}
\]
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

\[
\text{Element} \quad \text{List} \quad \text{List with ?a in front}
\]

\[
\text{(fact (insert ?a \(\text{List}\) (?a . ?r)))}
\]

\[
\text{Bigger list with ?a somewhere}
\]

\[
\text{(fact (insert ?a (?b . ?r) (?b . ?s)))}
\]

\[
\text{(insert ?a \(\text{List}\) ?r ?s)}
\]

\[
\text{List with ?a somewhere}
\]

\[
\text{(fact (anagram () \(\text{List}\)))}
\]

\[
\text{List with ?a somewhere}
\]

\[
\text{(fact (anagram (?a . ?r) ?b)}
\]

\[
\text{(insert ?a \(\text{List}\) ?s ?b)}
\]

\[
\text{(anagram ?r ?s)}
\]
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.

• The first element of the list inserted into an anagram of the rest of the list.

\[
\text{fact (insert } ?a \ ?r \ (\text{?a . ?r}))
\]

\[
\text{fact (insert } ?a \ (\text{?b . ?r}) \ (\text{?b . ?s}))\]

\[
\text{fact (anagram } () \ ()))\]

\[
\text{fact (anagram } (?a . ?r) \ ?b)\]

\[
\text{fact (anagram } ?r \ ?s))\]

Element | List | List with ?a in front

Bigger list with ?a somewhere

List with ?a somewhere
Anagrams in Logic

A permutation (i.e., anagram) of a list is:
• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
(fact (anagram () ()))
(fact (anagram (?a . ?r) ?b)
  (insert ?a ?s ?b)
  (anagram ?r ?s))
A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

\[
\text{fact (insert } ?a \ ?r \ (?a \ . \ ?r))
\]

\[
\text{fact (insert } ?a \ (?b \ . \ ?r) \ (?b \ . \ ?s))
\]

\[
\text{fact (anagram } () \ ()())
\]

\[
\text{fact (anagram } (?a \ . \ ?r) \ ?b)
\]

\[
\text{(insert } ?a \ ?s \ ?b)
\]

\[
\text{(anagram } ?r \ ?s))
\]
Anagrams in Logic

A permutation (i.e., anagram) of a list is:
• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
(fact (insert ?a (?b . ?r) (?b . ?s))
  (insert ?a ?r ?s))
(fact (anagram () ()
  (fact (anagram (?a . ?r) ?b)
    (insert ?a ?s ?b)
    (anagram ?r ?s)))

Element  List  List with ?a in front

Bigger list with ?a somewhere

List with ?a somewhere

a  r  t
r  t
a  r  t
Anagrams in Logic

A permutation (i.e., anagram) of a list is:
• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

\[
\text{(fact (insert ?a ?r (?a . ?r)))}
\]

\[
\text{(fact (insert ?a (?b . ?r) (?b . ?s))}
\]

\[
\text{(fact (anagram () ()))}
\]

\[
\text{(fact (anagram (?a . ?r) ?b)}
\]

\[
\text{(insert ?a ?s ?b)}
\]

\[
\text{(anagram ?r ?s))}
\]
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.

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\[
\text{(fact (insert ?a ?r (?a . ?r)))}
\]

\[
\text{(fact (insert ?a (?b . ?r) (?b . ?s))}
\]

\[
\text{(anagram () ()))}
\]

\[
\text{(fact (anagram (?a . ?r) ?b)}
\]

\[
\text{(insert ?a ?s ?b)}
\]

\[
\text{(anagram ?r ?s))}
\]
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.
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\[
\text{(fact (insert ?a ?r (?a . ?r)))}
\]

\[
\text{(fact (insert ?a (?b . ?r) (?b . ?s))}
\]

\[
\text{(fact (anagram (?a . ?r) ?b) (insert ?a ?s ?b) (anagram ?r ?s))}
\]
A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

\[
\text{fact (insert } \text{?a } \text{?r } (?a . ?r))
\]

\[
\text{fact (insert } \text{?a } (?b . ?r) (?b . ?s))
\]

\[
\text{fact (anagram } () )()
\]

\[
\text{fact (anagram } (?a . ?r) \text{?b)}
\]

\[
\text{fact (insert } \text{?a } \text{?s } \text{?b)}
\]

\[
\text{fact (anagram } \text{?r } \text{?s))}
\]
Anagrams in Logic

A permutation (i.e., anagram) of a list is:
• The empty list for an empty list.
• The first element of the list inserted into an anagram of the rest of the list.

\[
\text{(fact (insert } ?a ?r (?a . ?r)))
\]

\[
\text{(fact (insert } ?a (?b . ?r) (?a . ?r) ((?a . ?s))) (insert } ?a ?r ?s)
\]

\[
\text{(fact (anagram } () ()})
\]

\[
\text{(fact (anagram } (?a . ?r) ?b) (insert } ?a ?s ?b) (anagram } ?r ?s))
\]
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

• The empty list for an empty list.

• The first element of the list inserted into an anagram of the rest of the list.

(fact (insert ?a ?r (?a . ?r)))
(fact (insert ?a (?b . ?r) (?b . ?s))
 (insert ?a       ?r        ?s))
(fact (anagram () ()))
(fact (anagram (?a . ?r) ?b)
 (insert    ?a   ?s  ?b)
 (anagram    ?r   ?s))
Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

\[
\begin{align*}
\text{(fact (insert ?a ?r (?a . ?r)))} & \quad \text{Element} \\
\text{(fact (insert ?a (?b . ?r) (?b . ?s)))} & \quad \text{List} \\
\text{(fact (anagram () ()))} & \quad \text{List with ?a in front} \\
\text{(fact (anagram (?a . ?r) ?b) (insert ?a ?s ?b) (anagram ?r ?s))} & \quad \text{Bigger list with ?a somewhere} \\
\text{(fact (anagram (?a . ?r) ?b) (insert ?a ?s ?b) (anagram ?r ?s))} & \quad \text{List with ?a somewhere} \\
\end{align*}
\]

(Demo)
Unification
Pattern Matching
Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.
Pattern Matching

The basic operation of the Logic interpreter is to attempt to unify two relations. Unification is finding an assignment to variables that makes two relations the same.
Pattern Matching

The basic operation of the Logic interpreter is to attempt to \textit{unify} two relations. Unification is finding an assignment to variables that makes two relations the same.

\[
( (a \ b) \ c \ (a \ b) )
\]
The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

\[
( (a \ b) \ c \ (a \ b) )
\]

\[
( \ ?x \ c \ ?x \ )
\]
Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations. Unification is finding an assignment to variables that makes two relations the same.

\[
\begin{align*}
& ( (a \ b) \ c \ (a \ b) ) \\
& ( \ ?x \ c \ ?x \ ) \quad \text{True, } \{x: (a \ b)\}
\end{align*}
\]
Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

\[
( (a \ b) \ c \ (a \ b) ) \quad \text{True, } \{x: (a \ b)\}
\]

\[
( ?x \ c \ ?x )
\]

\[
( (a \ b) \ c \ (a \ b) )
\]
The basic operation of the Logic interpreter is to attempt to unify two relations. Unification is finding an assignment to variables that makes two relations the same.

\[
\begin{align*}
& ( (a \ b) \ c \ (a \ b) ) \\
& ( \ ?x \ c \ ?x ) \quad \text{True, } \{ x: (a \ b) \}
\end{align*}
\]

\[
\begin{align*}
& ( (a \ b) \ c \ (a \ b) ) \\
& ( (a \ ?y) \ ?z \ (a \ b) )
\end{align*}
\]
Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations. Unification is finding an assignment to variables that makes two relations the same.

\[
\begin{align*}
&((a \ b) \ c \ (a \ b)) \quad \text{True, } \{x: (a \ b)\} \\
&(\ ?x \ c \ ?x ) \quad \text{True, } \{y: b, z: c\}
\end{align*}
\]
Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

\[
\begin{align*}
( (a \ b) \ c \ (a \ b) ) & \quad \text{True, } \{x: (a \ b)\} \\
( \ ?x \ c \ ?x ) & \\
( (a \ b) \ c \ (a \ b) ) & \quad \text{True, } \{y: b, z: c\} \\
( (a \ ?y) \ ?z \ (a \ b) ) & \\
( (a \ b) \ c \ (a \ b) ) & \\
( \ ?x \ ?x \ ?x ) &
\end{align*}
\]
Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

- \((a \ b) \ c \ (a \ b)\)  \quad \text{True, } \{x: (a \ b)\}
- \((a \ ?x) \ ?z (a \ b)\)  \quad \text{True, } \{y: b, z: c\}
- \((a \ ?y) \ ?z (a \ b)\)  \quad \text{True, } \{y: b, z: c\}
- \((a \ b) \ c \ (a \ b)\)  \quad \text{False}
- \((a \ b) \ c \ (a \ b)\)  \quad \text{False}
- \((a \ b) \ c \ (a \ b)\)  \quad \text{False}
Unification
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{align*}
&\left( (a \ b) \ c \ (a \ b) \right) \\
&\left( \ ?x \ c \ ?x \right)
\end{align*}
\]

\{
\}

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{align*}
&\ ( (a \ b) \ c \ (a \ b) ) \\
&\ ( ?x \ c \ ?x )
\end{align*}
\]

\{\}
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{align*}
& ( (a \ b) \ c \ (a \ b) ) \\
& ( \ ?x \ c \ ?x \ ) \\
\end{align*}
\]

\[
\{ \ x: (a \ b) \ \}
\]
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{array}{c}
(a \ b) \ c \ (a \ b) \\
?x \ c \ ?x
\end{array}
\]

\{
    \begin{align*}
    x: (a \ b)
    \end{align*}
\}
Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{align*}
& ( \text{(a b)} \ c \ (\text{a b}) ) \\
& ( \ ?x \ c \ ?x \ ) \\
\end{align*}
\]

\{ x: (a b) \}
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{align*}
& ( (a, b) \quad c \quad (a, b) ) \\
& ( ?x \quad c \quad ?x ) \\
& \{ \quad x: (a, b) \quad \} 
\end{align*}
\]
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{align*}
&\left( \begin{array}{c}
\begin{array}{c}
(a \ b)
\end{array}
\end{array} \right)
\quad \begin{array}{c}
\begin{array}{c}
c
\end{array}
\end{array}
\quad \left( \begin{array}{c}
\begin{array}{c}
(a \ b)
\end{array}
\end{array} \right)
\\
&\left( \begin{array}{c}
\begin{array}{c}
?x
\end{array}
\end{array} \right)
\quad \begin{array}{c}
\begin{array}{c}
c
\end{array}
\end{array}
\quad \left( \begin{array}{c}
\begin{array}{c}
?x
\end{array}
\end{array} \right)
\end{align*}
\]

\[
\{ \ x: (a \ b) \ \}
\]
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
( (a\ b), c, (a\ b) )
\]

\[
( ?x, c, ?x )
\]

\[
\begin{align*}
\text{Lookup}\\
(a\ b)\\
(a\ b)
\end{align*}
\]

\[
\{ x: (a\ b) \}
\]

Success!
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{array}{c}
(a \ b) \ c \ (a \ b) \\
?x \ c \ ?x \\
\end{array}
\]

\[
\begin{array}{c}
(a \ b) \\
(a \ b)
\end{array}
\]

Lookup

\[
\begin{array}{c}
(a \ b) \ c \ (a \ b) \\
?x \ ?x \ ?x
\end{array}
\]

\[
\begin{array}{c}
\{ x: (a \ b) \}
\end{array}
\]

Success!
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

```
(a b) c (a b)
(?x c ?x)
```

```
(a b) c (a b)
(?x ?x ?x)
```

```
{ x: (a b) }
```

Success!
Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

Unification succeeds! 

{ x: (a b) }
Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

Unification

Success!
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{array}{c}
\text{Lookup} \\
(a \ b) \\
(a \ b)
\end{array} \\
\{ \text{x: (a b)} \} \\
\text{Success!}
\]
Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

\[
\begin{align*}
\text{( (a b) c (a b) )} & \quad \text{ ( (a b) c (a b) )} \\
\text{( ?x c ?x )} & \quad \text{ ( ?x ?x ?x )}
\end{align*}
\]

\[
\begin{align*}
\{ \ x: (a b) \} & \quad \{ \ x: (a b) \}
\end{align*}
\]

Success!

Symbols/relations without variables only unify if they are the same.
Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.

Symbols/relations without variables only unify if they are the same

Success!

Failure.
Unifying Variables
Unifying Variables

Two relations that contain variables can be unified as well.
Two relations that contain variables can be unified as well.

\[
(\ ?x \ ?x \ )
\]

\[
((a \ ?y \ c) \ (a \ b \ ?z))
\]
Unifying Variables

Two relations that contain variables can be unified as well.

( ?x ?x )
((a ?y c) (a b ?z))

True, {
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{array}{c}
\text{( ?x ?x )} \\
\text{( (a ?y c) (a b ?z) )}
\end{array}
\]

True, {
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{array}{c}
( ?x & ?x ) \\
((a ?y c) (a b ?z))
\end{array}
\]

True, \{x: (a ?y c),
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
(\ ?x & \ ?x) \\
(a \ ?y \ c) & (a \ b \ ?z)
\end{align*}
\]

True, \(\{x: (a \ ?y \ c),\}\)
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
\begin{array}{c}
\text{(a ?y c)} \\
\text{(a b ?z)}
\end{array}
\end{align*}
\]

\[
\begin{array}{c}
\text{True, \{x: (a ?y c),}
\end{array}
\]

\[
\begin{array}{c}
\text{Lookup}
\end{array}
\]

\[
\begin{align*}
\text{(a ?y c)} \\
\text{(a b ?z)}
\end{align*}
\]
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
& (\text{?x} & \text{?x}) \\
& (\text{a ?y c} & \text{a b ?z}) \\
\end{align*}
\]

True, \( \{x: (a ?y c), \} \)

Lookup
Unifying Variables

Two relations that contain variables can be unified as well.

\[ (?x \quad ?x) \]
\[ ((a \ ?y \ c) \quad (a \ b \ ?z)) \]

True, \( \{ x: (a \ ?y \ c) \} \),
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
&\text{(a ?y c)} \quad \text{(a b ?z)} \\
\end{align*}
\]

True, \{x: (a ?y c), y: b, \}
Two relations that contain variables can be unified as well.

True, \{x: (a ?y c), y: b, \}
Unifying Variables

Two relations that contain variables can be unified as well.

```
(a ?y c) (a b ?z)
```

True, \{x: (a ?y c), y: b, z: c\}
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
&\{(\ ?x, \ ?x) \mid (a \ ?y \ c), (a \ b \ ?z)\} \\
\rightarrow & \quad \text{True, } \{x: (a \ ?y \ c), y: b, z: c\}
\end{align*}
\]
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
&\left( \text{?x} \atop (a \ ?y \ c) \right) \quad \left( \text{?x} \atop (a \ b \ ?z) \right) \\
&\text{True, } \{ x: (a \ ?y \ c), y: b, z: c \} \\
\end{align*}
\]

Substituting values for variables may require multiple steps.

This process is called grounding. Two unified expressions have the same grounded form.
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
\text{True, } & \{x: (a \ ?y \ c), \\
y: b, \\
z: c\} \\
\end{align*}
\]

Substituting values for variables may require multiple steps.

This process is called \textit{grounding}. Two unified expressions have the same grounded form.

\texttt{lookup('?x')}
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
\text{lookup}(\,?x\, \mapsto \ (a \ ?y \ c)\,) \\
\end{align*}
\]

Substituting values for variables may require multiple steps.

This process is called \textit{grounding}. Two unified expressions have the same grounded form.

\[
\text{lookup}(\,?x\, \mapsto \ (a \ ?y \ c)\,) \Rightarrow (a \ ?y \ c)
\]
Two relations that contain variables can be unified as well.

\[
\begin{align*}
(a \ ?y \ c) & \quad (a \ b \ ?z) \\
\end{align*}
\]

True, \{x: (a \ ?y \ c), y: b, z: c\}

Substituting values for variables may require multiple steps.

This process is called \textit{grounding}. Two unified expressions have the same grounded form.

\texttt{lookup('?x')} \Rightarrow \texttt{(a \ ?y \ c)} \quad \texttt{lookup('?y')}
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
&\text{(a ?y c)} \quad \text{(a b ?z)} \\
\end{align*}
\]

True, \( \{x: (a \ ?y \ c), y: b, z: c\} \)

Substituting values for variables may require multiple steps.

This process is called **grounding**. Two unified expressions have the same grounded form.

\[
\begin{align*}
\text{lookup('?x')} & \Rightarrow (a \ ?y \ c) \\
\text{lookup('?y')} & \Rightarrow b
\end{align*}
\]
**Unifying Variables**

Two relations that contain variables can be unified as well.

\[
\begin{align*}
(a \ ?y \ c) & \quad (a \ b \ ?z) \\
\end{align*}
\]

True, \( \{ x: (a \ ?y \ c), y: b, z: c \} \)

Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

\[
\begin{align*}
\text{lookup('?x')} & \Rightarrow (a \ ?y \ c) & \text{lookup('?y')} & \Rightarrow b & \text{ground('?x')} \\
\end{align*}
\]
Unifying Variables

Two relations that contain variables can be unified as well.

\[
\begin{align*}
\text{(a ?y c)} & \quad \text{True, } \{x: (a ?y c),} \\
\text{(a b ?z)} & \quad y: b,} \\
\text{(a ?y c)} & \quad z: c} \\
\text{(a b ?z)} & \quad \text{True, } \{x: (a ?y c),} \\
\text{(a b ?z)} & \quad y: b,} \\
\text{(a ?y c)} & \quad z: c} \\
\end{align*}
\]

Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

\[
\begin{align*}
\text{lookup('?x')} & \Rightarrow (a ?y c) & \text{lookup('?y')} & \Rightarrow b & \text{ground('?x')} & \Rightarrow (a b c)
\end{align*}
\]
Implementing Unification

```python
def unify(e, f, env):
e = lookup(e, env)
f = lookup(f, env)
if e == f:
    return True
elif isvar(e):
    env.define(e, f)
    return True
elif isvar(f):
    env.define(f, e)
    return True
elif scheme_atomp(e) or scheme_atomp(f):
    return False
else:
    return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

2. Establish new bindings to unify elements.
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment
2. Establish new bindings to unify elements.

Symbols/relations without variables only unify if they are the same.
Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

( (a b) c (a b) )

( ?x c ?x )
Implementing Unification

def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

Recursively unify the first and rest of any lists.

```
( (a b) c (a b) )
( ?x c ?x )
env: {
}
```
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
(a b) c (a b)
(?x) c (?x)
```

```
env: { x: (a b) }
```
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```
Implementing Unification

```python
def unify(e, f, env):
e = lookup(e, env)
f = lookup(f, env)

if e == f:
    return True
elif isvar(e):
    env.define(e, f)
    return True
elif isvar(f):
    env.define(f, e)
    return True
elif scheme_atomp(e) or scheme_atomp(f):
    return False
else:
    return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a b) c (a b) )
( ?x c ?x )
```

env: {
    x: (a b)
}
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

Recursively unify the first and rest of any lists.

env: `{ x: (a b) }`
Implementing Unification

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.
**Implementing Unification**

```python
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

env: `{ x: (a b) }`
Searching for Proofs
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app ( ) ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))

(app (?a . ?r) ?y (?a . ?z))
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))

(app (e . ?r) (c d) (e b c d))
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}

(app (?a . ?r) ?y (?a . ?z))

conclusion <- hypothesis

(app ?r (c d) (b c d))
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
    (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d))

(app (?a2 . ?r2) ?y2 (?a2 . ?z2))

(app (e . ?r) (c d) (e b c d))
The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app (e . ?r) (c d) (e b c d)))

Conclusion <- hypothesis

Variables are local to facts & queries
The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}

(app (?a . ?r) ?y (?a . ?z))
  conclusion <- hypothesis
(app ?r (c d) (b c d))
  {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}

(app (?a2 . ?r2) ?y2 (?a2 . ?z2))

Variables are local to facts & queries
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

\[
\begin{align*}
&\text{The query is}: \\
&(\text{app } (?a . ?r) \ ?y \ (?a . ?z)) \\
&\text{where } ?x \in \text{var} \\
&(\text{app } ?r \ ?y \ ?z) \\
&(\text{query } (\text{app } {?\text{left} } (c \ d) \ (e \ b \ c \ d))) \\
\end{align*}
\]

\[
\begin{align*}
&\text{Unifying facts are}: \\
&\text{(fact } (\text{app } () \ ?x \ ?x)) \\
&(\text{fact } (\text{app } (?a . ?r) \ ?y \ (?a . ?z))) \\
&(\text{app } ?r \ ?y \ ?z) \\
&(\text{query } (\text{app } {?\text{left} } (c \ d) \ (e \ b \ c \ d))) \\
\end{align*}
\]

\[
\begin{align*}
&(\text{app } ?\text{left} \ (c \ d) \ (e \ b \ c \ d)) \\
&\{a: e, \ y: (c \ d), \ z: (b \ c \ d), \ \text{left: } (?a . ?r)\} \\
&(\text{app } (?a . ?r) \ ?y \ (?a . ?z)) \\
&\text{conclusion } \leftarrow \text{hypothesis} \\
&(\text{app } ?r \ (c \ d) \ (b \ c \ d)) \\
&\{a2: b, \ y2: (c \ d), \ z2: (c \ d), \ r: (?a2 . ?r2)\} \\
&(\text{app } (?a2 . ?r2) \ ?y2 \ (?a2 . ?z2)) \\
\end{align*}
\]

Variables are local to facts & queries.
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

Variables are local to facts & queries
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
   (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
   {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
   conclusion <- hypothesis
(app ?r (c d) (b c d))
   {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
   conclusion <- hypothesis
(app ?r2 (c d) (c d))
(app () ?x ?x)

Variables are local to facts & queries
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
(app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

Variables are local to facts & queries
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

\[
\text{(fact (app () ?x ?x))}
\]
\[
\text{(fact (app (?a . ?r) ?y (?a . ?z))}
\]
\[
\text{(app ?r ?y ?z))}
\]
\[
\text{(query (app ?left (c d) (e b c d)))}
\]

\[
\text{(app ?left (c d) (e b c d))}
\]
\[
\text{(a: e, y: (c d), z: (b c d), left: (?a . ?r))}
\]
\[
\text{(app (?a . ?r) ?y (?a . ?z))}
\]
\[
\text{conclusion <- hypothesis}
\]
\[
\text{(app ?r (c d) (b c d))}
\]
\[
\text{(a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2))}
\]
\[
\text{(app (?a2 . ?r2) ?y2 (?a2 . ?z2))}
\]
\[
\text{conclusion <- hypothesis}
\]
\[
\text{(app ?r2 (c d) (c d))}
\]
\[
\text{(r2: (), x: (c d))}
\]
\[
\text{(app () (c d) (c d))}
\]
\[
\text{(app () ?x ?x)}
\]

Variables are local to facts & queries
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
conclusion <- hypothesis
(app ?r (c d) (b c d))
  {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
conclusion <- hypothesis
(app ?r2 (c d) (c d))
  {r2: (), x: (c d)}
(app () (c d) (c d))
(app () ?x ?x)

Variables are local to facts & queries

?left:
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))

(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z))

(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}

(app (?a . ?r) ?y (?a . ?z))

conclusion <- hypothesis

(app ?r (c d) (b c d))
  {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}

(app (?a2 . ?r2) ?y2 (?a2 . ?z2))

conclusion <- hypothesis

(app ?r2 (c d) (c d))
  {r2: (), x: (c d)}

(app () (c d) (c d))

(app () ?x ?x)

(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

(app (b . ?r2) (c d) (b c d))

?left:

Variables are local to facts & queries
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
conclusion <- hypothesis
(app ?r (c d) (b c d))
  {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
conclusion <- hypothesis
(app ?r2 (c d) (c d))
  {r2: (), x: (c d)}
(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))
(app (b . ?r2) (c d) (b c d))
(app () (c d) (c d))
```

Variables are local to facts & queries

?left:
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
 {a: e, y: (c d), z: (b c d), left: (?a . ?r)}

(app (?a . ?r) ?y (?a . ?z))

conclusion <- hypothesis

(app ?r (c d) (b c d))
 {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}

(app (?a2 . ?r2) ?y2 (?a2 . ?z2))

conclusion <- hypothesis

(app ?r2 (c d) (c d))
 {r2: (), x: (c d)}

(app () (c d) (c d))

(app () ?x ?x)

Variables are local to facts & queries

?left: (e .)
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), ?left: (?a . ?r)}

(app (?a . ?r) ?y (?a . ?z))
  conclusion <- hypothesis

(app ?r (c d) (b c d))
  {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}

(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
  conclusion <- hypothesis

(app ?r2 (c d) (c d))
  {r2: (), x: (c d)}

(app () ?x ?x)

Variables are local to facts & queries

?left: (e .)
?r:
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

\[
\text{(app } ?\text{left } (c \text{ d}) (e \text{ b c d}))
\]

\[
\{a: e\}, y: (c \text{ d}), z: (b \text{ c d}), \{\text{left: (?a . ?r)}\}
\]

\[
\text{(app } ?\text{a . ?r) ?y (?a . ?z)}
\]

\[
\text{conclusion <- hypothesis}
\]

\[
\text{(app } ?r (c \text{ d}) (b \text{ c d}))
\]

\[
\{a2: b\}, y2: (c \text{ d}), z2: (c \text{ d}), r: (?a2 . ?r2)
\]

\[
\text{(app } ?\text{a2 . ?r2) ?y2 (?a2 . ?z2)}
\]

\[
\text{conclusion <- hypothesis}
\]

\[
\text{(app } ?r2 (c \text{ d}) (c \text{ d}))
\]

\[
\{r2: ()\}, x: (c \text{ d})
\]

\[
\text{(app } () (c \text{ d}) (c \text{ d}))
\]

\[
\text{(app } () \text{ ?x ?x)}
\]

\[
\text{(app } e . ?r) (c \text{ d}) (e \text{ b c d})
\]

\[
\text{(app } b . ?r2) (c \text{ d}) (b \text{ c d})
\]

\[
\text{?left: (e .)}
\]

\[
\text{?r: }
\]

Variables are local to facts & queries
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z))
(query (app ?left (c d) (e b c d)))

(app (?left (c d) (e b c d)))
{a: e, y: (c d), z: (b c d), ?left: (?a . ?r)}

(app (?a . ?r) ?y (?a . ?z))
conclusion <- hypothesis

(app ?r (c d) (b c d))
{a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}

(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
conclusion <- hypothesis

(app ?r2 (c d) (c d))
{r2: (), x: (c d)}

Variables are local to facts & queries

(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

?left: (e .

?r:}
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
{a: e, y: (c d), z: (b c d), left: (?a . ?r)}

(app (?a . ?r) ?y (?a . ?z))
conclusion <- hypothesis

(app ?r (c d) (b c d))
{a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}

(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
conclusion <- hypothesis

(app ?r2 (c d) (c d))
{r2: (), x: (c d)}

(app () (c d) (c d))
(app () (c d) (c d))

Variables are local to facts & queries

?left: (e .)

?r: (b .)
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

\[(\text{fact } (\text{app } () ?x ?x))\]
\[(\text{fact } (\text{app } (?a . ?r) ?y (?a . ?z))\]
\[(\text{app } ?r ?y ?z ))\]
\[(\text{query } (\text{app } ?\text{left} (c d) (e b c d)))\]

\[(\text{app } ?\text{left} (c d) (e b c d))\]
\[
\begin{array}{l}
\{a: e, y: (c d), z: (b c d), \text{left: } (?a . ?r)\}\n\end{array}
\]
\[(\text{app } (?a . ?r) ?y (?a . ?z))\]
\[
\begin{array}{l}
\text{conclusion } <- \text{ hypothesis}
\end{array}
\]
\[(\text{app } ?r (c d) (b c d))\]
\[
\begin{array}{l}
\{a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)\}
\end{array}
\]
\[(\text{app } (?a2 . ?r2) ?y2 (?a2 . ?z2))\]
\[
\begin{array}{l}
\text{conclusion } <- \text{ hypothesis}
\end{array}
\]
\[(\text{app } ?r2 (c d) (c d))\]
\[
\begin{array}{l}
\{r2: (), x: (c d)\}
\end{array}
\]
\[(\text{app } () (c d) (c d))\]
\[(\text{app } () (c d) (c d))\]

Variables are local to facts & queries

?\text{left}: (e .)

?\text{r}: (b .)
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

\[
\text{(app ?left (c d) (e b c d))}
\]

\[
\{a: e, y: (c d), z: (b c d), \text{left: (?a . ?r)}\}
\]

\[
\text{(app (?a . ?r) ?y (?a . ?z))}
\]

conclusion <- hypothesis

\[
\text{(app ?r (c d) (b c d))}
\]

\[
\{a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)\}
\]

\[
\text{(app (?a2 . ?r2) ?y2 (?a2 . ?z2))}
\]

conclusion <- hypothesis

\[
\text{(app ?r2 (c d) (c d))}
\]

\[
\{r2: (), x: (c d)\}
\]

\[
\text{(app () (c d) (c d))}
\]

\[
\text{(app () ?x ?x)}
\]

Variables are local to facts & queries

?left: (e .)

?r: (b . ())
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))
(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))
  {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))

conclusion <- hypothesis

(app ?r (c d) (b c d))
  {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))

conclusion <- hypothesis

(app ?r2 (c d) (c d))
  {r2: (), x: (c d)}
(app () (c d) (c d))
(app () ?x ?x)

Variables are local to facts & queries

?left: (e .)
?r: (b . ())  \(\rightarrow\) (b)
Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

(fact (app () ?x ?x))

(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))

(query (app ?left (c d) (e b c d)))

(app ?left (c d) (e b c d))

{a: e, y: (c d), z: (b c d), ?left: (?a . ?r)}

APPEND (app (?a . ?r) ?y (?a . ?z))

conclusion <- hypothesis

(app ?r (c d) (b c d))

{a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}

APPEND (app (?a2 . ?r2) ?y2 (?a2 . ?z2))

conclusion <- hypothesis

(app ?r2 (c d) (c d))

{r2: (), x: (c d)}

APPEND (app () (c d) (c d))

(app () ?x ?x)

Variables are local to facts & queries

# Variables are local to facts & queries

?left: (e . (b))

?r: (b . ())  (b)
Searching for Proofs

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(fact (app (?a . ?r) ?y (?a . ?z))
  (app ?r ?y ?z ))
(query (app ?left (c d) (e b c d)))

Variables are local to facts & queries

?left: (e . (b)) ➞ (e b)
?r: (b . ()) ➞ (b)
Depth-First Search
Depth-First Search

The space of facts is searched exhaustively, starting from the query and following a *depth-first* exploration order.
**Depth-First Search**

The space of facts is searched exhaustively, starting from the query and following a *depth-first* exploration order.

Depth-first search: Each proof approach is explored exhaustively before the next.
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```python
def search(clauses, env):
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def search(clauses, env):
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\begin{verbatim}
def search(clauses, env):
    for fact in facts:
        env_head = an environment extending env
\end{verbatim}
Depth-First Search

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def search(clauses, env):
    for fact in facts:
        env_head = an environment extending env
        if unify(conclusion of fact, first clause, env_head):
Depth-First Search

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```python
def search(clauses, env):
    for fact in facts:
        env_head = an environment extending env
        if unify(conclusion of fact, first clause, env_head):
            # Environment now contains new unifying bindings
```

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```python
def search(clauses, env):
    for fact in facts:
        env_head = an environment extending env
        if unify(conclusion of fact, first clause, env_head):
            for env_rule in search(hypotheses of fact, env_head):
```

Environment now contains new unifying bindings
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    for fact in facts:
        env_head = an environment extending env
        if unify(conclusion of fact, first clause, env_head):
            for env_rule in search(hypotheses of fact, env_head):
                for result in search(rest of clauses, env_rule):
```

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        if unify(conclusion of fact, first clause, env_head):
            for env_rule in search(hypotheses of fact, env_head):
                for result in search(rest of clauses, env_rule):
                    yield each successful result
```

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• Limiting depth of the search avoids infinite loops.

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- Limiting depth of the search avoids infinite loops.
- Each time a fact is used, its variables are renamed.
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(Demo)
Addition

(Demo)