Announcements

- Homework 12 due Tuesday 12/10 @ 11:59pm.
  - All you have to do is vote on your favorite recursive art.
- 29 review sessions next week! Come learn about the topics that interest you the most.
  - See [http://inst.eecs.berkeley.edu/~cs61a/fa13/exams/final.html](http://inst.eecs.berkeley.edu/~cs61a/fa13/exams/final.html) for the schedule.
- The final exam is on Friday 12/20 @ 11:30am in the RSF gym, emphasizing:
  - Higher-order functions
  - Sequences (tuples, lists, recursive lists, Scheme lists)
  - Non-local assignment and mutation
  - Object-oriented programming
  - Recursion and recursive data
  - Iterators, generators, and streams
Implicit Sequences Example
Example: Numerical Approximations

Is \( \sqrt{51} - 4 \ < \ \pi \)?

No calculators/interpreters allowed!

Let's say we have a computer that can +, −, *, /. How do we answer this question?
Approximating Square Roots

Is $\sqrt{51} - 4 < \pi$?

No calculators/interpreters allowed!

Let's say we have a computer that can $+, - , *, /$. How do we answer this question?

(A) A sequence of approximations (SoA) to $y$ is an infinite sequence that converges to $y$. Implicitly define a SoA to $\sqrt{a}$.

```python
def sqrt(a):
    x = 1
    while _________________ True _________________:
        yield _______________
        x = _________________ (x + a/x)/2

>>> for x in sqrt(2):
    ...       print(x)
1
1.5
1.4166666666666665
1.4142156862745097
...```

How to compute square_root(a):

**Idea:** Iteratively refine a guess $x$ about the square root of $a$.

$$x = \frac{x + \frac{a}{x}}{2}$$

*From lecture 6*
Approximating Pi

Is $\sqrt{51} - 4 < \pi$ ?

(B) Define a sequence of approximations to $\pi$.

```python
def sqrt(a):
    x = 1
    while True:
        yield x
        x = (x + a/x)/2

def pi():
    total, k = 0, 1
    while True:
        yield total
        total += 8/(4*k-3) * (4*k-1)
        k += 1
```

```
>>> for x in pi():
...     print(x)
0
2.6666666666666665
2.895238095238095
2.976046176046176
3.017071817071817
3.041839618929401
3.058402765927331
...`

$\sum_{k=1}^{\infty} \frac{8}{(4k - 3) \cdot (4k - 1)} = \pi$

From lecture 4
(C) Assume that $s$ is a SoA to $y$ and each element of $s$ is closer to $y$ than the last. Define $\text{less}_0(s)$ that returns True if it is certain that $y < 0$.

```python
def sqrt(a):
    x = 1
    while True:
        yield x
        x = (x + a/x)/2

def pi():
    total, k = 0, 1
    while True:
        yield total
        total += 8/((4*k-3)*(4*k-1))
        k += 1

def four():
    while True:
        yield 4

def subtract(x, y):
    while True:
        yield next(x)-next(y)

def less_than_0(s):
    current = next(s)
    while True:
        last, current = current, next(s)
        if last < 0 and current < last:
            return True

a = subtract(sqrt(51), four())
less_than_0(subtract(a, pi()))
```
Computer Science
61A was Designed to Introduce the Big Ideas in Computer Science

What are functions, data, sequences, trees, programs, languages, and interpreters.
How to write legible programs, use recursion, measure complexity, and solve problems.
Different programming paradigms: functional, object-oriented, and declarative.

What's left to learn in CS?
• Designing and testing software
• Algorithms for solving known problems
• Low-level representations of data and programs
• Discrete mathematics and analysis of programs
• Programming languages
• User interface design
• Networking
• Systems
• Artificial intelligence
• Lots of other subfields: graphics, theory, scientific computing, security, etc.
Life
Important Ideas Take a Long Time to Learn

- It's a good idea to study subjects other than computer science.
- Who you spend your time with is important.
- Ideas come from people, and people think from experience.
- Don't compare.
- Contribute to the world.
Thanks for being amazing!

Please stay for the HKN survey.