61A Lecture 5

Wednesday, September 10

Announcements

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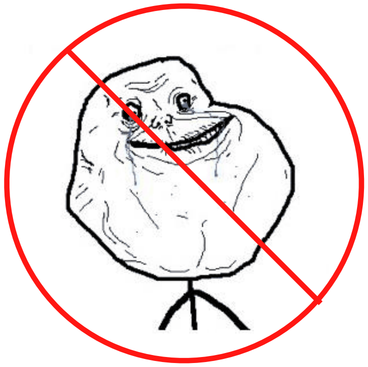

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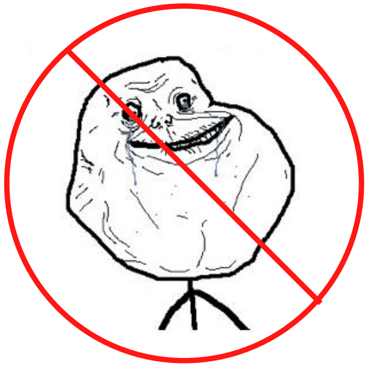

http://cs61a.org/staff.html

Environments for Higher-Order Functions

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Environment diagrams describe how higher-order functions work!

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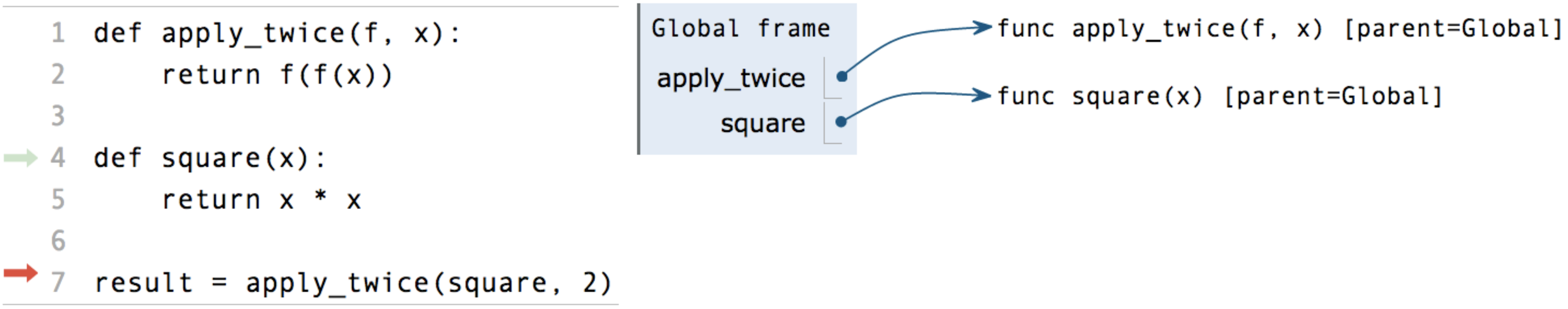
(Demo)

Names can be Bound to Functional Arguments



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def apply_twice(f, x):
    return f(f(x))

def square(x):
    return x * x

result = apply_twice(square, 2)
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Interactive Diagram

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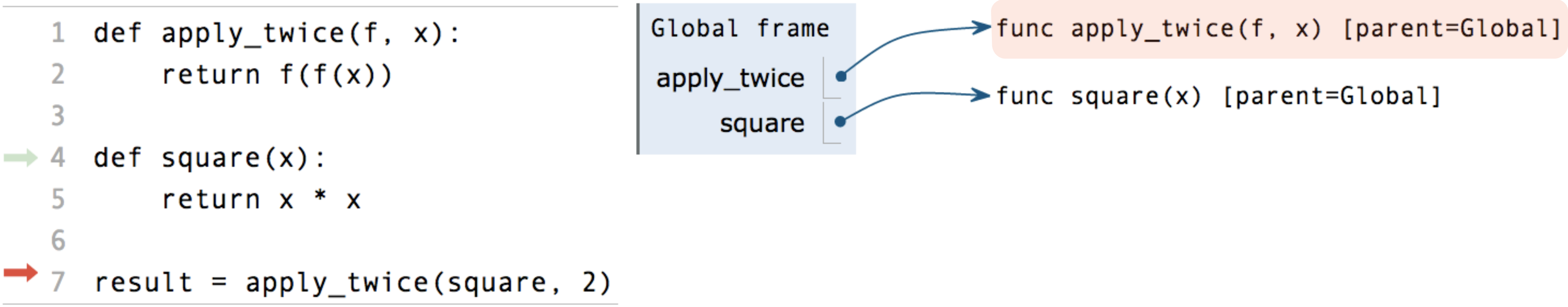

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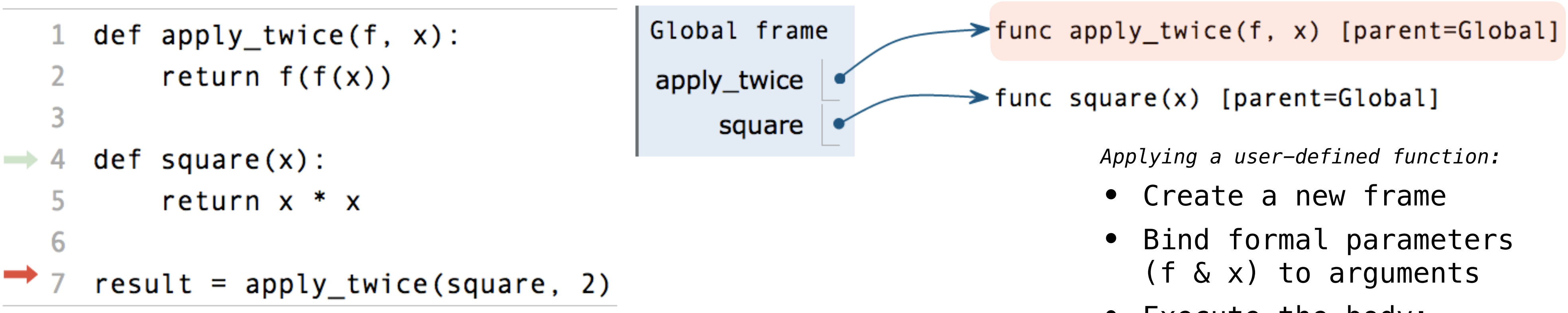

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- Create a new frame
- Bind formal parameters (f & x) to arguments
- Execute the body: return f(f(x))

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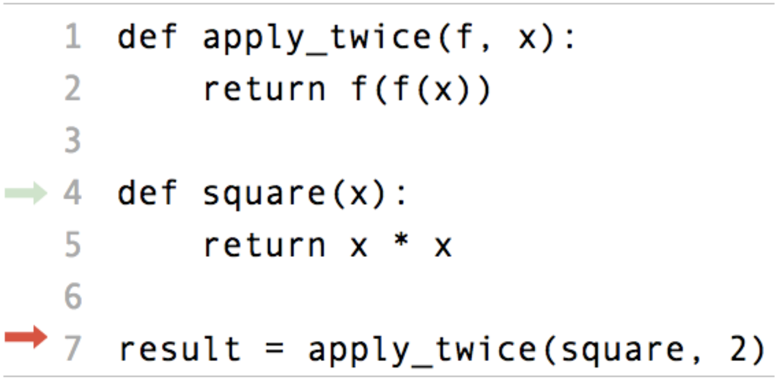

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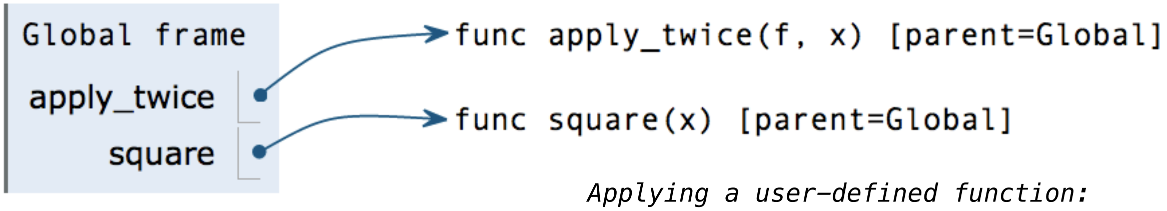
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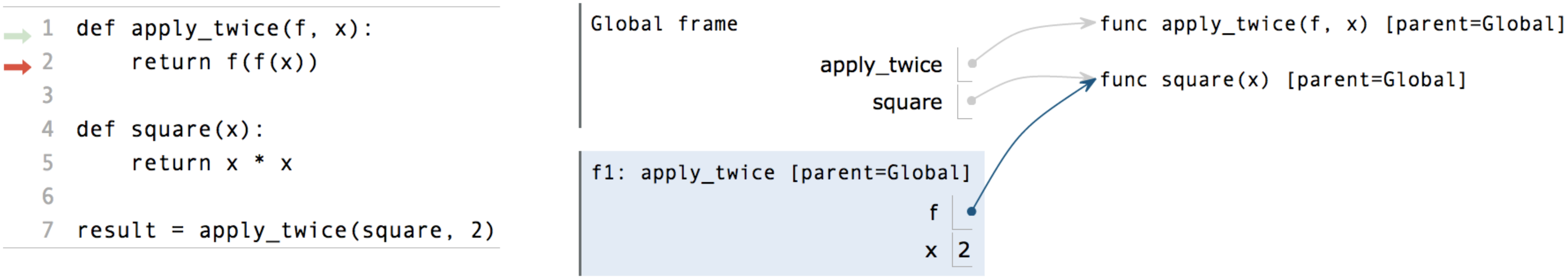

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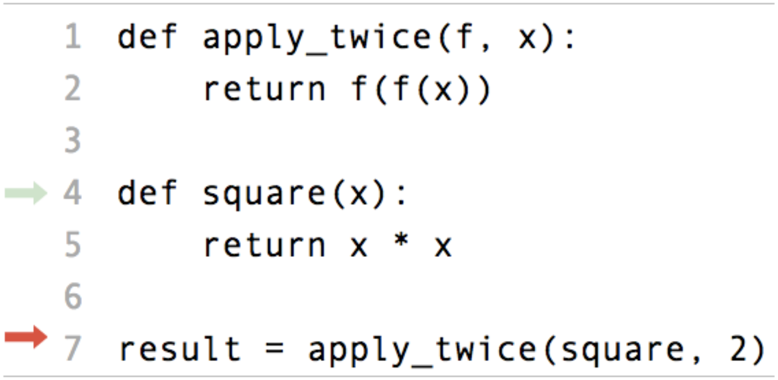

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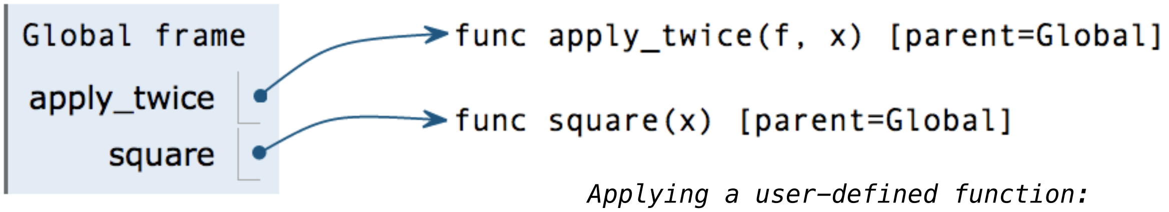
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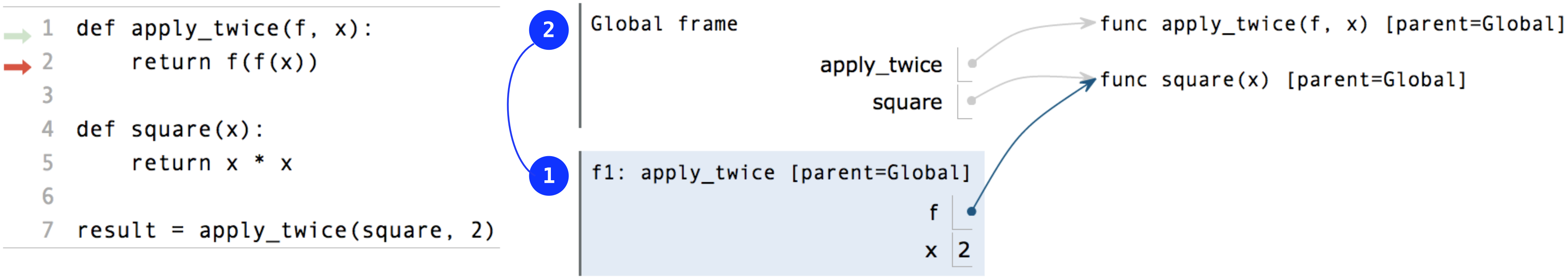

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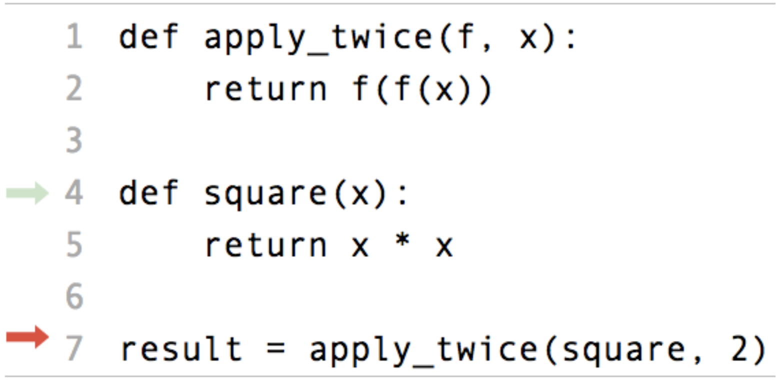

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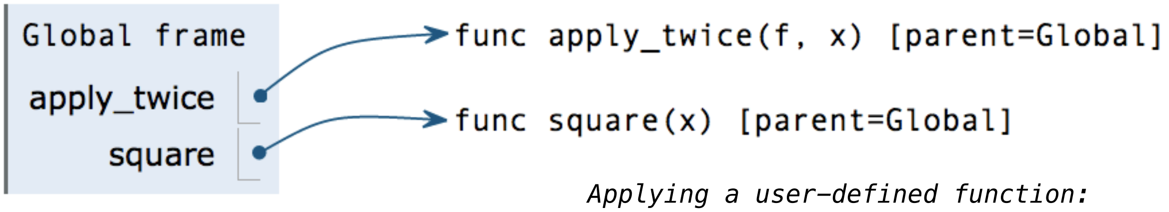

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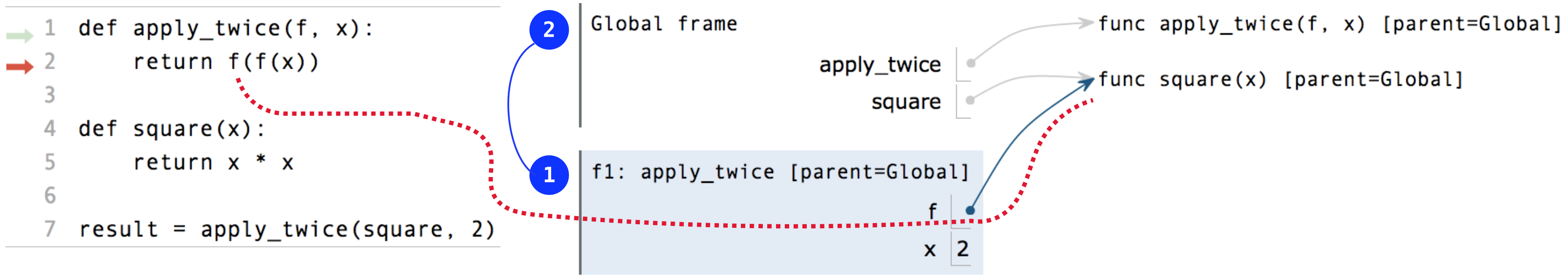

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    return x

def g(y):
    return (y + 5) // 3

result = repeat(g, 5)
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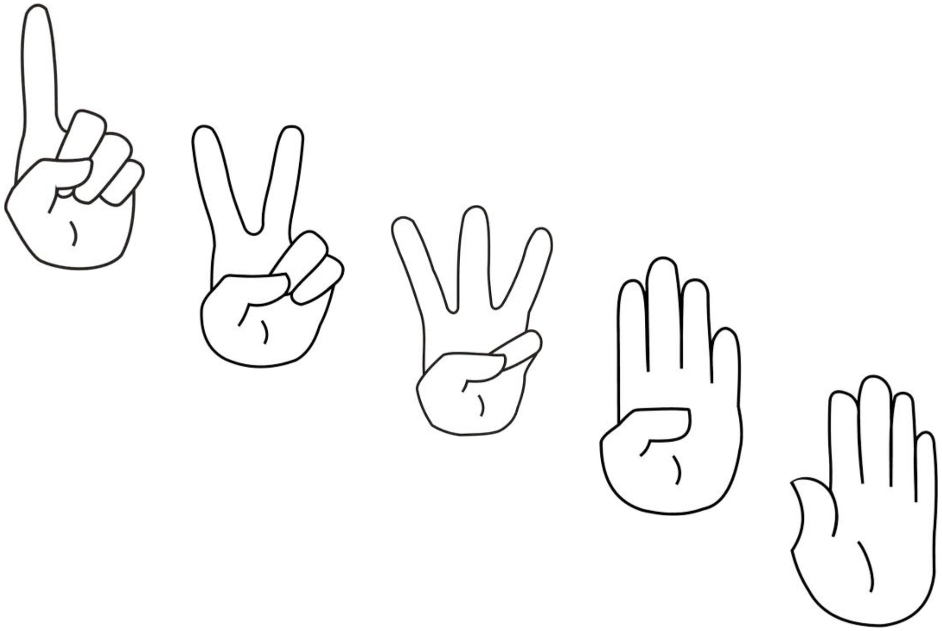

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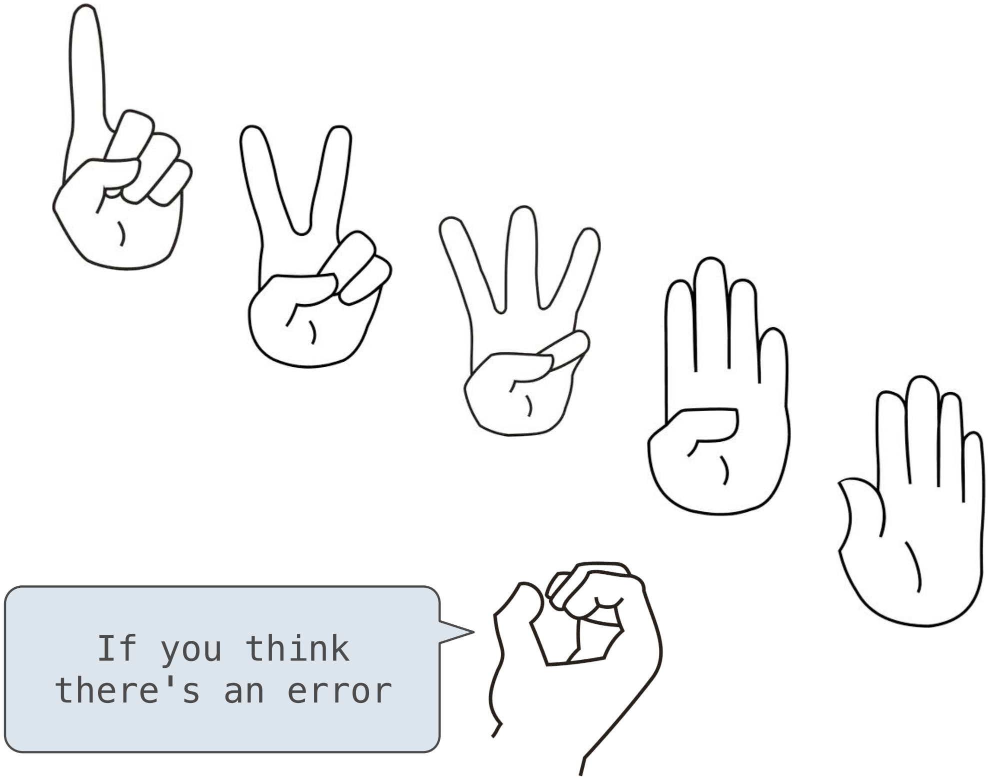
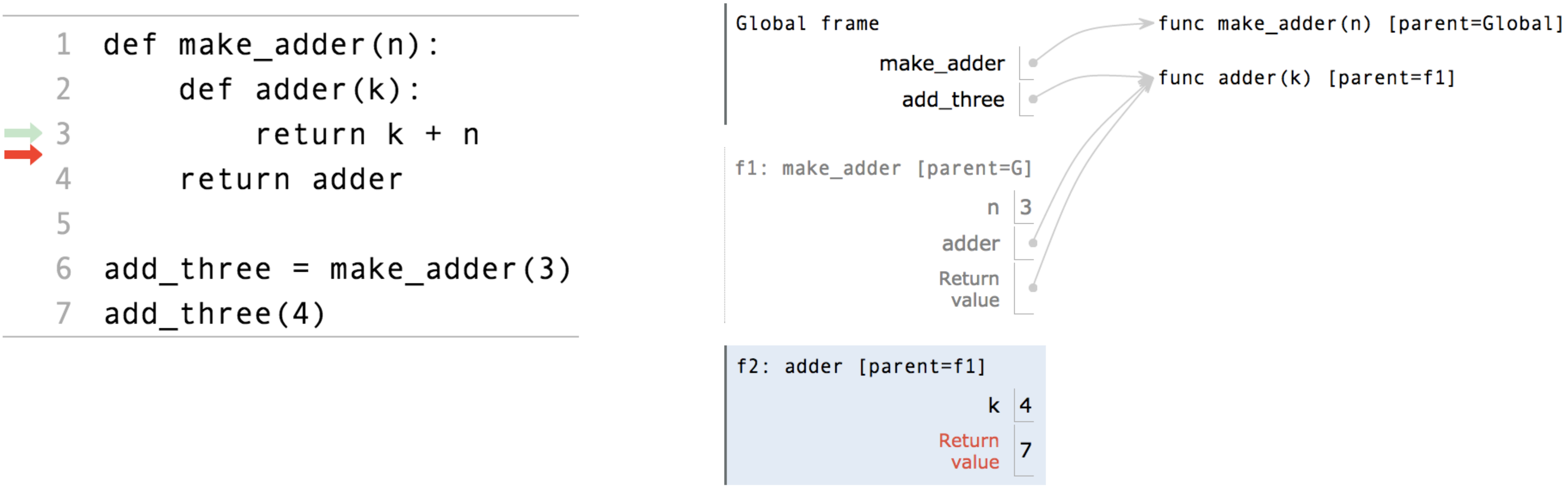


Interactive Diagram

Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements



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def make_adder(n):
    def adder(k):
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add_three = make_adder(3)
add_three(4)
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make_adder

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func adder(k) [parent=f1]

f1: make_adder [parent=G]

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adder

Return value

f2: adder [parent=f1]

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Interactive Diagram

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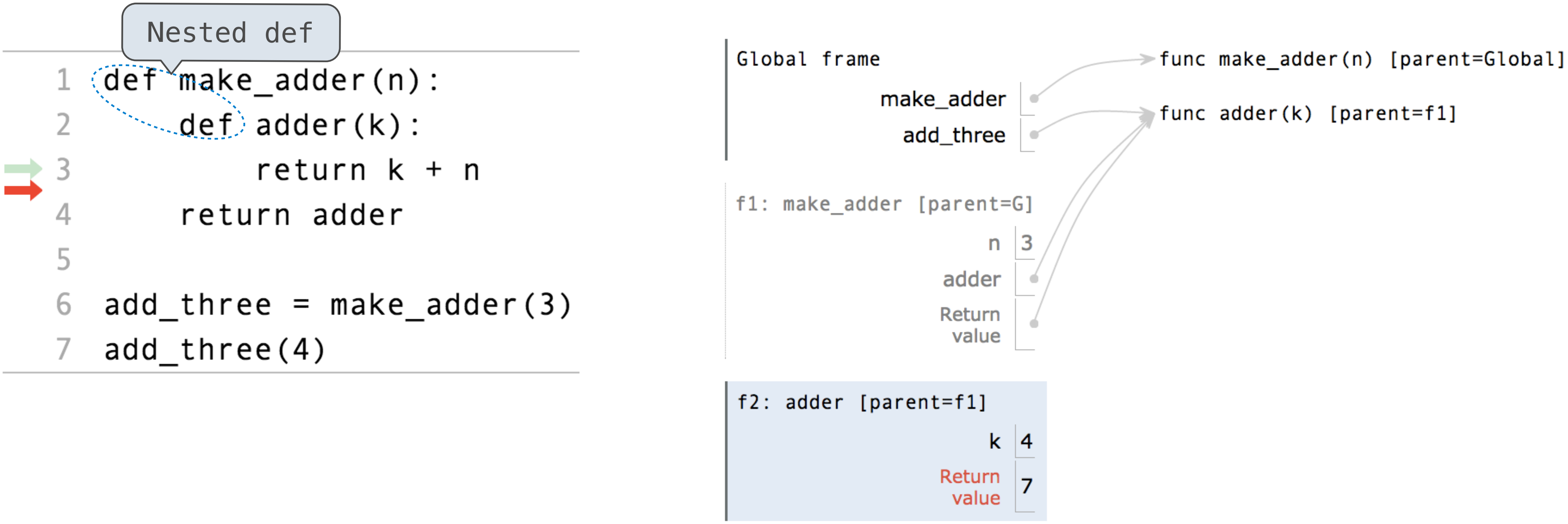

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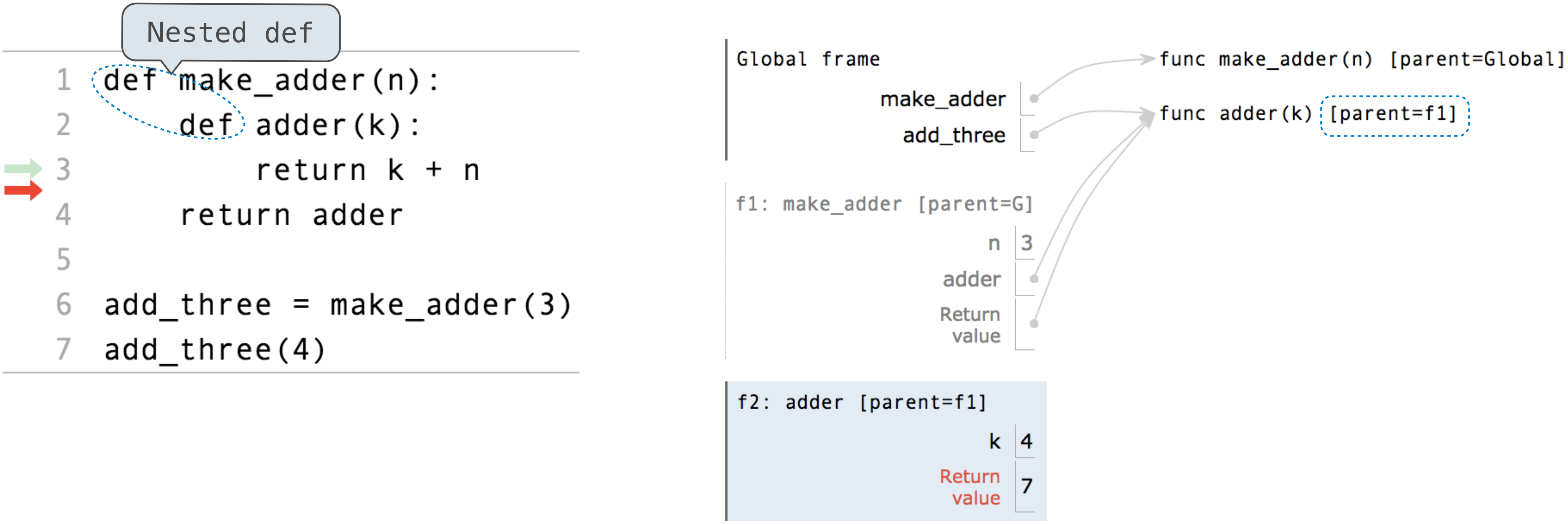

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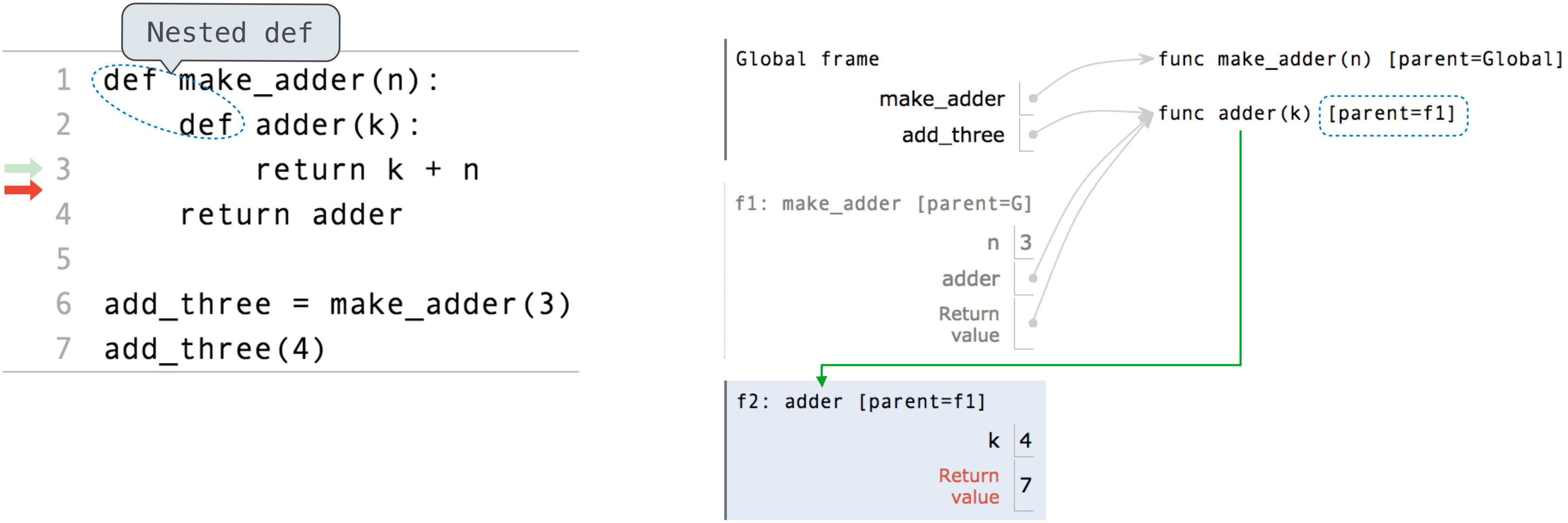

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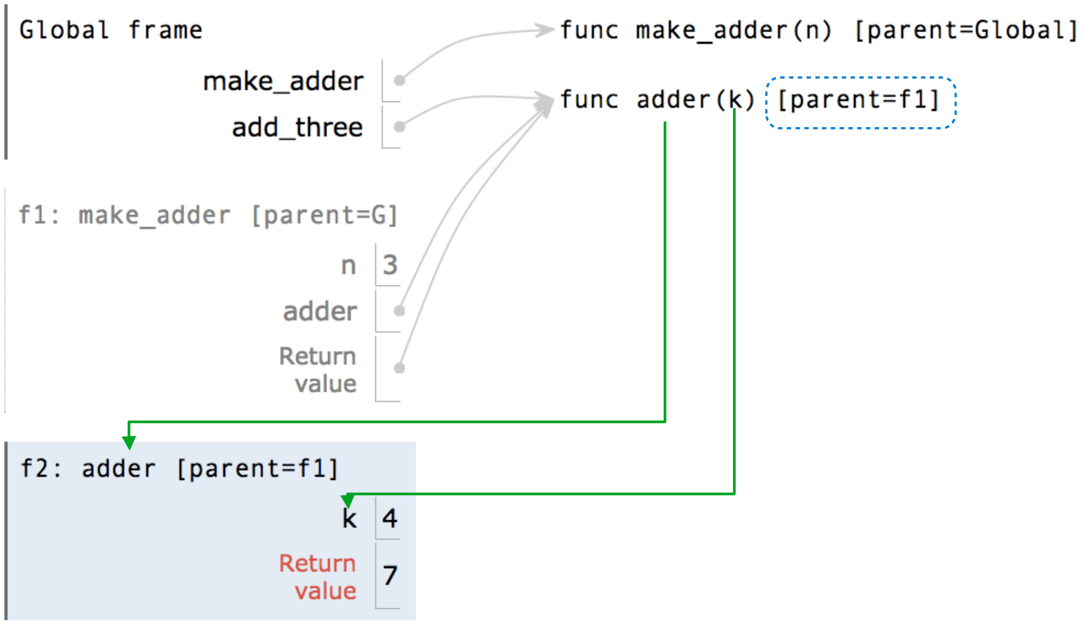

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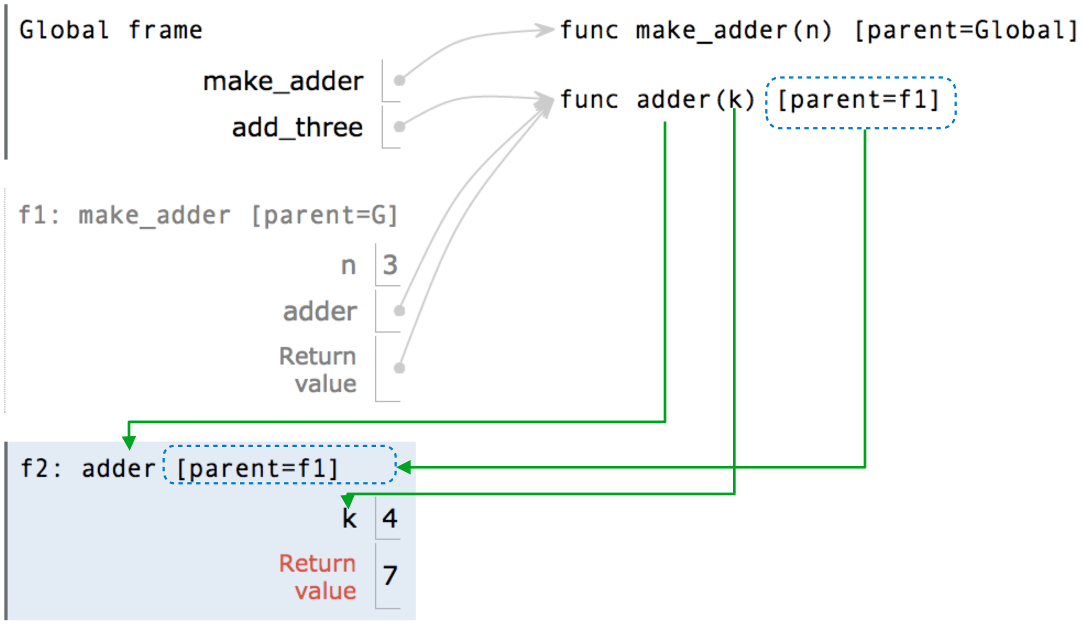

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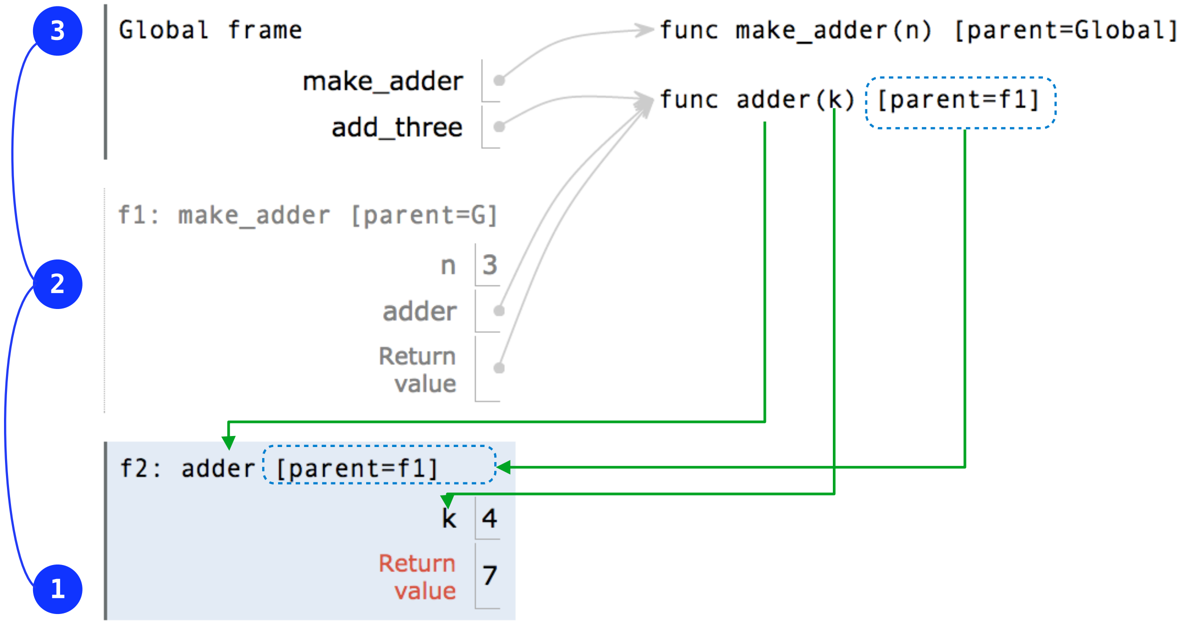

3

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2

f1: make_adder [parent=G]

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adder

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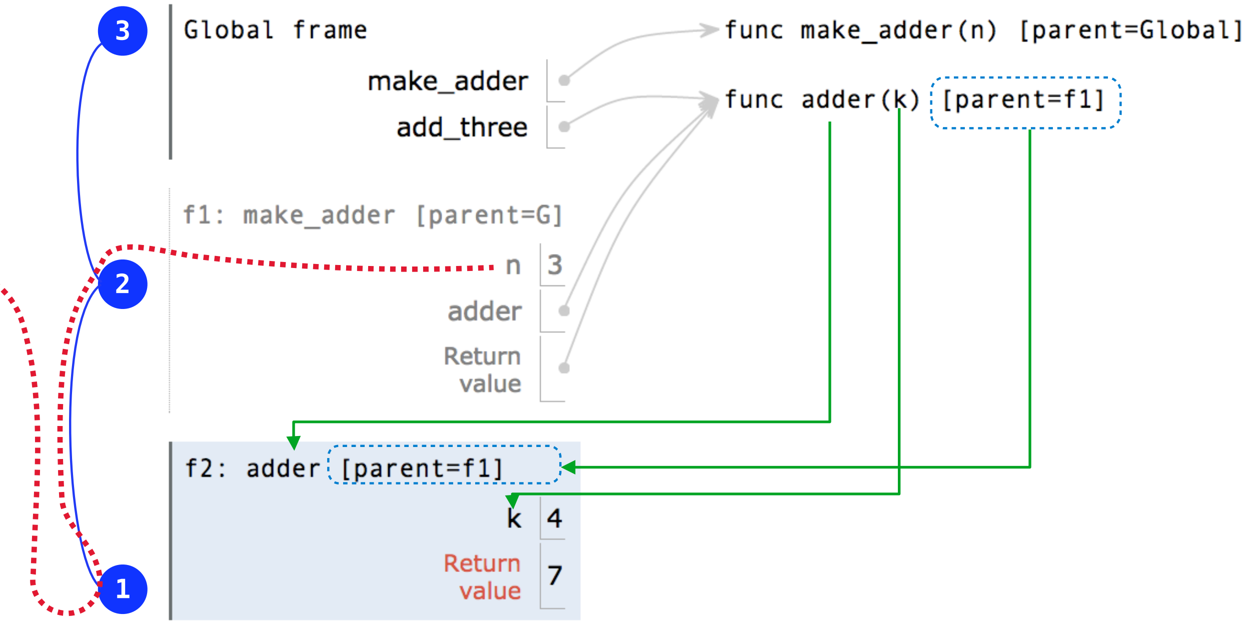

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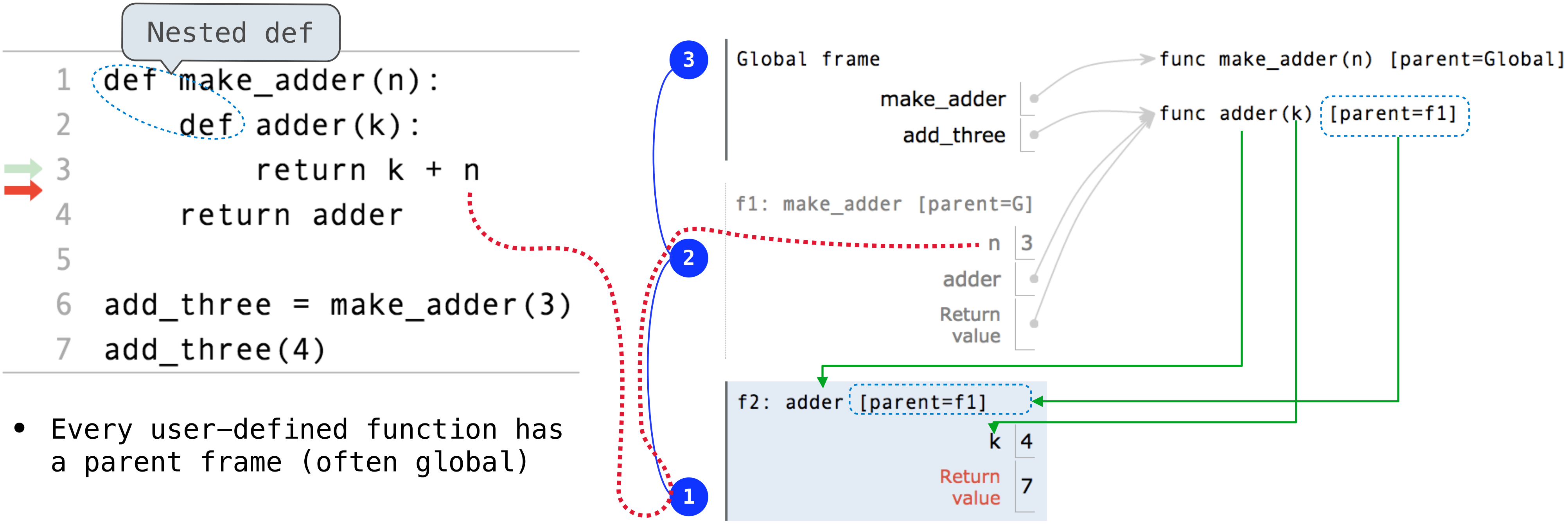

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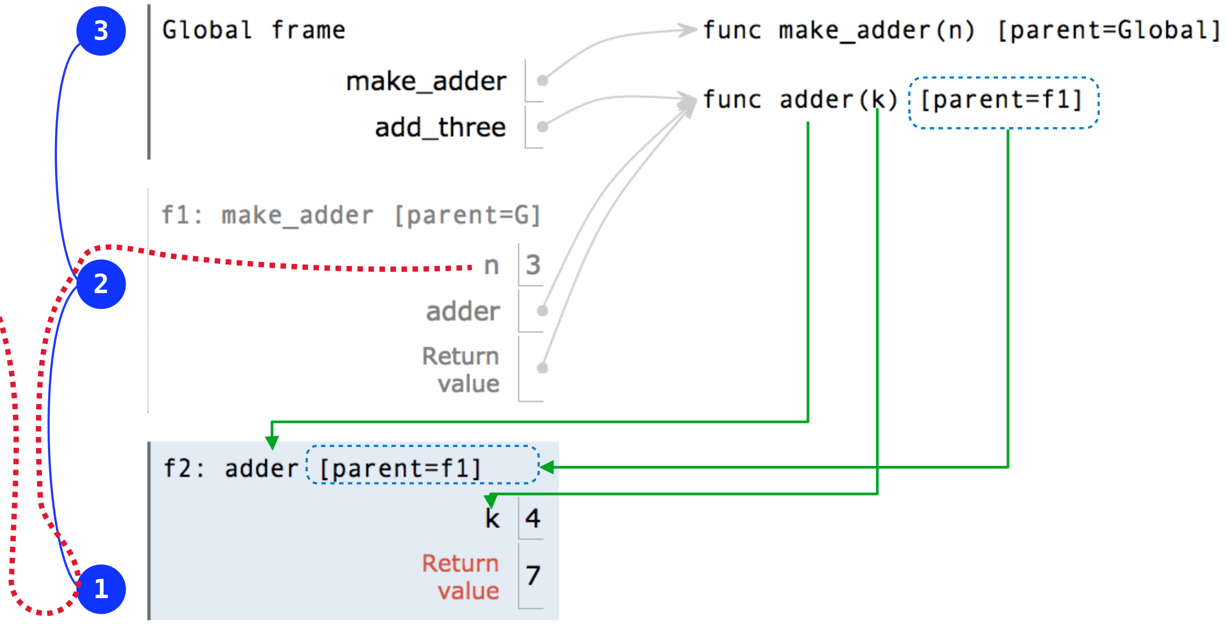

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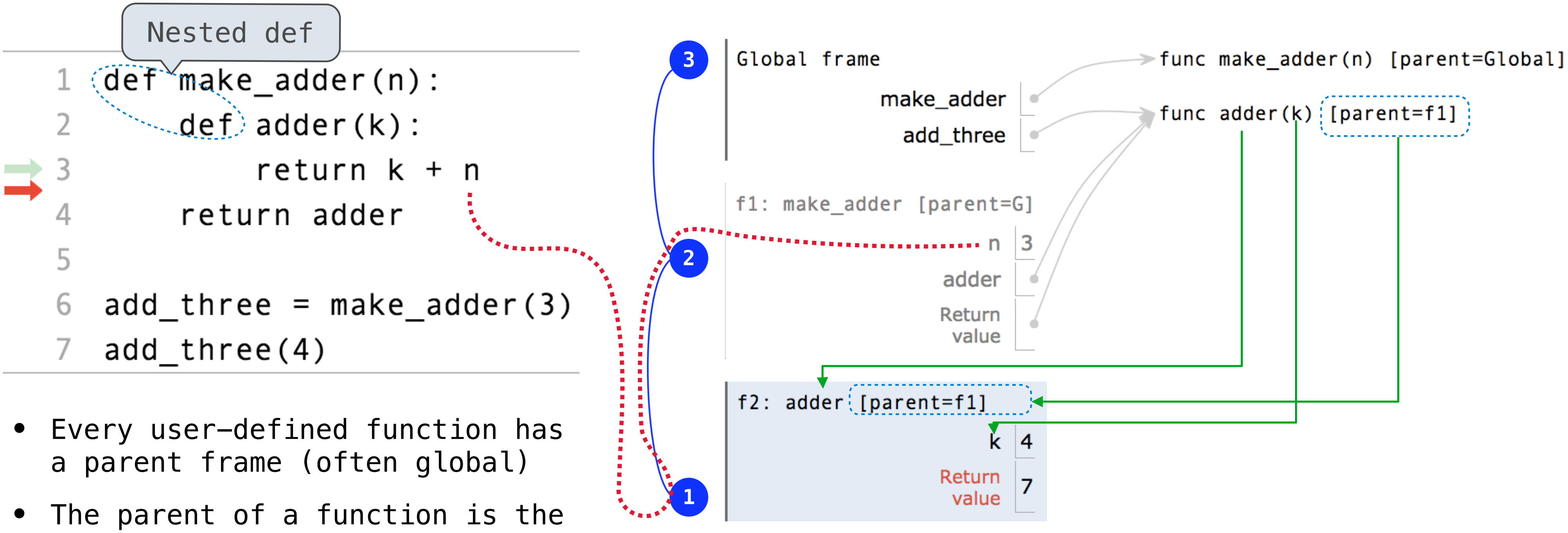
- The parent of a function is the frame in which it was defined

Interactive Diagram

Environment Diagrams for Nested Def Statements



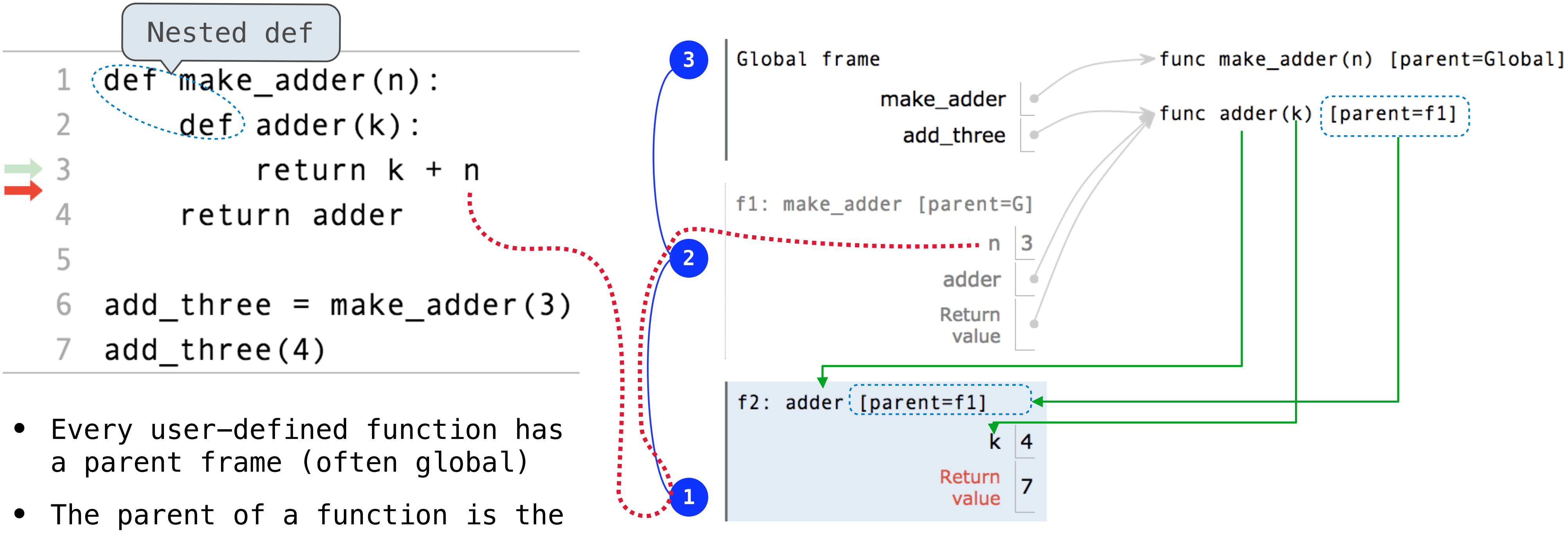
```
def make_adder(n):
    def adder(k):
        return k + n
    return adder

add_three = make_adder(3)
add_three(4)
```

- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)

Interactive Diagram

Environment Diagrams for Nested Def Statements



```
def make_adder(n):
    def adder(k):
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    return adder

add_three = make_adder(3)
add_three(4)
```

- Every user-defined function has a parent frame (often global)
- The parent of a function is the frame in which it was defined
- Every local frame has a parent frame (often global)
- The parent of a frame is the parent of the function called

Interactive Diagram

How to Draw an Environment Diagram

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When a function is defined:

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Create a function value: func <name>(<formal parameters>) [parent=<label>]

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When a function is defined:

Create a function value: `func <name>(<formal parameters>) [parent=<label>]`

Its parent is the current frame.

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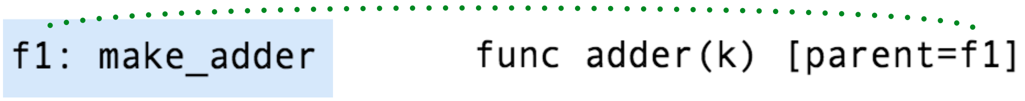

f1: make_adder

func adder(k) [parent=f1]

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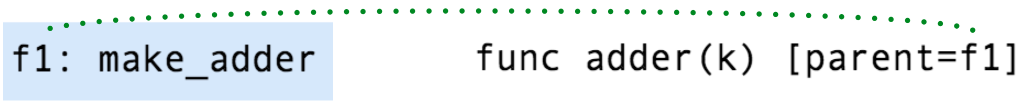

f1: make_adder

func adder(k) [parent=f1]

Bind <name> to the function value in the current frame

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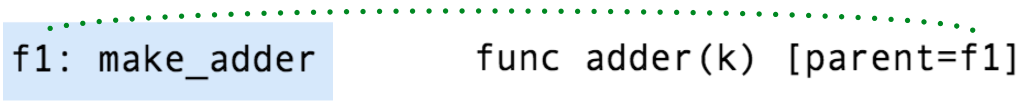

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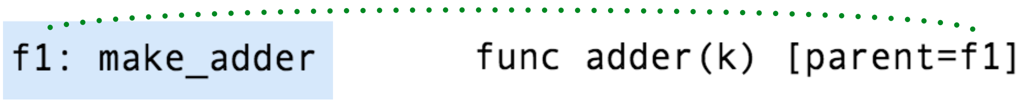


f1: make_adder

func adder(k) [parent=f1]

Bind <name> to the function value in the current frame

When a function is called:

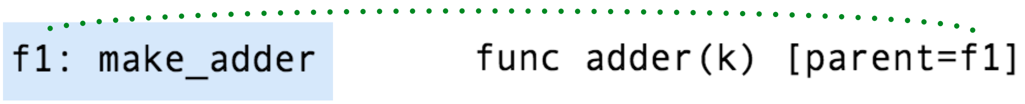
1. Add a local frame, titled with the <name> of the function being called.

How to Draw an Environment Diagram

When a function is defined:

Create a function value: func <name>(<formal parameters>) [parent=<label>]

Its parent is the current frame.



f1: make_adder

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Bind <name> to the function value in the current frame

When a function is called:

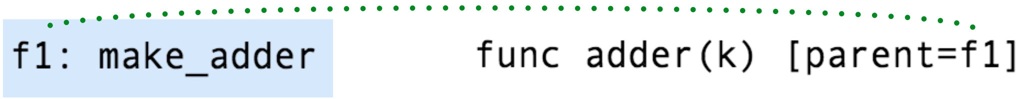
1. Add a local frame, titled with the <name> of the function being called.
2. Copy the parent of the function to the local frame: [parent=<label>]

How to Draw an Environment Diagram

When a function is defined:

Create a function value: func <name>(<formal parameters>) [parent=<label>]

Its parent is the current frame.



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Create a function value: func <name>(<formal parameters>) [parent=<label>]

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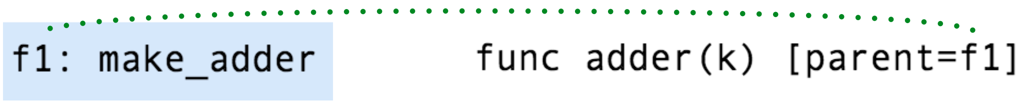


f1: make_adder

func adder(k) [parent=f1]

Bind <name> to the function value in the current frame

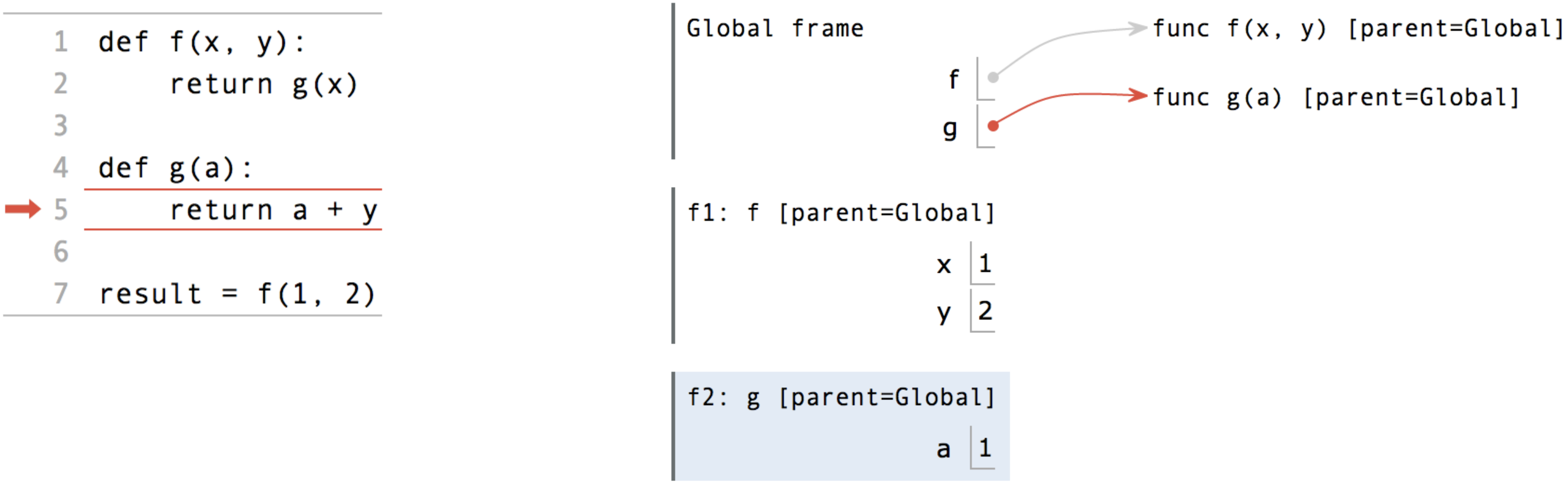
When a function is called:

1. Add a local frame, titled with the <name> of the function being called.
2. Copy the parent of the function to the local frame: [parent=<label>]
3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions



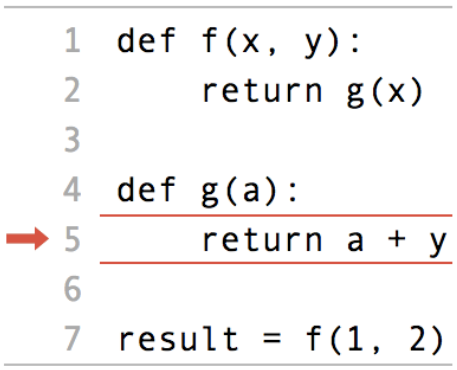
```
def f(x, y):
    return g(x)

def g(a):
    return a + y

result = f(1, 2)
```

Interactive Diagram

Local Names are not Visible to Other (Non-Nested) Functions



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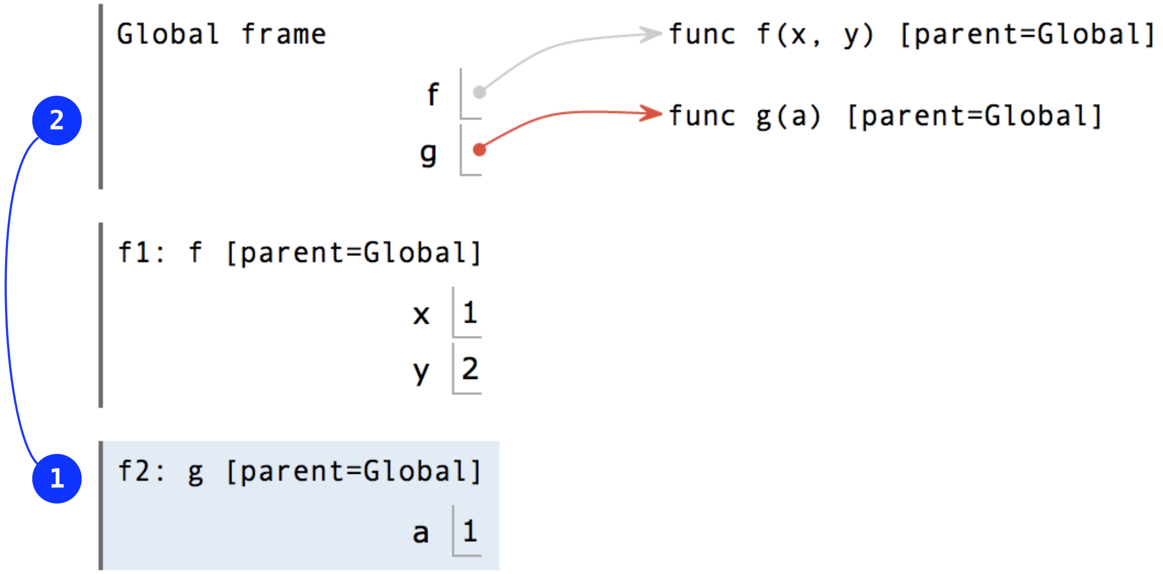
result = f(1, 2)
```



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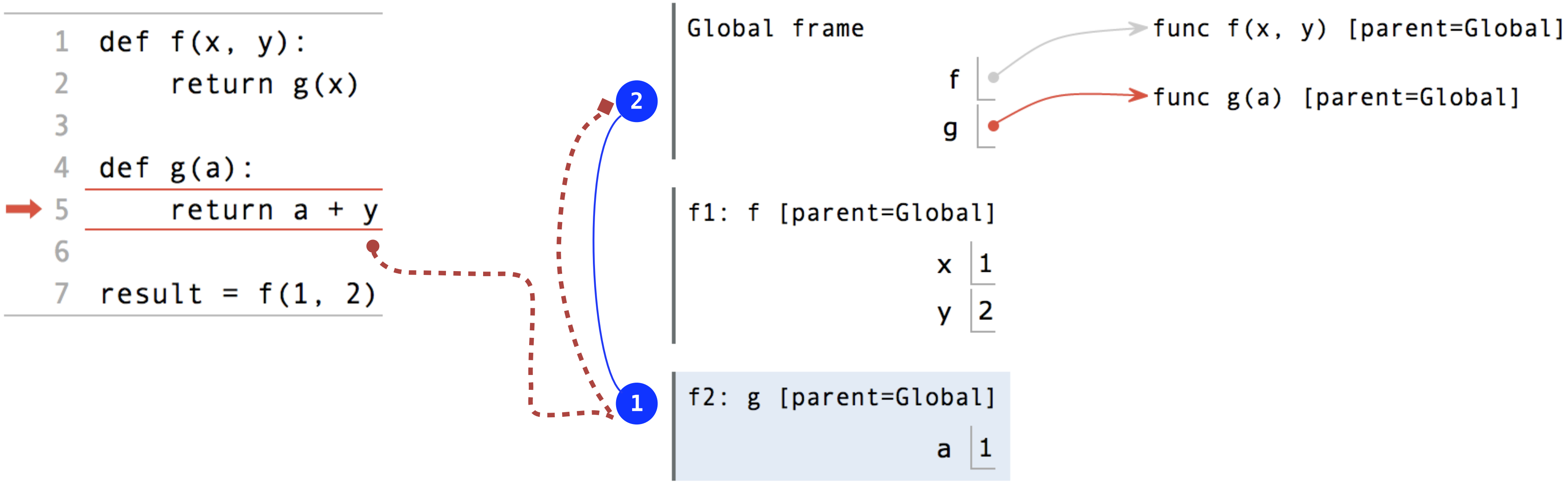


```
def f(x, y):
    return g(x)

def g(a):
    return a + y

result = f(1, 2)
```

Global frame

f → func f(x, y) [parent=Global]

g → func g(a) [parent=Global]

f1: f [parent=Global]

x 1

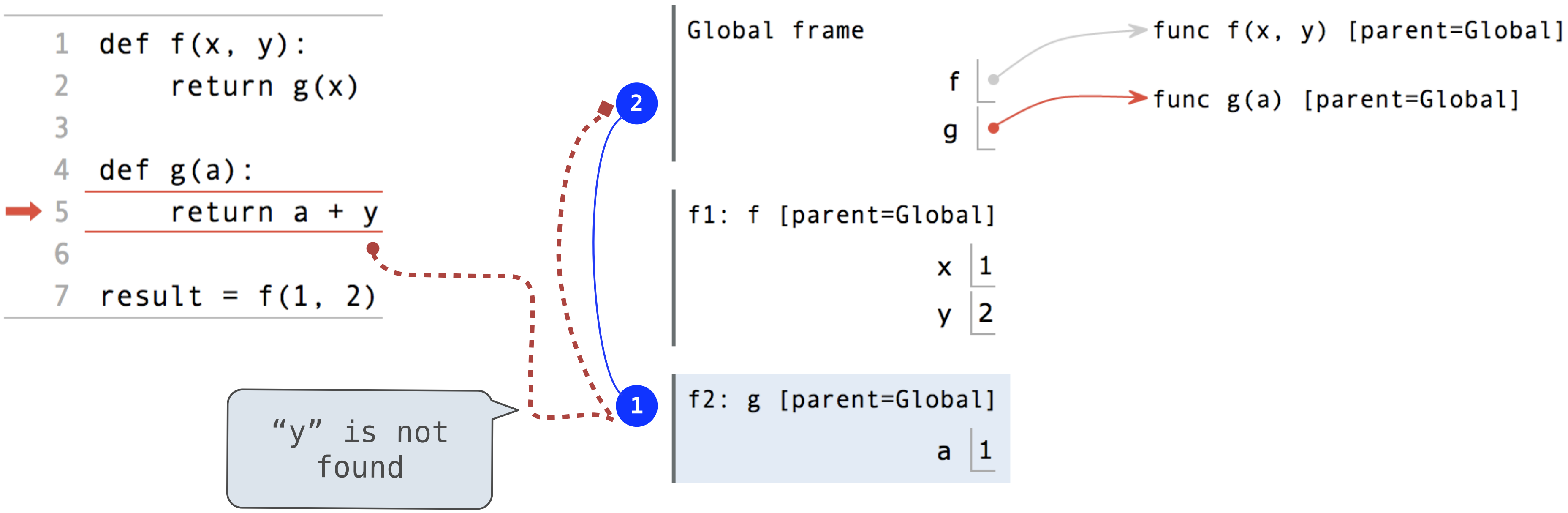
y 2

f2: g [parent=Global]

a 1

Interactive Diagram

Local Names are not Visible to Other (Non-Nested) Functions



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```

Interactive Diagram

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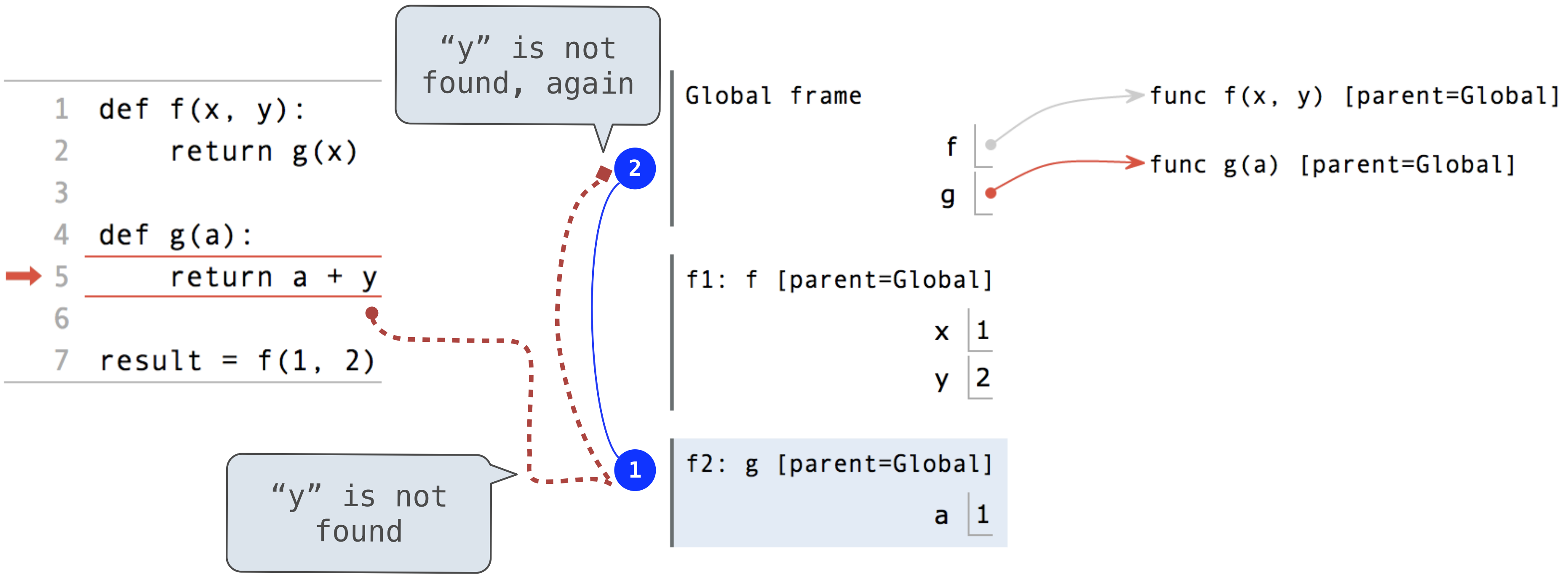


```
def f(x, y):
    return g(x)

def g(a):
    return a + y

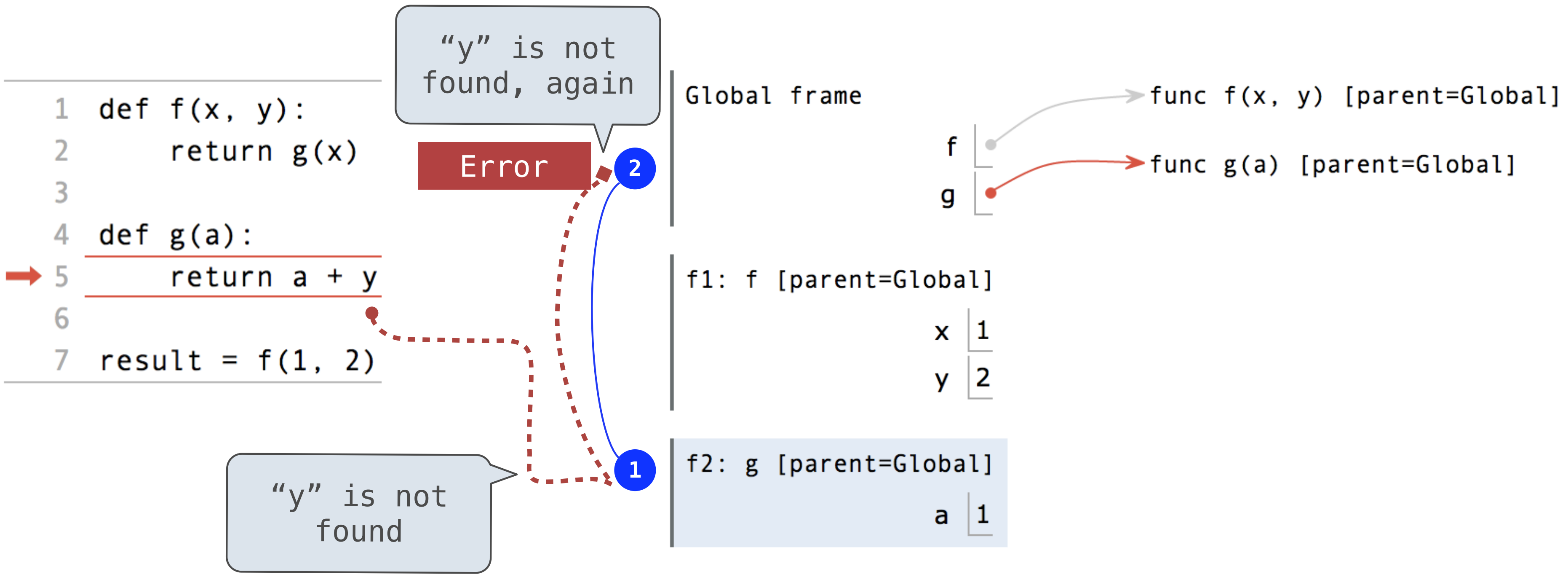
result = f(1, 2)
```

“y” is not found, again

“y” is not found

Interactive Diagram

Local Names are not Visible to Other (Non-Nested) Functions



```
def f(x, y):
    return g(x)

def g(a):
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result = f(1, 2)
```

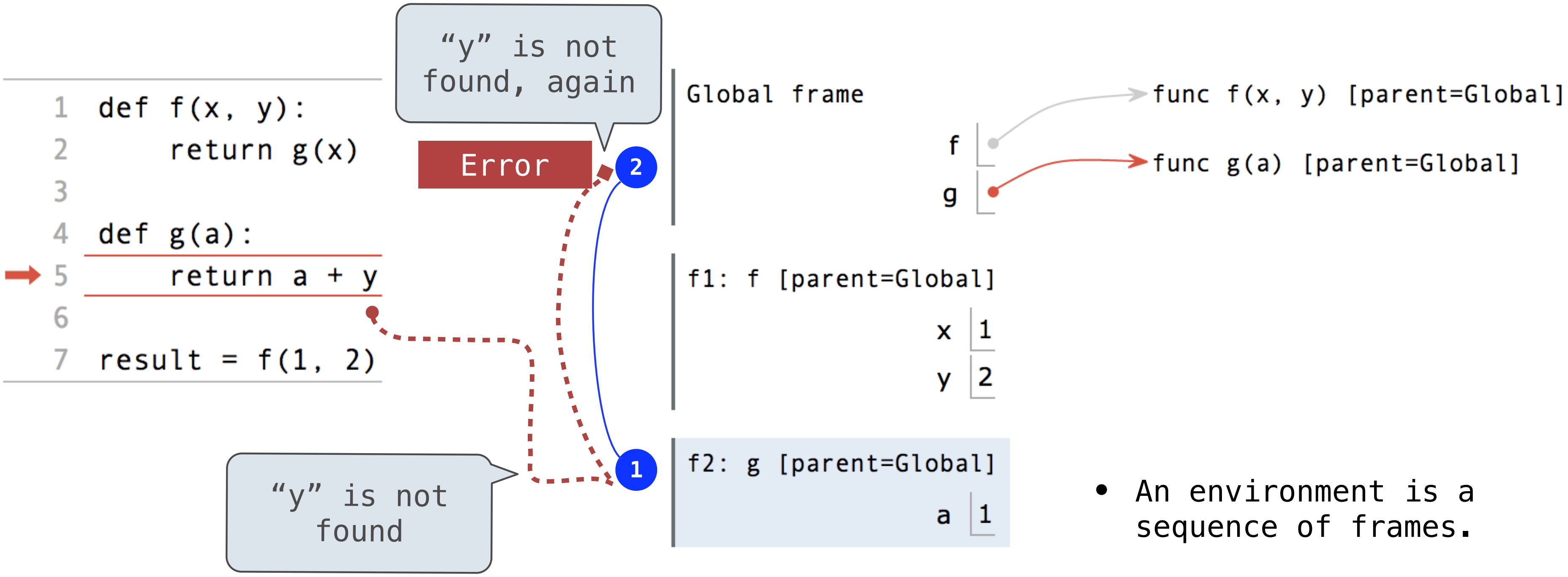
“y” is not found, again

Error

“y” is not found

Interactive Diagram

Local Names are not Visible to Other (Non-Nested) Functions



```
def f(x, y):
    return g(x)

def g(a):
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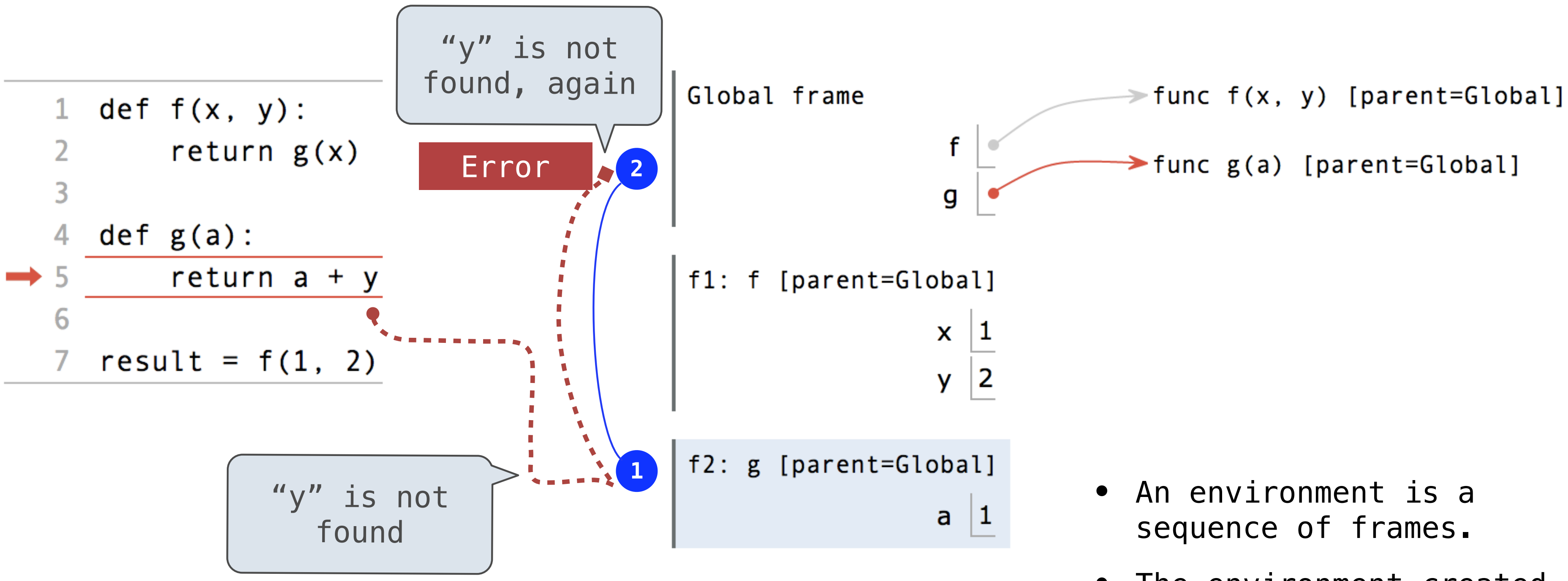
result = f(1, 2)
```

- An environment is a sequence of frames.

Interactive Diagram

Local Names are not Visible to Other (Non-Nested) Functions



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def f(x, y):
    return g(x)

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```

“y” is not found, again

Error

“y” is not found

```
Global frame
                f  → func f(x, y) [parent=Global]
                g  → func g(a) [parent=Global]

f1: f [parent=Global]
                x  1
                y  2

f2: g [parent=Global]
                a  1
```

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

Interactive Diagram

Function Composition

(Demo)

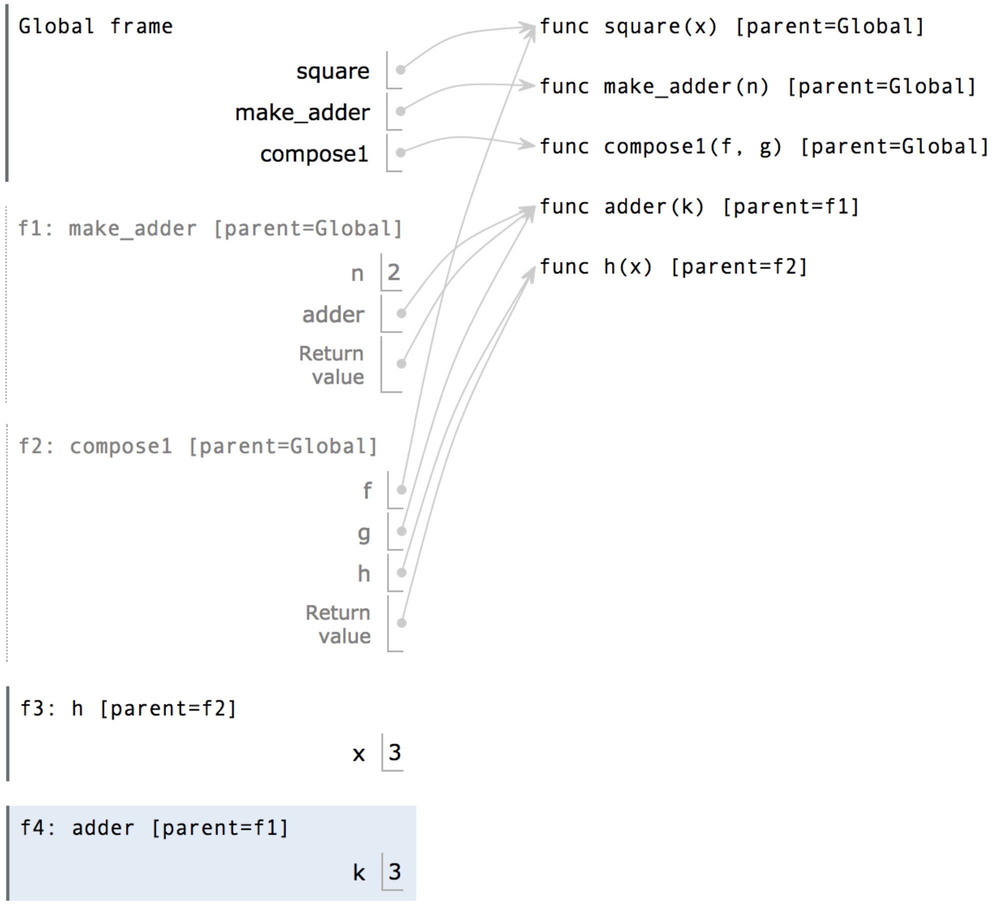
The Environment Diagram for Function Composition

```python
def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
    return adder

def compose1(f, g):
    def h(x):
        return f(g(x))
    return h

compose1(square, make_adder(2))(3)
```



Global frame
square
make_adder
compose1
func square(x) [parent=Global]
func make_adder(n) [parent=Global]
func compose1(f, g) [parent=Global]
func adder(k) [parent=f1]
func h(x) [parent=f2]
f1: make_adder [parent=Global]
n 2
adder
Return value
f2: compose1 [parent=Global]
f
g
h
Return value
f3: h [parent=f2]
x 3
f4: adder [parent=f1]
k 3

Interactive Diagram

The Environment Diagram for Function Composition

```
def square(x):
    return x * x

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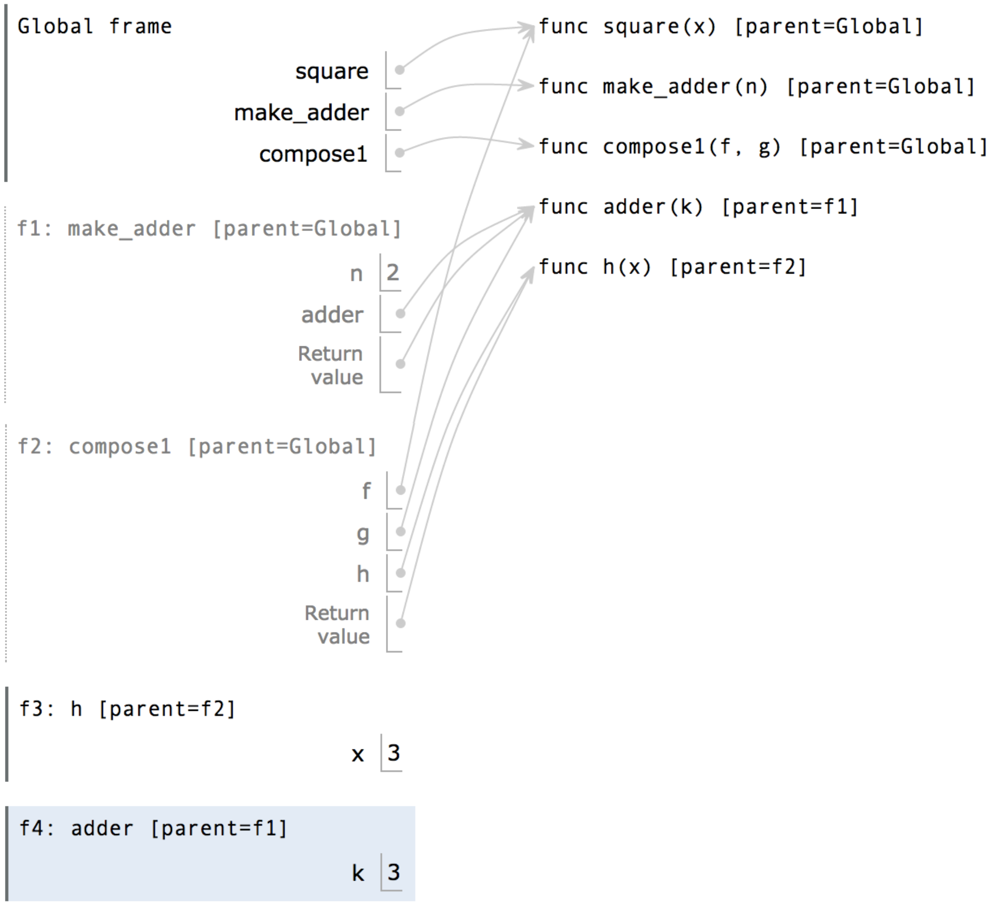

Global frame

square

make_adder

compose1

func square(x) [parent=Global]

func make_adder(n) [parent=Global]

func compose1(f, g) [parent=Global]

func adder(k) [parent=f1]

func h(x) [parent=f2]

f1: make_adder [parent=Global]

n	2
adder	
Return value	

f2: compose1 [parent=Global]

f

g

h

Return value

f3: h [parent=f2]

x	3

f4: adder [parent=f1]

k	3

Interactive Diagram

The Environment Diagram for Function Composition

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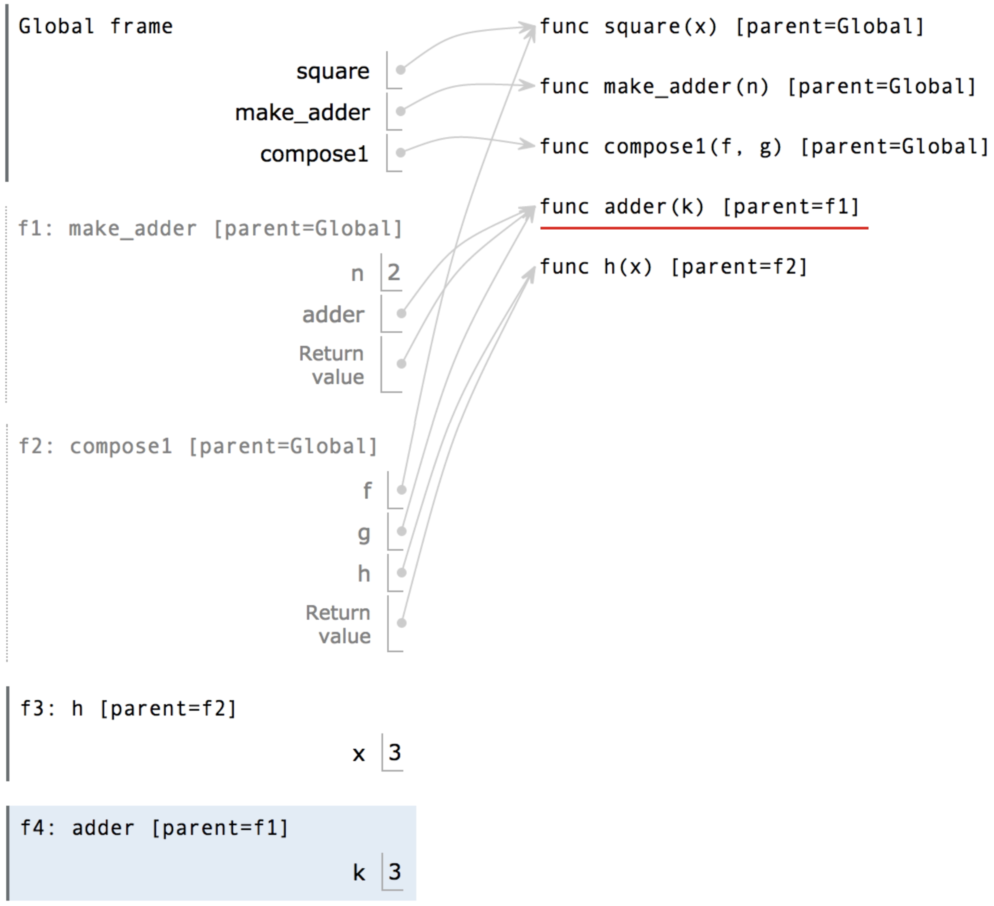

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n 2

adder

Return value

f2: compose1 [parent=Global]

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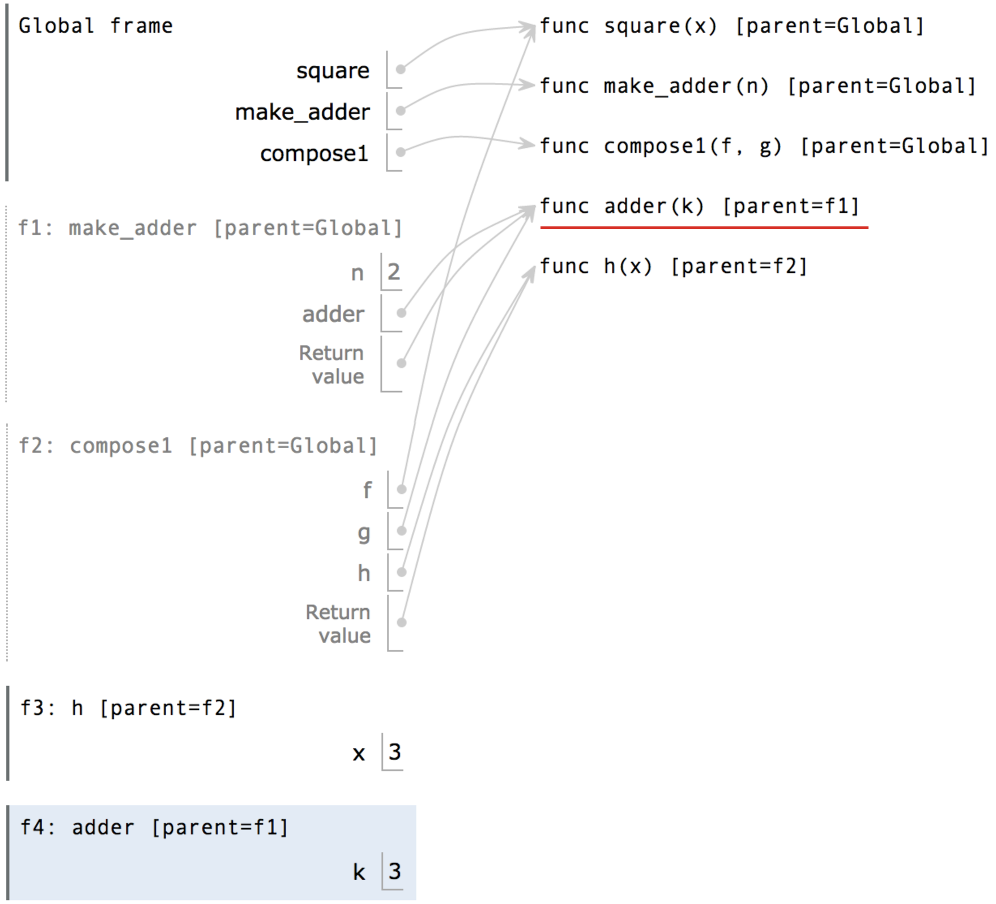

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compose1(square, make_adder(2))(3)
```
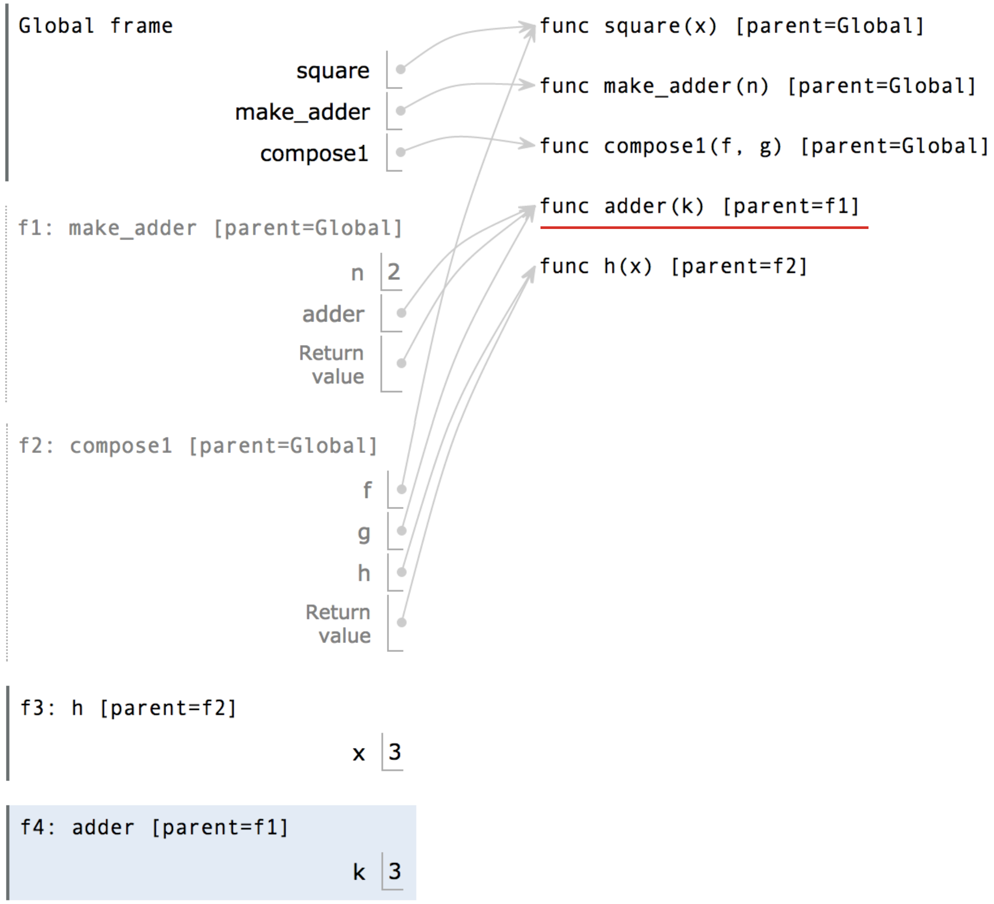

Return value of make_adder is an argument to compose1



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n 2

adder

Return value

f2: compose1 [parent=Global]

f

g

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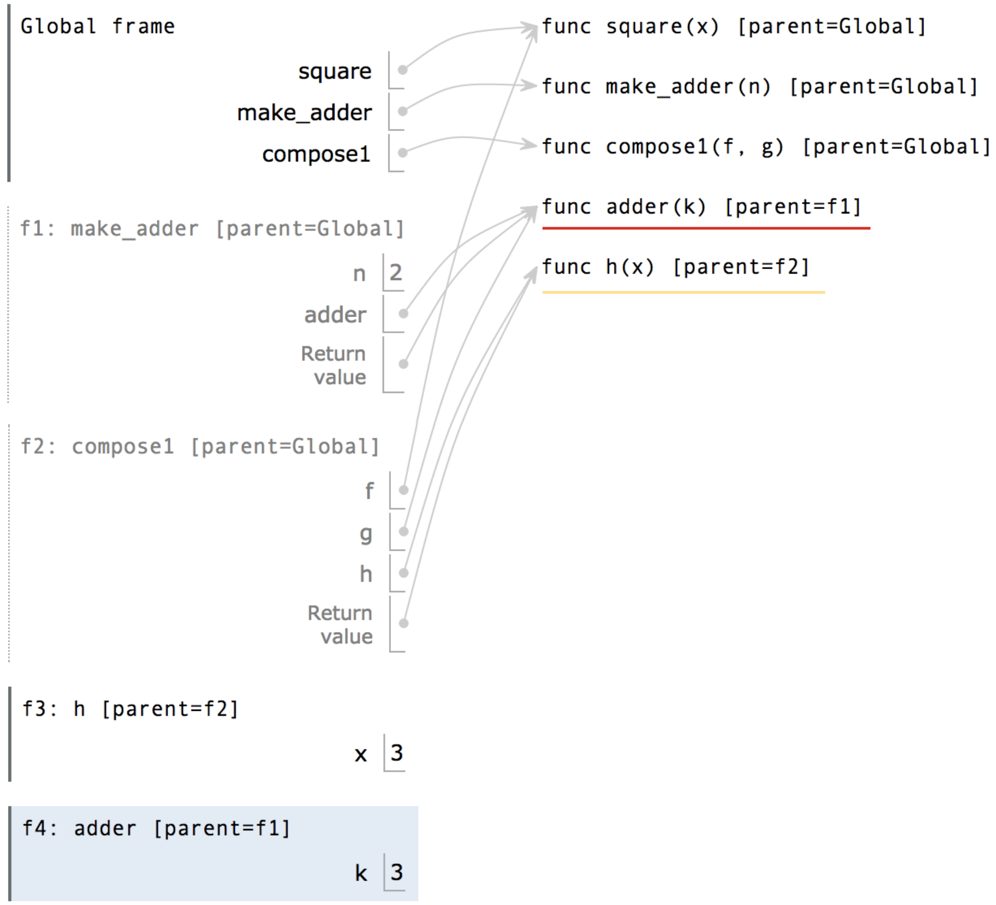

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n 2

adder

Return value

f2: compose1 [parent=Global]

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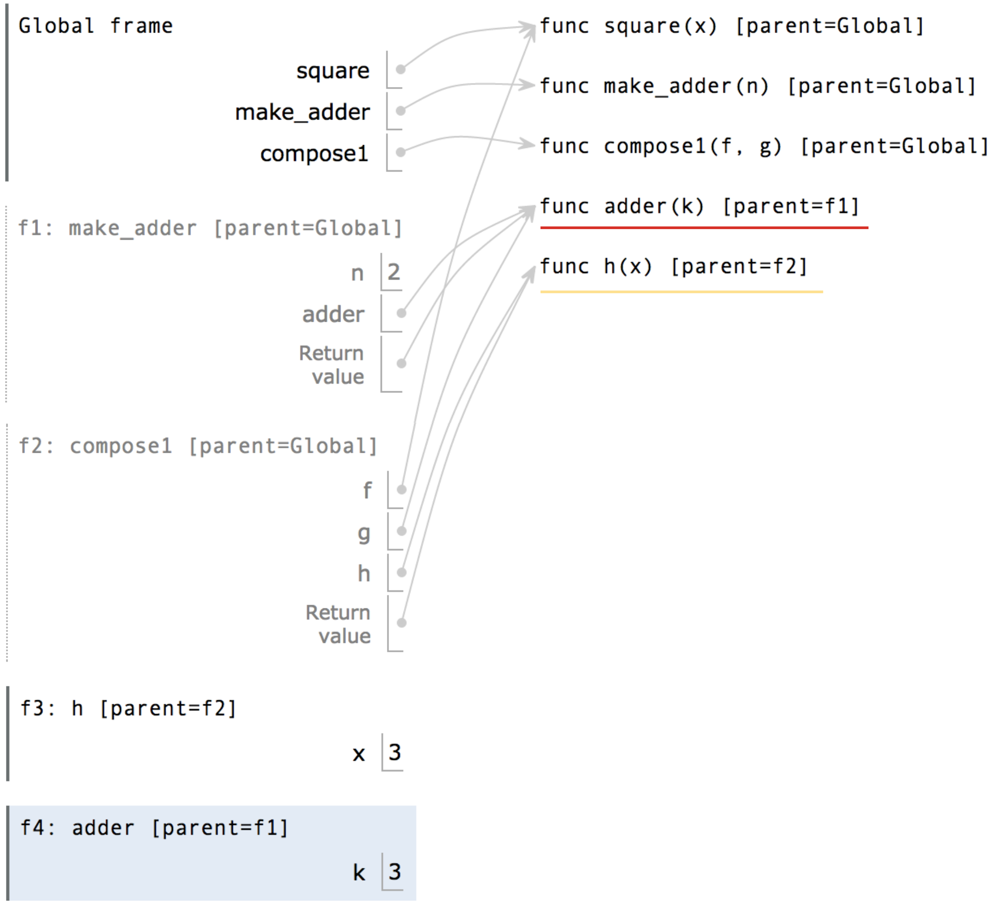

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n 2

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Return value

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f

g

h

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x 3

f4: adder [parent=f1]

k 3

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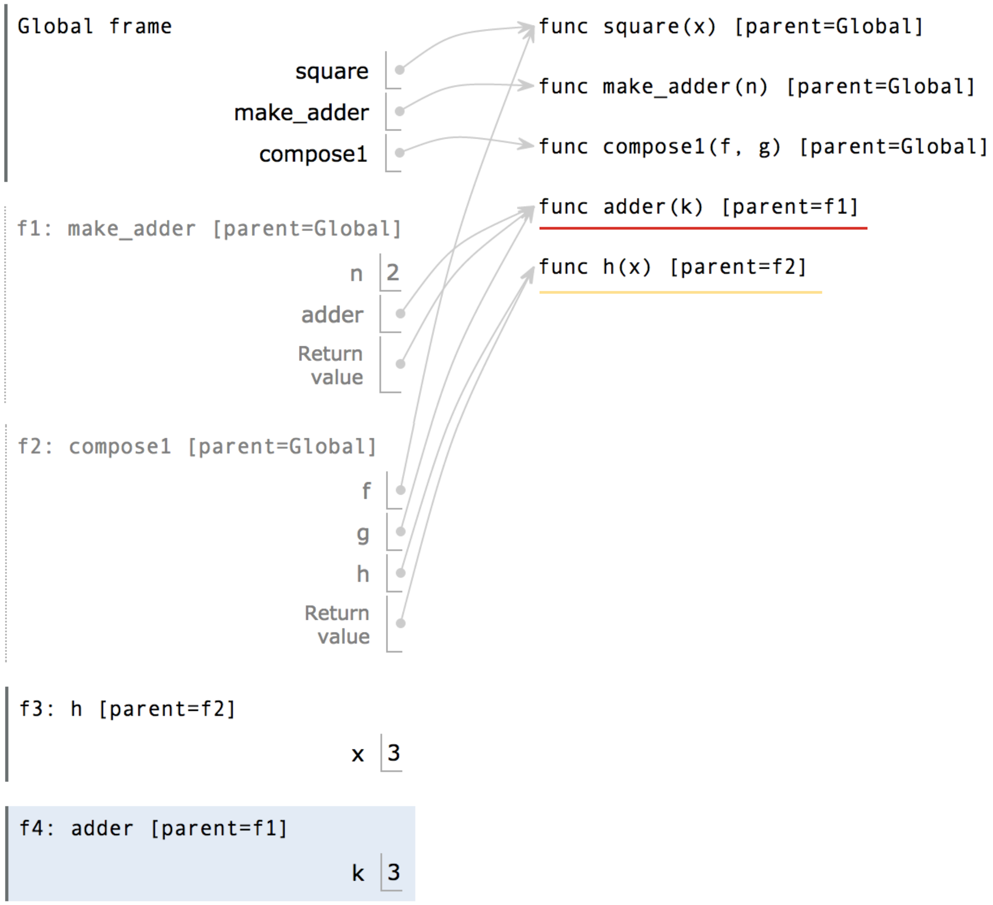

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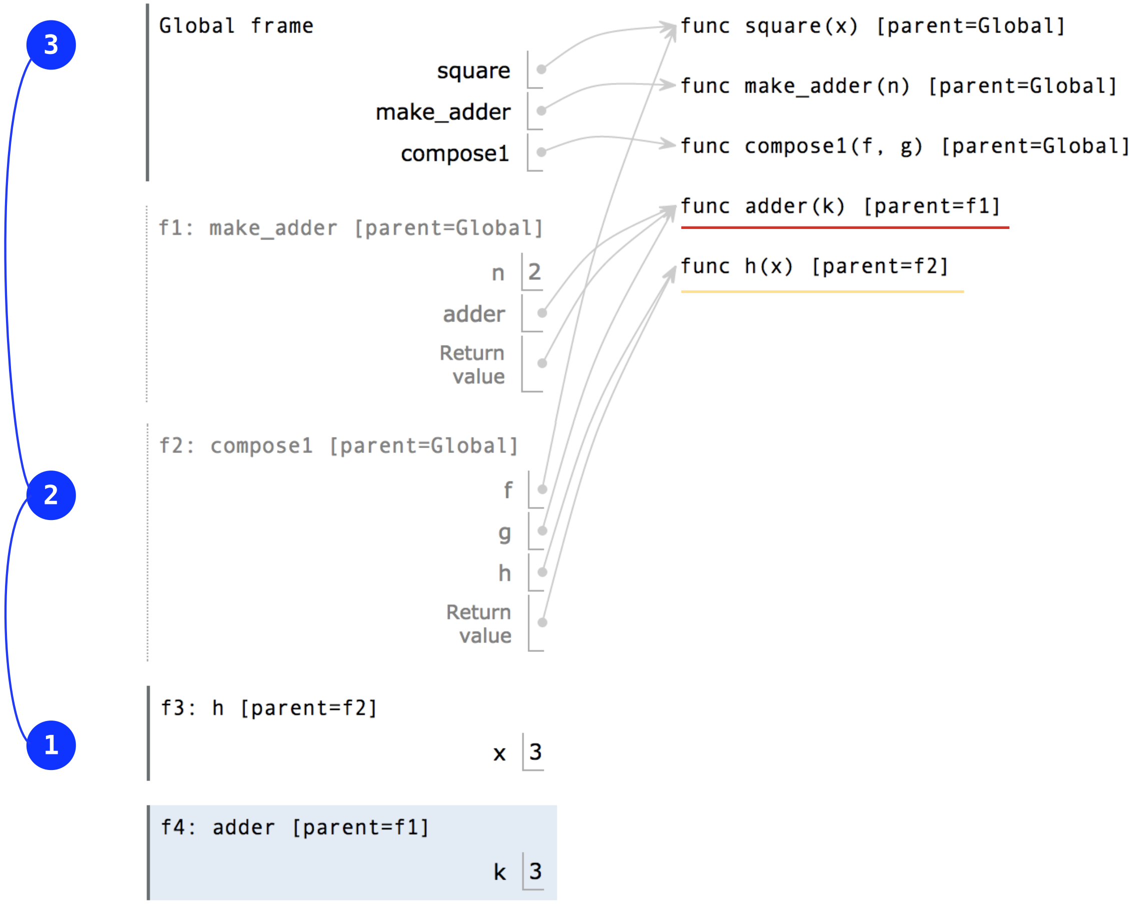

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adder
Return value

f2: compose1 [parent=Global]
f
g
h
Return value

f3: h [parent=f2]
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f4: adder [parent=f1]
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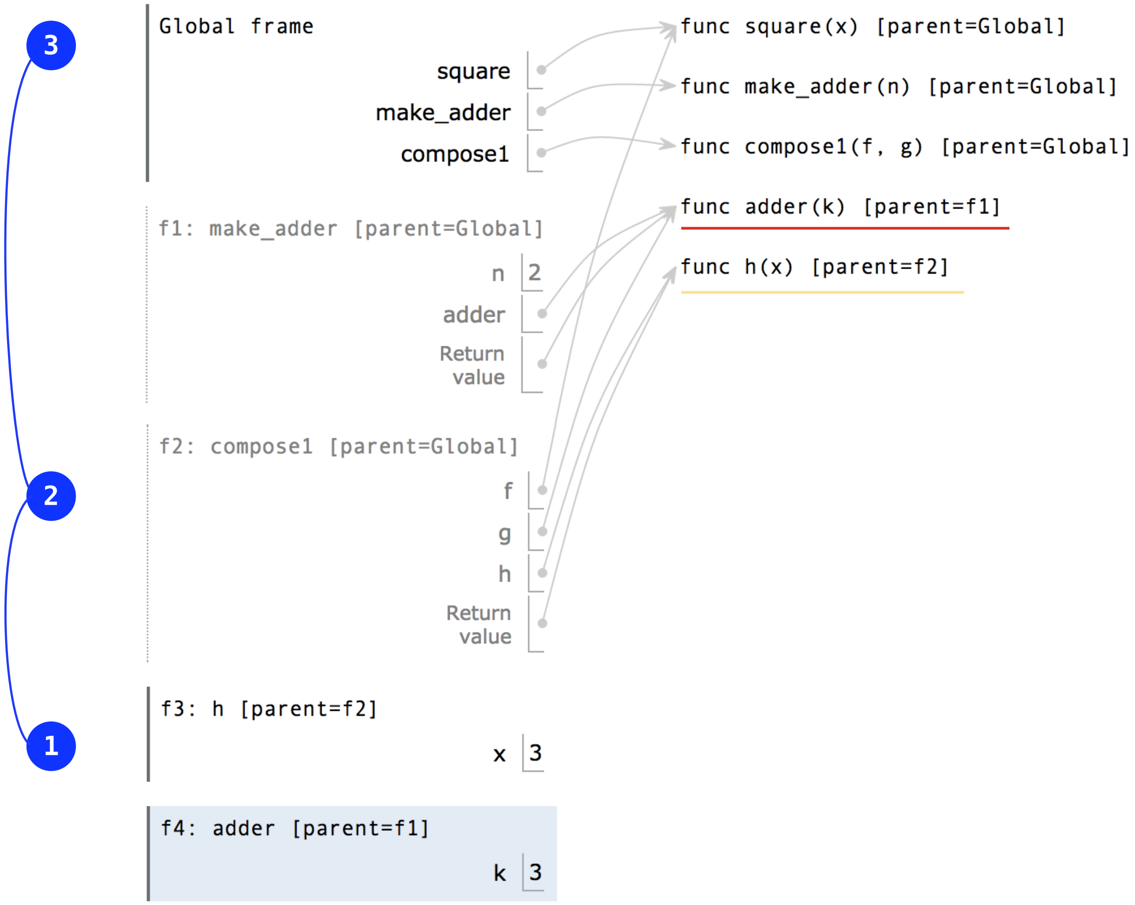

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adder
Return value

f2: compose1 [parent=Global]

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f3: h [parent=f2]

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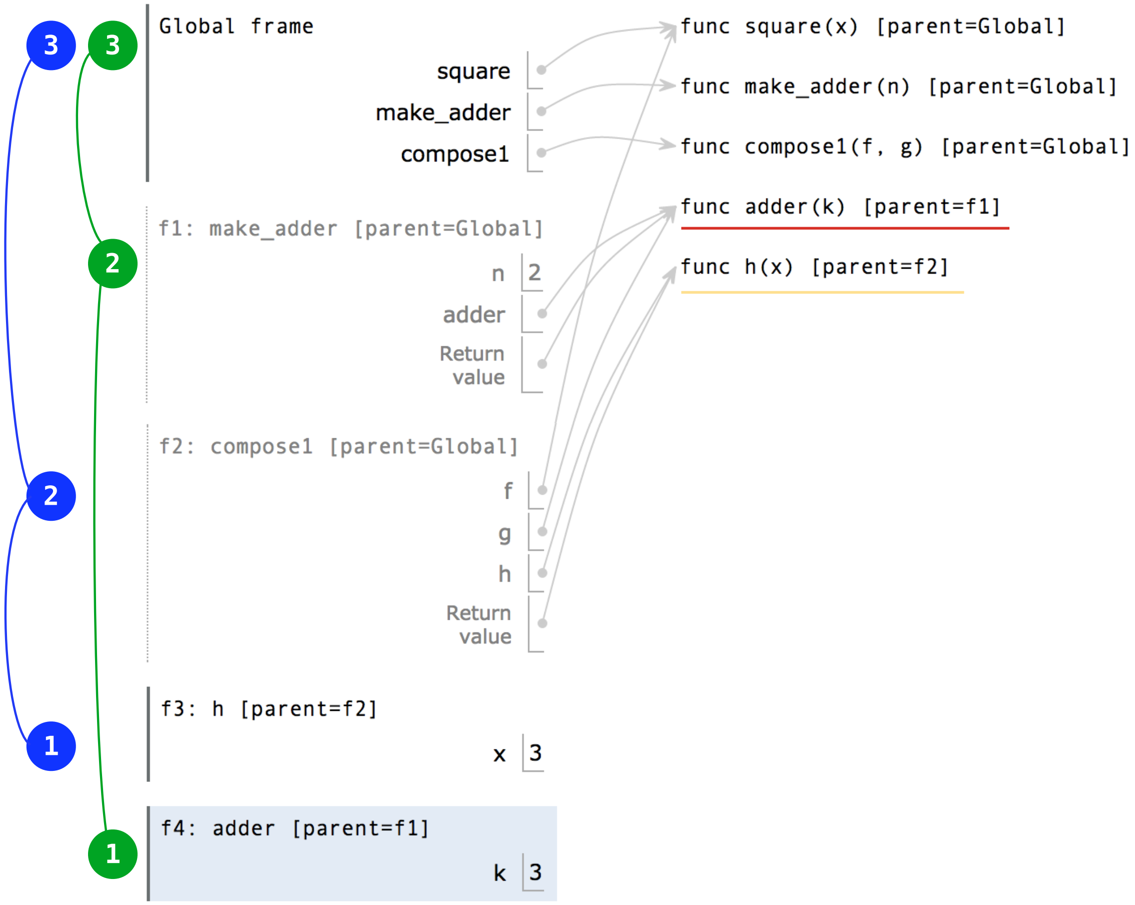

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func adder(k) [parent=f1]

func h(x) [parent=f2]

f1: make_adder [parent=Global]

n 2

adder

Return value

f2: compose1 [parent=Global]

f

g

h

Return value

f3: h [parent=f2]

x 3

f4: adder [parent=f1]

k 3

Interactive Diagram