Announcements

• Homework 3 due Wednesday 10/1 @ 11:59pm
  • Homework party on Monday evening, details TBD

• Optional Hog Contest entries due Wednesday 10/1 @ 11:59pm

• Composition scores for Project 1 will mostly be assigned this week
  • 3/3 is unusual on the first project
  • You can gain back composition points you lost on Project 1 by revising it (in November)

• Midterm 1 should be graded by Friday
  • Solutions to Midterm 1 will be posted after lecture

• Guerrilla section this Saturday 12–2 and 2:30–5 on recursion
Data
Data Types

Every value has a type

demo

Properties of native data types:

1. There are primitive expressions that evaluate to values of these types.
2. There are built-in functions, operators, and methods to manipulate those values.

Numeric Types in Python:

```python
>>> type(2)
<class 'int'> Represents integers exactly

>>> type(1.5)
<class 'float'> Represents real numbers approximately

>>> type(1+1j)
<class 'complex'>
```
Objects

(Demo)

• Objects represent information.
• They consist of data and behavior, bundled together to create abstractions.
• Objects can represent things, but also properties, interactions, & processes.
• A type of object is called a class; classes are first-class values in Python.
• Object-oriented programming:
  • A metaphor for organizing large programs
  • Special syntax that can improve the composition of programs
• In Python, every value is an object.
  • All objects have attributes.
  • A lot of data manipulation happens through object methods.
• Functions do one thing; objects do many related things.
Data Abstraction
Data Abstraction

• Compound objects combine objects together
  ▪ A date: a year, a month, and a day
  ▪ A geographic position: latitude and longitude

• An abstract data type lets us manipulate compound objects as units

• Isolate two parts of any program that uses data:
  ▪ How data are represented (as parts)
  ▪ How data are manipulated (as units)

• Data abstraction: A methodology by which functions enforce an abstraction barrier between representation and use
Rational Numbers

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost!

Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number $x$
- `numer(x)` returns the numerator of $x$
- `denom(x)` returns the denominator of $x$
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

\[
\frac{3}{2} + \frac{3}{5} = \frac{21}{10}
\]

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

\[
\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}
\]
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))

def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

- `rational(n, d)` returns a rational number \( \frac{n}{d} \)
- `numer(x)` returns the numerator of \( x \)
- `denom(x)` returns the denominator of \( x \)

These functions implement an abstract data type for rational numbers.
Pairs
Representing Pairs Using Lists

>>> pair = [1, 2]  
A list literal:
Comma-separated expressions in brackets

>>> pair
[1, 2]

>>> x, y = pair
"Unpacking" a list

>>> x
1
>>> y
2

>>> pair[0]
Element selection using the selection operator
1
>>> pair[1]
2

>>> from operator importgetitem
Element selection function

>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2

More lists next lecture
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]

```python
from fractions import gcd

def rational(n, d):
    """Construct a rational number x that represents n/d."""
    g = gcd(n, d)
    return [n//g, d//g]
```
Abstraction Barriers
## Abstraction Barriers

<table>
<thead>
<tr>
<th>Parts of the program that...</th>
<th>Treat rationals as...</th>
<th>Using...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use rational numbers to perform computation</td>
<td>whole data values</td>
<td>add_rational, mul_rational, rationals_are_equal, print_rational</td>
</tr>
<tr>
<td>Create rationals or implement rational operations</td>
<td>numerators and denominators</td>
<td>rational, numer, denom</td>
</tr>
<tr>
<td>Implement selectors and constructor for rationals</td>
<td>two-element lists</td>
<td>list literals and element selection</td>
</tr>
</tbody>
</table>

*Implementation of lists*
Violating Abstraction Barriers

Does not use constructors

Twice!

add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]

No selectors!

And no constructor!
Data Representations
What is Data?

• We need to guarantee that constructor and selector functions work together to specify the right behavior.

• Behavior condition: If we construct rational number $x$ from numerator $n$ and denominator $d$, then $\text{numer}(x)/\text{denom}(x)$ must equal $n/d$.

• An abstract data type is some collection of selectors and constructors, together with some behavior condition(s).

• If behavior conditions are met, then the representation is valid.

You can recognize abstract data types by their behavior, not by their class.
Behavior Conditions of a Pair

To implement our rational number abstract data type, we used a two-element list. But is that the only way to make pairs of values? No!

Constructors, selectors, and behavior conditions:

If a pair \( p \) was constructed from elements \( x \) and \( y \), then

- \( \text{select}(p, 0) \) returns \( x \), and
- \( \text{select}(p, 1) \) returns \( y \).

Together, selectors are the inverse of the constructor

Generally true of container types.

(Demo)

Not true for rational numbers because of GCD
def pair(x, y):
    """Return a function that represents a pair."""
    def get(index):
        if index == 0:
            return x
        elif index == 1:
            return y
    return get

def select(p, i):
    """Return the element at index i of pair p."""
    return p(i)

point = pair(2, 4)
select(point, 1)
Using a Functionally Implemented Pair

```python
>>> p = pair(1, 2)
>>> select(p, 0)
1
>>> select(p, 1)
2
```

If a pair `p` was constructed from elements `x` and `y`, then

- `select(p, 0)` returns `x`, and
- `select(p, 1)` returns `y`.

This pair representation is valid!

As long as we do not violate the abstraction barrier, we don't need to know that pairs are just functions.