Wednesday, October 15
Announcements

- Guerrilla Section 4 on Sunday 10/19: Object-oriented programming and recursive data
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- Homework 6 due Monday 10/20 @ 11:59pm (small)
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• Project 3 due Thursday 10/23 @ 11:59pm (BIG!)
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  Emphasis: mutable data, object-oriented programming, recursion, and recursive data
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  ▪ Emphasis: mutable data, object-oriented programming, recursion, and recursive data
  ▪ Have an course conflict? Fill out the conflict form!
  ▪ Review session on Saturday 10/26 3pm-4:30pm and 4:30pm-6pm in 2050 VLSB
Generic Functions of Multiple Arguments
More Generic Functions
More Generic Functions

A function might want to operate on multiple data types
More Generic Functions

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Last lecture:
More Generic Functions

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• Polymorphic functions using shared messages
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- Interfaces: collections of messages that have specific behavior conditions
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• Operator overloading
• Type dispatching
• Type coercion

What's different? Today's generic functions apply to multiple arguments that don't share a common interface.
Rational Numbers
class Rational:
    """A rational number represented as a numerator and denominator."
    ""
    def __init__(self, numer, denom):
        g = gcd(numer, denom)
        self.numer = numer // g
        self.denom = denom // g

    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)
class Rational:
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    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)

    def add(self, other):
        nx, dx = self.numer, self.denom
        ny, dy = other.numer, other.denom
        return Rational(nx * dy + ny * dx, dx * dy)

    def mul(self, other):
        numer = self.numer * other.numer
        denom = self.denom * other.denom
        return Rational(numer, denom)
Rational Numbers

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        nx, dx = self.numer, self.denom
        ny, dy = other.numer, other.denom
        return Rational(nx * dy + ny * dx, dx * dy)

    def mul(self, other):
        numer = self.numer * other.numer
        denom = self.denom * other.denom
        return Rational(numer, denom)

(Demo)
Complex Numbers
Complex Numbers

class Complex:
    def add(self, other):
        return ComplexRI(self.real + other.real,
                          self.imag + other.imag)
    def mul(self, other):
        return ComplexMA(self.magnitude * other.magnitude,
                         self.angle + other.angle)
Complex Numbers

class Complex:
    def __init__(self, real, imag):
        self.real = real
        self.imag = imag
    def add(self, other):
        return ComplexRI(self.real + other.real,
                         self.imag + other.imag)
    def mul(self, other):
        return ComplexMA(self.magnitude * other.magnitude,
                         self.angle + other.angle)

class ComplexRI(Complex):
    """A rectangular representation."""
    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)
Complex Numbers

class Complex:
    def __init__(self, real, imag):
        self.real = real
        self.imag = imag
    def add(self, other):
        return ComplexRI(self.real + other.real, self.imag + other.imag)
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    @property
def angle(self):
        return atan2(self.imag, self.real)

class ComplexMA(Complex):
    """A polar representation."""
    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle
    @property
def real(self):
        return self.magnitude * cos(self.angle)
    @property
def imag(self):
        return self.magnitude * sin(self.angle)
Complex Numbers

class Complex:
    def add(self, other):
        return ComplexRI(self.real + other.real,
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    def mul(self, other):
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def angle(self):
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class ComplexMA(Complex):
    """A polar representation."""
    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

@property
def real(self):
    return self.magnitude * cos(self.angle)

@property
def imag(self):
    return self.magnitude * sin(self.angle)

(Demo)
Cross-Type Arithmetic Examples

Currently, we can add rationals to rationals, but not rationals to complex numbers
Cross-Type Arithmetic Examples

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```python
>>> Rational(3, 14).add(Rational(2, 7))
Rational(1, 2)
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\[
\frac{3}{14} + \frac{2}{7}
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>>> Rational(3, 14).add(Rational(2, 7))
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>>> ComplexRI(0, 1).mul(ComplexMA(1, 0.5 * pi))
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Operators

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Rational(1, 2)
```

```python
>>> ComplexRI(0, 1) * ComplexMA(1, 0.5 * pi)
ComplexMA(1, 1 * pi)
```

```python
>>> Rational(1, 2) + ComplexRI(0.5, 2)
ComplexRI(1, 2)
```
Cross-Type Arithmetic Examples

Currently, we can add rationals to rationals, but not rationals to complex numbers

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>>> ComplexRI(0, 1) * ComplexMA(1, 0.5 * pi)
ComplexMA(1, 1 * pi)

>>> Rational(1, 2) + ComplexRI(0.5, 2)
ComplexRI(1, 2)

>>> ComplexMA(2, 0.5 * pi) * Rational(3, 2)
ComplexMA(3, 0.5 * pi)
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Cross-Type Arithmetic Examples

Currently, we can add rationals to rationals, but not rationals to complex numbers

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```

```
3
-- + 2
14 7
```

```python
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ComplexMA(1, 1 * pi)
```

```
i \cdot i
```

```python
>>> Rational(3, 14) + Rational(2, 7)
Rational(1, 2)
```

```
3
-- + 2
14 7
```

```python
>>> ComplexRI(0, 1) * ComplexMA(1, 0.5 * pi)
ComplexMA(1, 1 * pi)
```

```
i \cdot i
```

```python
>>> Rational(1, 2) + ComplexRI(0.5, 2)
ComplexRI(1, 2)
```

```
1
-- + (0.5 + 2 \cdot i)
2
```

```python
>>> ComplexMA(2, 0.5 * pi) * Rational(3, 2)
ComplexMA(3, 0.5 * pi)
```

```
2 \cdot i \cdot 3
- -
2
```
Cross-Type Arithmetic Examples

Currently, we can add rationals to rationals, but not rationals to complex numbers

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Operators

>>> Rational(3, 14) + Rational(2, 7)
Rational(1, 2)

>>> ComplexRI(0, 1) * ComplexMA(1, 0.5 * pi)
ComplexMA(1, 1 * pi)

Cross-type arithmetic

>>> Rational(1, 2) + ComplexRI(0.5, 2)
ComplexRI(1, 2)

>>> ComplexMA(2, 0.5 * pi) * Rational(3, 2)
ComplexMA(3, 0.5 * pi)
```

\[
\begin{align*}
\text{Rational}(3, 14) + \text{Rational}(2, 7) &= \frac{3}{14} + \frac{2}{7} \\
\text{ComplexRI}(0, 1) \cdot \text{ComplexMA}(1, 0.5 \cdot \pi) &= i \cdot i \\
\text{Rational}(3, 14) + \text{Rational}(2, 7) &= \frac{3}{14} + \frac{2}{7} \\
\text{ComplexRI}(0, 1) \cdot \text{ComplexMA}(1, 0.5 \cdot \pi) &= i \cdot i \\
\text{Rational}(1, 2) + \text{ComplexRI}(0.5, 2) &= \frac{1}{2} + (0.5 + 2 \cdot i) \\
\text{ComplexMA}(2, 0.5 \cdot \pi) \cdot \text{Rational}(3, 2) &= 2 \cdot i \cdot \frac{3}{2}
\end{align*}
\]
Special Method Names
Special Method Names in Python
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Certain names are special because they have built-in behavior
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These names always start and end with two underscores
Special Method Names in Python

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```python
__init__
```
Special Method Names in Python

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`__init__` Method invoked automatically when an object is constructed.
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__init__ Method invoked automatically when an object is constructed
__repr__
Special Method Names in Python

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`__init__` Method invoked automatically when an object is constructed

`__repr__` Method invoked to display an object as a string
Special Method Names in Python

Certain names are special because they have built-in behavior.

These names always start and end with two underscores:

- `__init__` - Method invoked automatically when an object is constructed.
- `__repr__` - Method invoked to display an object as a string.
- `__add__` -
Special Method Names in Python

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`__init__`          Method invoked automatically when an object is constructed
`__repr__`          Method invoked to display an object as a string
`__add__`           Method invoked to add one object to another
**Special Method Names in Python**

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Special Method Names in Python

Certain names are special because they have built-in behavior.

These names always start and end with two underscores:

- **__init__** — Method invoked automatically when an object is constructed
- **__repr__** — Method invoked to display an object as a string
- **__add__** — Method invoked to add one object to another
- **__bool__** — Method invoked to convert an object to True or False

```python
>>> zero, one, two = 0, 1, 2
```
Special Method Names in Python

Certain names are special because they have built-in behavior.

These names always start and end with two underscores

- `__init__`: Method invoked automatically when an object is constructed.
- `__repr__`: Method invoked to display an object as a string.
- `__add__`: Method invoked to add one object to another.
- `__bool__`: Method invoked to convert an object to True or False.

```python
>>> zero, one, two = 0, 1, 2
>>> one + two
3
```
Special Method Names in Python

Certain names are special because they have built-in behavior.

These names always start and end with two underscores:

- `__init__`: Method invoked automatically when an object is constructed.
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```python
>>> zero, one, two = 0, 1, 2
>>> one + two
3
>>> bool(zero), bool(one)
(False, True)
```
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- `__init__`  Method invoked automatically when an object is constructed
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Special Method Names in Python

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```py
__init__  Method invoked automatically when an object is constructed
__repr__  Method invoked to display an object as a string
__add__   Method invoked to add one object to another
__bool__  Method invoked to convert an object to True or False
```

```py
>>> zero, one, two = 0, 1, 2
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>>> zero, one, two = 0, 1, 2
>>> one + two
3
>>> bool(zero), bool(one)
(False, True)
```

```python
>>> zero, one, two = 0, 1, 2
>>> one.__add__(two)
3
>>> zero.__bool__(), one.__bool__()
(False, True)
```
Special Methods
Special Methods

Adding instances of user-defined classes invokes the `__add__` method
Special Methods

Adding instances of user-defined classes invokes the __add__ method

class Number:
    """A number."""
    def __add__(self, other):
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    def __mul__(self, other):
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Special Methods

Adding instances of user-defined classes invokes the __add__ method

class Number:
    """A number."""
    def __add__(self, other):
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class Rational(Number):
    def add(self, other):
        ...
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>>> Rational(1, 3) + Rational(1, 6)
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.want{10036.15161016213}
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We can also __add__ complex numbers, even with multiple representations (Demo)
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We can also __add__ complex numbers, even with multiple representations (Demo)


http://docs.python.org/py3k/reference/datamodel.html#special-method-names
Type Dispatching
The Independence of Data Types
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Data abstraction and class definitions keep types separate
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Some operations need access to the implementation of two different abstractions
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Data abstraction and class definitions keep types separate.

Some operations need access to the implementation of two different abstractions.

How do we add a complex number and a rational number together?

Rational numbers as numerators & denominators & Complex numbers as two-dimensional vectors

def add_complex_and_rational(c, r):
    """Return c + r for complex c and rational r."""
    return ComplexRI(c.real + r.numer/r.denom, c.imag)
Type Dispatching
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Define a different function for each possible combination of types for which an operation (e.g., addition) is valid
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Same tag: same interface
Defer to add method
All forms of cross-type addition for self
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adders = {("com", "rat"): add_complex_and_rational,
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(Demo)
Type Dispatching Analysis
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Minimal violation of abstraction barriers: we define cross-type functions as necessary
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**Type Dispatching Analysis**

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**Question:** How many cross-type implementations are required for \( m \) types and \( n \) operations?
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```plaintext
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```
  m         n         m \cdot n       m^2 \cdot n       m^2 \cdot n^2
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$m \cdot (m - 1) \cdot n$
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Type Coercion
Coercion
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Idea: Some types can be converted into other types
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Takes advantage of structure in the type system
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```python
def rational_to_complex(r):
    """Return complex equal to rational."""
    return ComplexRI(r.numer/r.denom, 0)
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Question: Is coercion exact?
Applying Operators with Coercion
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class Number:

Applying Operators with Coercion

class Number:
    def __add__(self, other):
        x, y = self.coerce(other)
        return x.add(y)
Applying Operators with Coercion

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Always defer to add method
Applying Operators with Coercion

class Number:
    def __add__(self, other):
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    def coerce(self, other):

        Always defer to add method
Applying Operators with Coercion

class Number:
    def __add__(self, other):
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        return x.add(y)

    def coerce(self, other):
        if self.type_tag == other.type_tag:
            return self, other

Always defer to add method
Applying Operators with Coercion

class Number:
    def add (self, other):
        x, y = self.coerce(other)
        return x.add(y)

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Same interface: no change required
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            # Always defer to add method
        else:
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class Number:
    def __add__(self, other):
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coercions = {('rat', 'com'): rational_to_complex}
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class Number:
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            return (self.coerce_to(other.type_tag), other)

    def coerce_to(self, other_tag):
        coercion_fn = self.coercions[(self.type_tag, other_tag)]
        return coercion_fn(self)

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(Demo)
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