61A Lecture 21

Monday, October 20
Announcements
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• Homework 6 is due Monday 10/20 @ 11:59pm
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• Project 3 is due Thursday 10/23 @ 11:59pm
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• Project 3 is due Thursday 10/23 @ 11:59pm

  - Project/Homework party on Monday 10/20 6pm–8pm in 155 Dwinelle
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• Project 3 is due Thursday 10/23 @ 11:59pm
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• Midterm 2 is on Monday 10/27 7pm–9pm
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  ▪ Project/Homework party on Monday 10/20 6pm–8pm in 155 Dwinelle

• Midterm 2 is on Monday 10/27 7pm–9pm
  ▪ Class Conflict? Fill out the conflict form at the top of http://cs61a.org
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• Homework 6 is due Monday 10/20 @ 11:59pm

• Project 3 is due Thursday 10/23 @ 11:59pm
  • Project/Homework party on Monday 10/20 6pm–8pm in 155 Dwinelle

• Midterm 2 is on Monday 10/27 7pm–9pm
  • Class Conflict? Fill out the conflict form at the top of http://cs61a.org
  • Review session on Saturday 10/25 3pm–6pm in 2050 VLSB
The Consumption of Time
Implementations of the same functional abstraction can require different amounts of time
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Problem: How many factors does a positive integer $n$ have?
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A factor \( k \) of \( n \) is a positive integer that evenly divides \( n \)
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Implementations of the same functional abstraction can require different amounts of time.

Problem: How many factors does a positive integer \( n \) have?

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def factors(n):

    Slow: Test each k from 1 through n
Implementations of the same functional abstraction can require different amounts of time.

**Problem:** How many factors does a positive integer $n$ have?

A factor $k$ of $n$ is a positive integer that evenly divides $n$.

```python
def factors(n):
    
    **Slow:** Test each $k$ from 1 through $n$
    
    **Fast:** Test each $k$ from 1 to square root $n$
    For every $k$, $n/k$ is also a factor!
```
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Greatest integer less than \( \sqrt{n} \)
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Active environments:

• Environments for any function calls currently being evaluated
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- Parent environments of functions named in active environments
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Active environments:

- Environments for any function calls currently being evaluated.
- Parent environments of functions named in active environments.

(Demo)

Interactive Diagram
Fibonacci Space Consumption
Fibonacci Space Consumption

fib(5)
Fibonacci Space Consumption

fib(5)

fib(3)
Fibonacci Space Consumption

```
   fib(5)
   /   
fib(3)   fib(4)
```
Fibonacci Space Consumption

```
fib(5)
  /    |
/fib(3)  \
  /      |
/fib(1)  fib(2)
      /    |
      /     |
      1      fib(0)  fib(1)
        /    |
        /     |
        0      1
```
Fibonacci Space Consumption

```
   fib(5)
  /     |
|       |
fib(3)  fib(4)
 /     /     |
|     |     |
fib(1) fib(2) fib(3)
 |     |
1 fib(0) fib(1)
 |   |     |
0 fib(0) fib(1) fib(2)
 |       |
0 fib(0) fib(1) fib(2)
 |       |
0 fib(0) fib(1) fib(2)
 |   |
0 fib(0) fib(1)
 |   |
0 fib(0) fib(1)
```

Fibonacci Space Consumption

Assume we have reached this step.
Fibonacci Space Consumption

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Fibonacci Space Consumption

Assume we have reached this step.
Fibonacci Space Consumption

Assume we have reached this step

Has an active environment
Can be reclaimed
Fibonacci Space Consumption

Assume we have reached this step

Has an active environment
Can be reclaimed
Hasn't yet been created
Order of Growth
Order of Growth
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A method for bounding the resources used by a function by the "size" of a problem
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\[ n: \text{ size of the problem} \]
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\[ n: \quad \text{size of the problem} \]

\[ R(n): \quad \text{measurement of some resource used (time or space)} \]
Order of Growth

A method for bounding the resources used by a function by the "size" of a problem

**n:** size of the problem

**R(n):** measurement of some resource used (time or space)

\[ R(n) = \Theta(f(n)) \]
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means that there are positive constants \( k_1 \) and \( k_2 \) such that
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\[ k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n) \]
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for all \( n \) larger than some minimum \( m \)
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Counting Factors
Counting Factors

Number of operations required to count the factors of $n$ using factors_fast is $\Theta(\sqrt{n})$
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Number of operations required to count the factors of n using factors_fast is $\Theta(\sqrt{n})$

def factors_fast(n):
    sqrt_n = sqrt(n)
    k, total = 1, 0
    while k < sqrt_n:
        if divides(k, n):
            total += 2
            k += 1
        if k * k == n:
            total += 1
    return total
Counting Factors

Number of operations required to count the factors of $n$ using $\text{factors\_fast}$ is $\Theta(\sqrt{n})$

To check the lower bound, we choose $k_1 = 1$:

```python
def factors_fast(n):
    sqrt_n = sqrt(n)
    k, total = 1, 0
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Number of operations required to count the factors of \( n \) using \texttt{factors_fast} is \( \Theta(\sqrt{n}) \)

To check the lower bound, we choose \( k_1 = 1 \):

- Statements outside the \texttt{while}: 4 or 5

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def factors_fast(n):
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To check the \textit{upper bound}

- Maximum statements executed: \( 5 + 4\sqrt{n} \)

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Assumption: every statement, such as addition-then-assignment using the += operator, takes some fixed number of operations to execute
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- Maximum statements executed: $5 + 4\sqrt{n}$
- Maximum operations required per statement: some $p$
- We choose $k_2 = 5p$ and $m = 25$

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Assumption: every statement, such as addition-then-assignment using the `+=` operator, takes some fixed number of operations to execute
Order of Growth of Counting Factors

Implementations of the same functional abstraction can require different amounts of time.

Problem: How many factors does a positive integer $n$ have?

A factor $k$ of $n$ is a positive integer that evenly divides $n$.

```python
def factors(n):
    
    **Slow:** Test each $k$ from 1 through $n$

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    Time Space
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    # Θ(\sqrt{n}) | Θ(1)
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<tr>
<td>( \Theta(\sqrt{n}) )</td>
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Assumption: integers occupy a fixed amount of space
Exponentiation
Exponentiation
Exponentiation

**Goal:** one more multiplication lets us double the problem size
Exponentiation

**Goal:** one more multiplication lets us double the problem size

```python
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)
```
Exponentiation

**Goal:** one more multiplication lets us double the problem size

```python
def exp(b, n):
    if n == 0:
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```

\[
b^n = \begin{cases} 
1 & \text{if } n = 0 \\
 b \cdot b^{n-1} & \text{otherwise}
\end{cases}
\]
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\]

\[
b^n = \begin{cases} 
1 & \text{if } n = 0 \\
\left(b^{\frac{1}{2}}n\right)^2 & \text{if } n \text{ is even} \\
b \cdot b^{n-1} & \text{if } n \text{ is odd}
\end{cases}
\]
Exponentiation

Goal: one more multiplication lets us double the problem size

\[
\begin{align*}
    b^n &= \begin{cases} 
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\end{align*}
\]

```python
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)

def square(x):
    return x**x

def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)
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\end{cases} \]

(Demo)
Exponentiation

**Goal:** one more multiplication lets us double the problem size

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def exp(b, n):
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    if n == 0:
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        return square(exp_fast(b, n//2))
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        return b * exp_fast(b, n-1)
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    if n == 0: 
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    elif n % 2 == 0:  
        return square(exp_fast(b, n//2))  
    else:  
        return b * exp_fast(b, n-1)
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        return b * exp_fast(b, n-1)
```

<table>
<thead>
<tr>
<th>Time</th>
<th>Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta(n)$</td>
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</tr>
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<td>$\Theta(\log n)$</td>
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</tbody>
</table>
Comparing Orders of Growth
Properties of Orders of Growth
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**Constants:** Constant terms do not affect the order of growth of a process
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*Outer: length of a*
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*If a and b are both length n, then overlap takes \( \Theta(n^2) \) steps.*
Comparing orders of growth (n is the problem size)
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\( \Theta(b^n) \)
Comparing orders of growth (n is the problem size)

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  - Incrementing the problem scales $R(n)$ by a factor

- $\Theta(n^2)$: Quadratic growth. E.g., `overlap`
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