61A Lecture 8

Friday, February 6
Announcements
Abstraction
Functional Abstractions
Functional Abstractions

def square(x):
    return mul(x, x)
Functional Abstractions

```python
def square(x):
    return mul(x, x)

def sum_squares(x, y):
    return square(x) + square(y)
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def square(x):
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What does sum_squares need to know about square?
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• Square takes one argument.
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What does sum_squares need to know about square?

• Square takes one argument. Yes
• Square has the intrinsic name square. No
• Square computes the square of a number.
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- Square computes the square of a number. **Yes**
- Square computes the square by calling `mul`.
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def square(x):
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**Functional Abstractions**

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If the name “square” were bound to a built-in function, `sum_squares` would still work identically.
Choosing Names
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Names typically don’t matter for correctness

but

they matter a lot for composition
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Names should convey the meaning or purpose of the values to which they are bound.
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Which Values Deserve a Name

Reasons to add a new name
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Repeated compound expressions:
Which Values Deserve a Name

**Reasons to add a new name**

*Repeated compound expressions:*

```python
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More Naming Tips

- Names can be long if they help document your code:

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average_age = average(age, students)
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is preferable to

```python
# Compute average age of students
aa = avg(a, st)
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  f, g, h - Usually functions
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Testing
Test-Driven Development
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Write the test of a function before you write the function.
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A test will clarify the domain, range, & behavior of a function.
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(Demo)
Currying
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def make_adder(n):
    return lambda k: n + k
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def make_adder(n):
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>>> make_adder(2)(3)
5
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Function Currying

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def make_adder(n):
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>>> add(2, 3)
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```

There's a general relationship between these functions.
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**Curry**: Transform a multi-argument function into a single-argument, higher-order function.
Decorators
Function Decorators

(Demo)
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(Demo)

@trace1
def triple(x):
    return 3 * x
Function Decorators

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Function decorator

Decorated function
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Why not just use this?
Review
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The print function returns None. It also displays its arguments (separated by spaces) when it is called.
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from operator import add, mul

def square(x):
    return mul(x, x)

print(print(5))
None

print(5)
5

print(print(5))
None
```

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def square(x):
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print(print(5))
None

print(5)
5

print(print(5))
None

This expression | Evaluates to | Interactive Output
--- | --- | ---
5 | 5 | 5
print(5) | None | 5
print(print(5)) | None | 5 None
What Would Python Print?

The print function returns None. It also displays its arguments (separated by spaces) when it is called.

```python
from operator import add, mul
def square(x):
    return mul(x, x)

print(print(5))  # None
print(5)         # None 5
print(print(5))  # None
```

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None
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The print function returns None. It also displays its arguments (separated by spaces) when it is called.

```python
from operator import add, mul

def square(x):
    return mul(x, x)

def delay(arg):
    print('delayed')
    def g():
        return arg
    return g
```

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<td>5</td>
</tr>
<tr>
<td></td>
<td>None</td>
<td>5 None</td>
</tr>
</tbody>
</table>

```python
print(5)
```

```
5
```

```python
print(print(5))
```

```
5
```

```python
def g():
    return arg
```

```
None
```
What Would Python Print?

The print function returns None. It also displays its arguments (separated by spaces) when it is called.

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from operator import add, mul

def square(x):
    return mul(x, x)

def delay(arg):
    print('delayed')
    return g

def g():
    return arg

return g
```

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<td>5</td>
</tr>
<tr>
<td>delay(delay)()(6)()</td>
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None

14
What Would Python Print?

The print function returns None. It also displays its arguments (separated by spaces) when it is called.

```python
from operator import add, mul

def square(x):
    return mul(x, x)

def delay(arg):
    print('delayed')
    def g():
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    return g

print(print(5))
print(delay(delay)()(6))()
```

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<td>5</td>
</tr>
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<td>delay(delay)()()(6)()</td>
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<td>5 None</td>
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Names in nested def statements can refer to their enclosing scope.
What Would Python Print?

The print function returns None. It also displays its arguments (separated by spaces) when it is called.

```python
from operator import add, mul

def square(x):
    return mul(x, x)

A function that takes any argument and returns a function that returns that arg

def delay(arg):
    print('delayed')
    def g():
        return arg
    return g

def square(x):
    return mul(x, x)

from operator import add, mul

def square(x):
    return mul(x, x)

print(print(5))
None
print(print(5))
None
print(5)
5
print(5)
5
None
5
None
```

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from operator import add, mul
def square(x):
    return mul(x, x)

def delay(arg):
    print('delayed')
    def g():
        return arg
    return g

print(print(5))
print(print(5))
```

This expression  |  Evaluates to  |  Interactive Output
--- | --- | ---
5 | 5 | 5
print(5) | None | 5
print(print(5)) | None | 5
```
def delay(delay)(6)()
```

None

A function that takes any argument and returns a function that returns that arg

Names in nested def statements can refer to their enclosing scope
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The print function returns None. It also displays its arguments (separated by spaces) when it is called.

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# Names in nested def statements can refer to their enclosing scope

delay(delay)()(6)()
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def delay(arg):
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Names in nested def statements can refer to their enclosing scope
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def square(x):
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# A function that takes any argument and returns a function that returns that arg

def delay(arg):
    print('delayed')
    def g():
        return arg
    return g

# This expression Evaluates to Interactive Output
5 5
print(5) None 5
print(print(5)) None 5
(delay(delay)()(6))() delayed
```
What Would Python Print?

The print function returns None. It also displays its arguments (separated by spaces) when it is called.

```python
from operator import add, mul
def square(x):
    return mul(x, x)
```

A function that takes any argument and returns a function that returns that arg

```python
def delay(arg):
    print('delayed')
    def g():
        return arg
    return g
```

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    def g():
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<tr>
<td>print(print(5))</td>
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<td>5 None</td>
</tr>
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<td>delay(delay)(6)()</td>
<td>delayed delayed delayed</td>
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def square(x):
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    def g():
        return arg
    return g

def g():
    return
c
```

A function that takes any argument and returns a function that returns that arg

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<tr>
<td>print(print(5))</td>
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<td>5</td>
</tr>
<tr>
<td>delay(delay)()()</td>
<td>6</td>
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A function that takes any argument and returns a function that returns that arg

def delay(arg):
    print('delayed')
    def g():
        return arg
    return g

delay(delay)(6)()

Names in nested def statements can refer to their enclosing scope

print(delay(print)()(4))
```

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<td>5</td>
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<tr>
<td>print(print(5))</td>
<td>None</td>
<td>5 None</td>
</tr>
<tr>
<td>delay(delay)(6)()</td>
<td>6</td>
<td>delayed 6</td>
</tr>
<tr>
<td>print(delay(print)()(4))</td>
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def square(x):
    return mul(x, x)
```

A function that takes any argument and returns a function that returns that arg

```python
def delay(arg):
    print('delayed')
    def g():
        return arg
    return g
```

Names in nested def statements can refer to their enclosing scope

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<td>delayed 6</td>
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A function that takes any argument and returns a function that returns that arg

def delay(arg):
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    return g

def g():
    return
    return

A function that takes any argument and returns a function that returns that arg

Names in nested def statements can refer to their enclosing scope

This expression		Evaluates to		Interactive Output

5		5		5

print(5)		None		5

print(print(5))	
None

print(delay(delay)()())

delayed

print(delay(print)()())

delayed

delayed

5

None

5

None

5

None

5

None

5

None

5
What Would Python Print?

The print function returns None. It also displays its arguments (separated by spaces) when it is called.

```python
from operator import add, mul
def square(x):
    return mul(x, x)
def delay(arg):
    print('delayed')
    def g():
        return arg
    return g

def g():
    return arg

print(print(5))

print(print(print(5)))

print(delay(print)()(4))
```

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<td>5</td>
</tr>
<tr>
<td>print(print(5))</td>
<td>None</td>
<td>5</td>
</tr>
<tr>
<td>delay(delay)()()()</td>
<td>6</td>
<td>delayed</td>
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<tr>
<td>print(delay(print)())()</td>
<td>delayed</td>
<td>delayed</td>
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<td></td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>delayed</td>
<td>delayed</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>None</td>
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The print function returns None. It also displays its arguments (separated by spaces) when it is called.

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from operator import add, mul

def square(x):
    return mul(x, x)

# A function that takes any argument and returns a function that returns that arg

def delay(arg):
    print('delayed')
    def g():
        return arg
    return g

# Names in nested def statements can refer to their enclosing scope

def g():
    return delay(6)

print(g())
```

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<td>print(print(5))</td>
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<td>delay(delay)()()</td>
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<td>delayed 6</td>
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<tr>
<td>delay(print)()()()</td>
<td>None</td>
<td>delayed 4 None</td>
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</table>
def horse(mask):
    horse = mask
    def mask(horse):
        return horse
    return horse(mask)

mask = lambda horse: horse(2)
horse(mask)
```python
def horse(mask):
    horse = mask
    def mask(horse):
        return horse
    return horse(mask)

mask = lambda horse: horse(2)
horse(mask)
```
```python
def horse(mask):
    horse = mask
    return horse

mask = lambda horse: horse(2)
horse(mask)
```

```
def horse(mask):
    horse = mask
    return horse

mask = lambda horse: horse(2)
horse(mask)
```
def horse(mask):
    horse = mask
    return horse
    
def mask(horse):
        return horse(mask)
    
mask = lambda horse: horse(2)

horse(mask)
def horse(mask):
    horse = mask
    def mask(horse):
        return horse
    return horse(mask)

mask = lambda horse: horse(2)
horse(mask)
def horse(mask):
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    def mask(horse):
        return horse
    return horse(mask)

mask = lambda horse: horse(2)
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horse(mask)
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mask = lambda horse: horse(2)
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def horse(mask):
    horse = mask
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    return horse(mask)
mask = lambda horse: horse(2)
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def horse(mask):
    horse = mask
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        return horse
    return horse(mask)

mask = lambda horse: horse(2)
horse(mask)
```python
def horse(mask):
    horse = mask
    def mask(horse):
        return horse
    return horse(mask)
mask = lambda horse: horse(2)
horse(mask)
```

Diagram:
```
Global frame
  horse
  mask

f1: horse [parent=Global]
  horse
  Return Value

f2: λ [parent=Global]
  horse
  Return Value

f3: mask [parent=f1]
  horse 2
  Return Value 2
```
def horse(mask):
    horse = mask
    def mask(horse):
        return horse
    return horse(mask)

mask = lambda horse: horse(2)
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def horse(mask):
    horse = mask
    def mask(horse):
        return horse
    return horse(mask)

mask = lambda horse: horse(2)
horse(mask)