Announcements
Data Abstraction
Data Abstraction
Data Abstraction

• Compound values combine other values together
Data Abstraction

- Compound values combine other values together
  - A date: a year, a month, and a day
Data Abstraction

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  ▪ A date: a year, a month, and a day
  ▪ A geographic position: latitude and longitude
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• Data abstraction lets us manipulate compound values as units
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- Isolate two parts of any program that uses data:
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• Data abstraction: A methodology by which functions enforce an abstraction barrier between representation and use
Rational Numbers
Rational Numbers

numerator
---------
denominator
Rational Numbers

\[
\text{numerator} \quad \frac{\text{numerator}}{\text{denominator}} \quad \text{denominator}
\]

Exact representation of fractions
Rational Numbers

[numerator]

_________

denominator

Exact representation of fractions

A pair of integers
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:
Rational Numbers

\[
\text{numerator} \quad \frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \text{rational}(n, d) \text{ returns a rational number } x
Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions
A pair of integers
As soon as division occurs, the exact representation may be lost! (Demo)
Assume we can compose and decompose rational numbers:

- \text{rational}(n, d) \text{ returns a rational number } x
- \text{numer}(x) \text{ returns the numerator of } x
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \text{rational}(n, d) \text{ returns a rational number } x
- \text{numer}(x) \text{ returns the numerator of } x
- \text{denom}(x) \text{ returns the denominator of } x
Rational Numbers

\[
\begin{array}{c}
\text{numerator} \\
\hline \\
\text{denominator}
\end{array}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

\begin{itemize}
  \item \texttt{rational(n, d)} returns a rational number \texttt{x}
  \item \texttt{numer(x)} returns the numerator of \texttt{x}
  \item \texttt{denom(x)} returns the denominator of \texttt{x}
\end{itemize}
Rational Numbers

Exact representation of fractions
A pair of integers
As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \text{rational}(n, d): \text{return}s a rational number \( x \)
- \text{num}(x): \text{return}s the numerator of \( x \)
- \text{den}(x): \text{return}s the denominator of \( x \)
Rational Number Arithmetic

Example

General Form
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5}
\]
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

Example

\[
\frac{nx}{dx} \times \frac{ny}{dy}
\]

General Form
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

Example

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]
Rational Number Arithmetic

Example

\[
\frac{3}{2} \ast \frac{3}{5} = \frac{9}{10}
\]

General Form

\[
\frac{nx}{dx} \ast \frac{ny}{dy} = \frac{nx \ast ny}{dx \ast dy}
\]
Rational Number Arithmetic

\[
\begin{align*}
\frac{3}{2} \times \frac{3}{5} &= \frac{9}{10} \\
\frac{3}{2} + \frac{3}{5} &= \frac{21}{10}
\end{align*}
\]

Example

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
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Example

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
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General Form
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\[
\frac{3}{2} + \frac{3}{5} = \frac{21}{10}
\]

Example

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

\[
\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}
\]
Rational Number Arithmetic Implementation

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
- denom(x) returns the denominator of x
Rational Number Arithmetic Implementation

def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))

- rational(n, d) returns a rational number x
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\text{rational}(n, d) \text{ returns a rational number } x
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- `rational(n, d)` returns a rational number `x`
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- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
- denom(x) returns the denominator of x

These functions implement an abstract representation for rational numbers.
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))

def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

- `rational(n, d)` returns a rational number $\frac{n}{d}$
- `numer(x)` returns the numerator of $x$:
  $$\frac{nx \times dy + ny \times dx}{dx \times dy}$$
- `denom(x)` returns the denominator of $x$:
  $$\frac{nx \times ny}{dx \times dy}$$

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    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
```

- `rational(n, d)` returns a rational number \( x \)
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    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rations_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

- `rational(n, d)` returns a rational number
- `numer(x)` returns the numerator of `x`
- `denom(x)` returns the denominator of `x`
Pairs
Representing Pairs Using Lists
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets

```python
[1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
```

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
>>> x
1
```

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
gap> pair = [1, 2]  
gap> pair
[1, 2]

gap> x, y = pair
gap> x
1
gap> y
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
1
2

>>> pair[0]
1

>>> pair[1]
2

>>> from operator importgetitem
```
Representing Pairs Using Lists

>>> pair = [1, 2]
>>> pair
[1, 2]

"Unpacking" a list

>>> x, y = pair
>>> x
1
>>> y
2

Element selection using the selection operator

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator
Representing Pairs Using Lists

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>>> x, y = pair
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1
>>> y
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>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator

Element selection function
Representing Pairs Using Lists

>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
1
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator

Element selection function

More lists next lecture
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
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    return [n, d]
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
Representing Rational Numbers

def rational(n, d):
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def rational(n, d):
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    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

(Demo)
Reducing to Lowest Terms

Example:
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]

\[
\frac{15}{6} \times \frac{1}{3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\begin{align*}
\frac{3}{2} \times \frac{5}{3} &= \frac{5}{2} \\
\frac{2}{5} + \frac{1}{10} &= \frac{5}{10} \\
\frac{15}{6} \times \frac{1/3}{1/3} &= \frac{5}{2}
\end{align*}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
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Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]

\[
\frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
\]

\[
\frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]

from fractions import gcd
Reduction to Lowest Terms

**Example:**

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]

```python
from fractions import gcd

def rational(n, d):
    return gcd(n, d)
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""

Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)

Reducing to Lowest Terms

Example:

\[
\begin{array}{ccc}
\frac{3}{2} & \times & \frac{5}{3} & = & \frac{5}{2} \\
\frac{2}{5} & + & \frac{1}{10} & = & \frac{1}{2}
\end{array}
\]

\[
\begin{array}{ccc}
\frac{15}{6} & \times & \frac{1}{3} & = & \frac{5}{2} \\
\frac{25}{50} & \times & \frac{1}{25} & = & \frac{1}{2}
\end{array}
\]
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)
    return [n//g, d//g]
Reducing to Lowest Terms

Example:

\[
\begin{align*}
\frac{3}{2} \times \frac{5}{3} &= \frac{5}{2} \quad & \frac{2}{5} + \frac{1}{10} &= \frac{1}{2} \\
\frac{15}{6} \times \frac{1/3}{1/3} &= \frac{5}{2} \\
\frac{25}{50} \times \frac{1/25}{1/25} &= \frac{1}{2}
\end{align*}
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Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \text{and} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \text{and} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
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def rational(n, d):
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(Demo)
Abstraction Barriers
Abstraction Barriers
### Abstraction Barriers

<table>
<thead>
<tr>
<th>Parts of the program that...</th>
<th>Treat rationals as...</th>
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### Abstraction Barriers

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Create rationals or implement rational operations
### Abstraction Barriers

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**Implementation of lists**
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*Implementation of lists*
Violating Abstraction Barriers

```python
add_rational([1, 2], [1, 4])

def divide_rational(x, y):
    return [x[0] * y[1], x[1] * y[0]]
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Violating Abstraction Barriers

Does not use constructors

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Twice!

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Violating Abstraction Barriers

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Data Representations
What is Data?
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(Demo)
Rationals Implemented as Functions
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
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            return d
    return select

def numer(x):
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Interactive Diagram:

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