Announcements
Mutable Functions
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100.
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

>>> withdraw(25)
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```python
>>> withdraw(25)
75
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A Function with Behavior That Varies Over Time

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A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

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Argument: amount to withdraw
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)
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>>> withdraw(25)
50

Argument: amount to withdraw
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)
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Argument: amount to withdraw

Second withdrawal of the same amount
Let's model a bank account that has a balance of $100.

Return value: remaining balance

>>> withdraw(25)
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Different return value!

>>> withdraw(25)
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Argument: amount to withdraw

Second withdrawal of the same amount.
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Argument: amount to withdraw
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>>> withdraw(25)  
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Different return value!

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Second withdrawal of the same amount

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A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

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>>> withdraw(25)
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Different return value!

Second withdrawal of the same amount

>>> withdraw(60)
'Insufficient funds'
A Function with Behavior That Varies Over Time

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Different return value!

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Where's this balance stored?

>>> withdraw = make_withdraw(100)
A Function with Behavior That Varies Over Time

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Within the parent frame of the function!
Let's model a bank account that has a balance of $100

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'Insufficient funds'

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Argument: amount to withdraw

Second withdrawal of the same amount

Where's this balance stored?

>>> withdraw = make_withdraw(100)

Within the parent frame of the function!

A function has a body and a parent environment
Persistent Local State Using Environments

**Interactive Diagram**

```
Global frame

make_withdraw
withdraw

func make_withdraw(balance) [parent=Global]

func withdraw(amount) [parent=f1]

f1: make_withdraw [parent=Global]

  balance 50
  withdraw
  Return value

f2: withdraw [parent=f1]

  amount 25
  Return value 75

f3: withdraw [parent=f1]

  amount 25
  Return value 50
```
Persistent Local State Using Environments

The parent frame contains the balance, the local state of the withdraw function.
Persistent Local State Using Environments

The parent frame contains the balance, the local state of the withdraw function.

Every call decreases the same balance by (a possibly different) amount.
Persistent Local State Using Environments

All calls to the same function have the same parent.

Every call decreases the same balance by (a possibly different) amount.

The parent frame contains the balance, the local state of the withdraw function.

Interactive Diagram
Reminder: Local Assignment

```python
def percent_difference(x, y):
    difference = abs(x-y)
    return 100 * difference / x

diff = percent_difference(40, 50)
```

**Global frame**

- `percent_difference` (parent=Global)

**f1: percent_difference [parent=Global]**

<p>| | |</p>
<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>40</td>
</tr>
<tr>
<td>y</td>
<td>50</td>
</tr>
<tr>
<td>difference</td>
<td>10</td>
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**Interactive Diagram**
Reminder: Local Assignment

```python
def percent_difference(x, y):
    difference = abs(x-y)
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diff = percent_difference(40, 50)
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Assignment binds name(s) to value(s) in the first frame of the current environment.

Interactive Diagram
Reminder: Local Assignment

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**Interactive Diagram**
Reminder: Local Assignment

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**Execution rule for assignment statements:**
Reminder: Local Assignment

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right
2. Bind the names on the left to the resulting values in the current frame
Non-Local Assignment & Persistent Local State
Non-Local Assignment & Persistent Local State

```python
def make_withdraw(balance):
```

7
Non-Local Assignment & Persistent Local State

def make_withdraw(balance):

    """Return a withdraw function with a starting balance."""

Non-Local Assignment & Persistent Local State

```python
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
```

7
Non-Local Assignment & Persistent Local State

def make_withdraw(balance):
    '''Return a withdraw function with a starting balance.'''

    def withdraw(amount):
        nonlocal balance
Non-Local Assignment & Persistent Local State

```python
def make_withdraw(balance):
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def withdraw(amount):
    nonlocal balance
    if amount > balance:
```
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def make_withdraw(balance):
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(Demo)
Non-Local Assignment
The Effect of Nonlocal Statements

nonlocal <name>
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**Effect:** Future assignments to that name change its pre-existing binding in the first non-local frame of the current environment in which that name is bound.
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Python Docs: an "enclosing scope"
The Effect of Nonlocal Statements

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*From the Python 3 language reference:*
The Effect of Nonlocal Statements

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From the Python 3 language reference:

Names listed in a nonlocal statement must refer to pre-existing bindings in an enclosing scope.
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http://docs.python.org/release/3.1.3/reference/simple_stmts.html#the-nonlocal-statement
The Effect of Nonlocal Statements

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[http://www.python.org/dev/peps/pep-3104/](http://www.python.org/dev/peps/pep-3104/)
The Many Meanings of Assignment Statements

\[ x = 2 \]
The Many Meanings of Assignment Statements

<table>
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<td>$x = 2$</td>
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## The Many Meanings of Assignment Statements

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**Effect**

Create a new binding from name "x" to object 2 in the first frame of the current environment

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- No nonlocal statement
- "x" is bound locally

Re-bind name "x" to object 2 in the first frame of the current environment

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- nonlocal x
- "x" is bound in a non-local frame
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Python Particulars
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Python pre-computes which frame contains each name before executing the body of a function.
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def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw

wd = make_withdraw(20)
wd(5)
```
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Interactive Diagram
Python Particulars

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w(5)
```

UnboundLocalError: local variable 'balance' referenced before assignment
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

withdraw = make_withdraw_list(100)
withdraw(25)
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

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**Interactive Diagram**
Mutable values & Persistent Local State

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Mutable Values & Persistent Local State

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Interactive Diagram
Multiple Mutable Functions

(Demo)
Referential Transparency, Lost

Interactive Diagram
Referential Transparency, Lost

- Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.
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\[
mul(add(2, mul(4, 6)), add(3, 5))
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Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

\[
\text{mul}(\text{add}(2, \text{mul}(4, 6)), \text{add}(3, 5))
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\[
\text{mul}(\text{add}(2, 24), \text{add}(3, 5))
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Expressions are *referentially transparent* if substituting an expression with its value does not change the meaning of a program.

mul(add(2, mul(4, 6)), add(3, 5))

mul(add(2, 24), add(3, 5))

mul(26, add(3, 5))
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*Mutation operations violate the condition of referential transparency because they do more than just return a value; they change the environment.*
Referential Transparency, Lost

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