

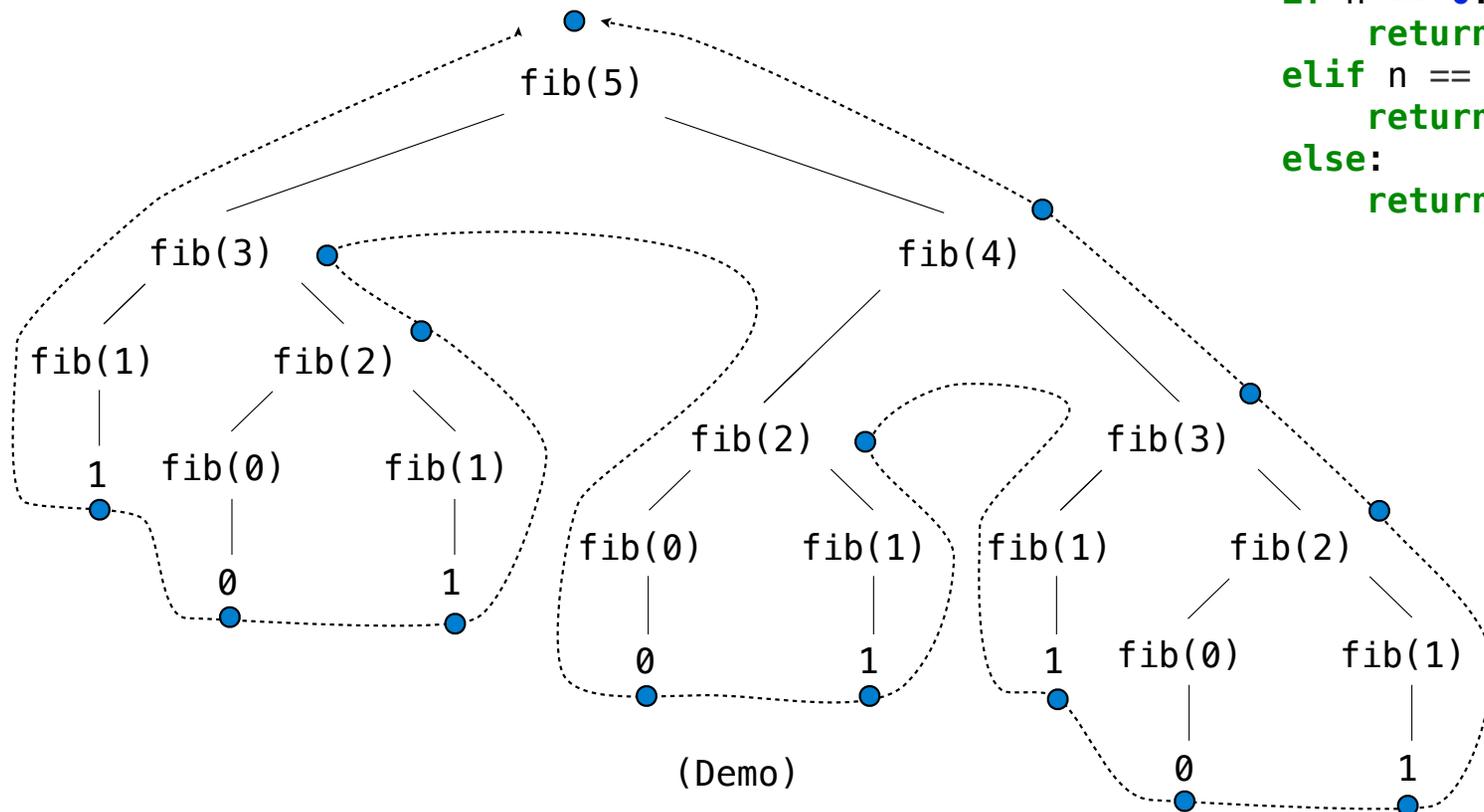
61A Lecture 18

Announcements

Measuring Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:



```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



Memoization

Memoization

Idea: Remember the results that have been computed before

```
def memo(f):  
    cache = {}  
    def memoized(n):  
        if n not in cache:  
            cache[n] = f(n)  
        return cache[n]  
    return memoized
```

Keys are arguments that map to return values

Same behavior as f, if f is a pure function

(Demo)

Tree Class

Tree Class

A Tree has an entry (any value) at its root and a list of branches

```
class Tree:
    def __init__(self, entry, branches=()):
        self.entry = entry
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

Built-in `isinstance` function:
returns True if `branch` has a class
that is or inherits from `Tree`

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        return Tree(left.entry + right.entry, (left, right))
```

(Demo)

Hailstone Trees

Hailstone Trees

Pick a positive integer n as the start

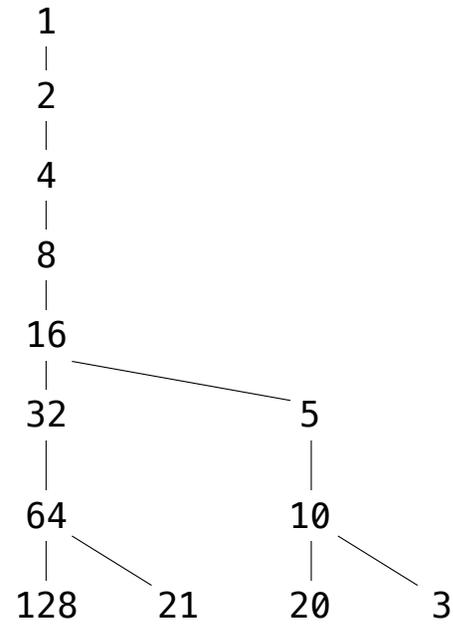
If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

```
def hailstone_tree(k, n=1):  
    """Return a Tree in which the paths from the  
    leaves to the root are all possible hailstone  
    sequences of length k ending in n."""
```

All possible n that start a
length-8 hailstone sequence



(Demo)