61A Lecture 26
Announcements
Scheme Recursive Art Contest: Start Early!
Scheme Recursive Art Contest: Start Early!

Fall 2012 Featherweight Winner
176 Scheme Tokens
Scheme Recursive Art Contest: Start Early!

Fall 2012 Featherweight Winner
176 Scheme Tokens

Fall 2013 Heavyweight Winner
1857 Scheme Tokens
Scheme Recursive Art Contest: Start Early!

Fall 2012 Featherweight Winner
176 Scheme Tokens

Fall 2013 Heavyweight Winner
1857 Scheme Tokens

Extra lecture on ray tracing
Monday 11/2 6:30pm
A1 Hearst Annex
Interpreting Scheme
The Structure of an Interpreter
The Structure of an Interpreter

Eval

Apply
The Structure of an Interpreter

Base cases:

Eval

Apply
The Structure of an Interpreter

Base cases:
  • Primitive values (numbers)
The Structure of an Interpreter

Base cases:
- Primitive values (numbers)

Recursive calls:
The Structure of an Interpreter

**Base cases:**
- Primitive values (numbers)

**Recursive calls:**
- Eval(operator, operands) of call expressions

---

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**Apply**
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Recursive calls:
• Eval(operator, operands) of call expressions
• Apply(procedure, arguments)
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Base cases:
• Built-in primitive procedures
The Structure of an Interpreter

Base cases:
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• Look up values bound to symbols

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Evaluate (Eval):
- Primitive values (numbers)
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Apply (Apply):
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Recursive calls:
- Eval(operator, operands) of call expressions
- Apply(procedure, arguments)
- Eval(sub-expressions) of special forms
The Structure of an Interpreter

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Recursive calls:
- Eval(operator, operands) of call expressions
- Apply(procedure, arguments)
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Base cases:
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Recursive calls:
- Eval(body) of user-defined procedures
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- Eval(operator, operands) of call expressions
- Apply(procedure, arguments)
- Eval(sub-expressions) of special forms

Requires an environment for symbol lookup

**Apply**

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**Apply**

Requires an environment for symbol lookup

Base cases:
- Built-in primitive procedures

Recursive calls:
- Eval(body) of user-defined procedures

Creates a new environment each time a user-defined procedure is applied
Special Forms
Scheme Evaluation
Scheme Evaluation

The `scheme_eval` function chooses behavior based on expression form:
Scheme Evaluation

The scheme_eval function chooses behavior based on expression form:

- Symbols are looked up in the current environment
Scheme Evaluation

The scheme_eval function choose behavior based on expression form:
• Symbols are looked up in the current environment
• Self-evaluating expressions are returned as values
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The scheme_eval function chooses behavior based on expression form:

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- Self-evaluating expressions are returned as values
- All other legal expressions are represented as Scheme lists, called combinations
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\[(\text{if } <\text{predicate}> <\text{consequent}> <\text{alternative}>)\]
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(\texttt{if \ <predicate> \ <consequent> \ <alternative>})

(\texttt{lambda\ (<formal-parameters>) \ <body>})
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(lambda (<formal-parameters>) <body>)

(define <name> <expression>)
```
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```
(if <predicate> <consequent> <alternative>)
```

```
(lambda (<formal-parameters>) <body>)
```

```
(define <name> <expression>)
```

```
(<operator> <operand 0> ... <operand k>)
```
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The `scheme_eval` function chooses behavior based on expression form:

- Symbols are looked up in the current environment
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\[
\text{(if \ <predicate> \ <consequent> \ <alternative>)}
\]

\[
\text{(lambda \ (<formal-parameters>) \ <body>)}
\]

\[
\text{(define \ <name> \ <expression>)}
\]

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\text{(<operator> \ <operand 0> \ ... \ <operand k>)}
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Special forms are identified by the first list element.
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\text{(if } \langle\text{predicate}\rangle \ \langle\text{consequent}\rangle \ \langle\text{alternative}\rangle) \\
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\text{(define} \langle\text{name}\rangle \ \langle\text{expression}\rangle) \\
\text{(operator} \ \langle\text{operand 0}\rangle \ \ldots \ \langle\text{operand k}\rangle)
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Any combination that is not a known special form is a call expression
Scheme Evaluation

The scheme_eval function choose behavior based on expression form:

• Symbols are looked up in the current environment
• Self-evaluating expressions are returned as values
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```
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s))))))
```

Special forms are identified by the first list element.

Any combination that is not a known special form is a call expression.
Scheme Evaluation

The scheme_eval function chooses behavior based on expression form:

- Symbols are looked up in the current environment
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\[
\begin{align*}
\text{(if} & \ <\text{predicate}> \ <\text{consequent}> \ <\text{alternative}> \\
\text{\lambda} & \ (<\text{formal-parameters}>) \ <\text{body}> \\
\text{(define} & \ <\text{name}> \ <\text{expression}> \\
\text{(operator} & \ <\text{operand} \, 0> \ \ldots \ <\text{operand} \, k>)
\end{align*}
\]

Special forms are identified by the first list element.

Any combination that is not a known special form is a call expression.

\[
\begin{align*}
\text{(define} & \ \text{(demo} \, s) \ \text{(if} \ \text{(null?} \, s) \, \text{'(3)} \ \text{(cons} \ \text{(car} \, s) \ \text{(demo} \ \text{(cdr} \, s))) \,)) \\
\text{(demo} & \ \text{(list} \, 1 \, 2))
\end{align*}
\]
Logical Forms
Logical Special Forms
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Logical forms may only evaluate some sub-expressions
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- **If** expression: \((\text{if } \text{<predicate>} \text{ <consequent> <alternative>})\)
Logical Special Forms

Logical forms may only evaluate some sub-expressions

- **If expression:** \((\text{if} \ <\text{predicate}>\ <\text{consequent}>\ <\text{alternative}>))\)
- **And and or:** \((\text{and} \ <\text{e1}>\ ...\ <\text{en}>),\ \ (\text{or} \ <\text{e1}>\ ...\ <\text{en}>))\)
Logical Special Forms

Logical forms may only evaluate some sub-expressions

- **If** expression: \( (\text{if } <\text{predicate}> <\text{consequent}> <\text{alternative}>) \)
- **And** and **or**: \( (\text{and } <e1> ... <en>), \quad (\text{or } <e1> ... <en>) \)
- **Cond** expression: \( (\text{cond } (<p1> <e1>) ... (<pn> <en>) \text{ (else } <e>)) \)
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- **If** expression:  \((\text{if } \text{<predicate> } \text{<consequent> } \text{<alternative>})\)

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- **Cond** expression:  \((\text{cond } (<\text{p1> } \text{<e1>}) ... (<\text{pn> } \text{<en>}) \ (\text{else } \text{<e>}))\)

The value of an if expression is the value of a sub-expression:
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- **If** expression: \((\text{if } <\text{predicate}> <\text{consequent}> <\text{alternative}>)\)
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The value of an if expression is the value of a sub-expression:

- Evaluate the predicate
Logical Special Forms

Logical forms may only evaluate some sub-expressions

- **If** expression: \((\text{if} \ <\text{predicate}> \ <\text{consequent}> \ <\text{alternative}>)\)
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- **Cond** expression: \((\text{cond} \ (<\text{p}1> \ <\text{e}1>) \ ... \ (<\text{p}n> \ <\text{e}n>) \ (\text{else} \ <\text{e}>)\))

The value of an if expression is the value of a sub-expression:

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Logical forms may only evaluate some sub-expressions

- **If** expression: \((\text{if } \langle\text{predicate}\rangle \ \langle\text{consequent}\rangle \ \langle\text{alternative}\rangle)\)
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- **Cond** expression: \((\text{cond } (\langle p_1 \rangle \ \langle e_1 \rangle) \ldots (\langle p_n \rangle \ \langle e_n \rangle) \ (\text{else } \langle e \rangle))\)

The value of an if expression is the value of a sub-expression:

- Evaluate the predicate
- Choose a sub-expression: \(<\text{consequent}>\) or \(<\text{alternative}>\)
- Evaluate that sub-expression to get the value of the whole expression
Logical Special Forms

Logical forms may only evaluate some sub-expressions

- **If** expression: \((\text{if} <\text{predicate}> <\text{consequent}> <\text{alternative}>)\)
- **And and or:** \((\text{and} <e_1> ... <e_n>), \text{(or} <e_1> ... <e_n>))\)
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The quote special form evaluates to the quoted expression, which is not evaluated
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(quote <expression>)
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(quote <expression>)  (quote (+ 1 2))  evaluates to the three-element Scheme list  (+ 1 2)
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+ 1 2

evaluates to the three-element Scheme list

The <expression> itself is the value of the whole quote expression
Quotation

The quote special form evaluates to the quoted expression, which is not evaluated

\[\text{(quote <expression>)} \quad \text{(quote (+ 1 2))}\]

\[\text{evaluates to the three-element Scheme list} \quad (+ 1 2)\]

The <expression> itself is the value of the whole quote expression

\'<expression> is shorthand for (quote <expression>)
Quotation

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(quote <expression>)  (quote (+ 1 2))  evaluates to the three-element Scheme list (+ 1 2)

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'(<expression> is shorthand for (quote <expression>))

(quote (1 2))  is equivalent to  '(1 2)
**Quotation**

The quote special form evaluates to the quoted expression, which is not evaluated

\[
\text{quote <expression>} \quad \text{quote (+ 1 2)} \quad \text{evaluates to the three-element Scheme list} \quad (+ 1 2)
\]

The <expression> itself is the value of the whole quote expression

'<expression> is shorthand for (quote <expression>)

\[
\text{quote (1 2)} \quad \text{is equivalent to} \quad '(1 2)
\]

The scheme_read parser converts shorthand ' to a combination that starts with quote
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(Demo)
Lambda Expressions
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Lambda expressions evaluate to user-defined procedures
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(lambda (<formal-parameters>) <body>)
Lambda Expressions

Lambda expressions evaluate to user-defined procedures

\[
\text{lambda} (<\text{formal-parameters}>) <\text{body}>
\]

\[
\text{lambda} (x) (* x x)
\]
Lambda Expressions

Lambda expressions evaluate to user-defined procedures

\[ \text{(lambda } (<\text{formal-parameters}>) \text{ <body>}) \]

\[ \text{(lambda } (x) (\times x x)) \]

class LambdaProcedure:

def __init__(self, formals, body, env):
    self.formals = formals
    self.body = body
    self.env = env
Lambda Expressions

Lambda expressions evaluate to user-defined procedures

\[
(\text{lambda} (<\text{formal-parameters}>) <\text{body}>)
\]

\[
(\text{lambda} (x) (* x x))
\]

class LambdaProcedure:

def __init__(self, formals, body, env):
    self.formals = formals  # A scheme list of symbols
    self.body = body
    self.env = env
Lambda Expressions

Lambda expressions evaluate to user-defined procedures

\[
\text{(lambda (<formal-parameters>) <body>)}
\]

\[
\text{(lambda (x) (* x x))}
\]

class LambdaProcedure:
    def __init__(self, formals, body, env):
        self.formals = formals  # A scheme list of symbols
        self.body = body        # A scheme list of expressions
        self.env = env
Lambda Expressions

Lambda expressions evaluate to user-defined procedures

```
(lambdasmall (<formal-parameters>) <body>)
```

```
(lambda small (x) (* x x))
```

class LambdaProcedure:
    def __init__(self, formals, body, env):
        self.formals = formals  # A scheme list of symbols
        self.body = body  # A scheme list of expressions
        self.env = env  # A Frame instance
Frames and Environments
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A frame represents an environment by having a parent frame
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Frames are Python instances with methods `lookup` and `define`.
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In Project 4, Frames do not hold return values.
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```
g: Global frame
   y   3
   z   5
```
Frames and Environments

A frame represents an environment by having a parent frame.

Frames are Python instances with methods **lookup** and **define**.

In Project 4, Frames do not hold return values.

```
g: Global frame
    y    3
    z    5

f1: [parent=g]
    x    2
    z    4
```
Frames and Environments

A frame represents an environment by having a parent frame.

Frames are Python instances with methods `lookup` and `define`.

In Project 4, Frames do not hold return values.

<table>
<thead>
<tr>
<th>g: Global frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
</tr>
<tr>
<td>z</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
<tr>
<td>z</td>
</tr>
</tbody>
</table>

(Demo)
Define Expressions
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Define binds a symbol to a value in the first frame of the current environment.
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\[(\text{define } <\text{name}> <\text{expression}>)\]
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(define <name> <expression>)

1. Evaluate the <expression>
Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

\[(\text{define} \ <\text{name}>\ <\text{expression}>\)]

1. Evaluate the \(<\text{expression}>\)
2. Bind \(<\text{name}>\) to its value in the current frame
Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

```
(define <name> <expression>)
```

1. Evaluate the <expression>

2. Bind <name> to its value in the current frame

```
(define x (+ 1 2))
```
Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

\[(\text{define } \text{<name>} \text{<expression>})\]

1. Evaluate the \text{<expression>}
2. Bind \text{<name>} to its value in the current frame

\[(\text{define } x (+ 1 2))\]

Procedure definition is shorthand of define with a lambda expression
Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

(\texttt{define} \texttt{<name>} \texttt{<expression>})

1. Evaluate the \texttt{<expression>}
2. Bind \texttt{<name>} to its value in the current frame

(\texttt{define} \texttt{x} (+ 1 2))

Procedure definition is shorthand of define with a lambda expression

(\texttt{define} \texttt{(\texttt{<name> \texttt{<formal parameters>}}) \texttt{<body>}})
Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

\[
\text{(define } \text{name} \text{ expression}) \\
\]

1. Evaluate the expression
2. Bind name to its value in the current frame

\[
\text{(define x (+ 1 2))} \\
\]

Procedure definition is shorthand of define with a lambda expression

\[
\text{(define (name formal parameters) body)} \\
\text{(define name (lambda formal parameters body))} \\
\]
Applying User-Defined Procedures
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To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the \texttt{env} attribute of the procedure.
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To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the `env` attribute of the procedure.

Evaluate the body of the procedure in the environment that starts with this new frame.
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Evaluate the body of the procedure in the environment that starts with this new frame:

```scheme
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s))))
```
Applying User-Defined Procedures

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```
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))))
```

<table>
<thead>
<tr>
<th>g: Global frame</th>
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</tr>
</thead>
<tbody>
<tr>
<td>demo</td>
<td></td>
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```

```
(demo (list 1 2))
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```
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s))))
  (demo (list 1 2)))
```

```
g: Global frame
    demo ___

LambdaProcedure instance [parent=g]
```

```
Pair   Pair
  1     2   nil
```
Applying User-Defined Procedures

To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the env attribute of the procedure.

Evaluate the body of the procedure in the environment that starts with this new frame.

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(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))))

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To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the \texttt{env} attribute of the procedure.

Evaluate the body of the procedure in the environment that starts with this new frame.

\begin{verbatim}
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))))

(demo (list 1 2))
\end{verbatim}
Eval/Apply in Lisp 1.5
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apply[fn;x;a] =
  [atom[fn] → [eq[fn;CAR] → caar[x];
    eq[fn;CDR] → cdar[x];
    eq[fn;CONS] → cons[car[x];cadr[x]];
    eq[fn;ATOM] → atom[car[x]];
    eq[fn;EQ] → eq[car[x];cadr[x]];
  T → apply[eval[fn;a];x;a]];

eq[car[fn];LAMBDA] → eval[caddr[fn];pairlis[cadr[fn];x;a]];
eq[car[fn];LABEL] → apply[caddr[fn];x;cons[cons[cadr[fn];
  caddr[fn]]];a]]

eval[e;a] = [atom[e] → cdr[assoc[e;a]];
  atom[car[e]] →
    [eq[car[e];QUOTE] → cadr[e];
    eq[car[e];COND] → evcon[cdr[e];a];
    T → apply[car[e];evlis[cdr[e];a];a];
  T → apply[car[e];evlis[cdr[e];a];a]]