Announcements
Dynamic Scope
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**Lexical scope:** The parent of a frame is the environment in which a procedure was defined
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**Lexical scope**: The parent of a frame is the environment in which a procedure was *defined*

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f  (λ (x) ...)
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f1: g [parent=global]

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f2: f [parent=global]
  x   6
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mu
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Special form to create dynamically scoped procedures (mu special form only exists in Project 4 Scheme)

```
4
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Global frame
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But... no for/while statements! Can we make basic iteration efficient? Yes!
Recursion and Iteration in Python

In Python, recursive calls always create new active frames

\[
\text{factorial}(n, k) \text{ computes: } n! \times k
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`factorial(n, k) computes: n! * k`

def factorial(n, k):
    if n == 0:
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How? Eliminate the middleman!

Should use resources like

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(Demo)
Tail Recursion Examples
Which Procedures are Tail Recursive?

Which of the following procedures run in constant space? $\Theta(1)$

;; Compute the length of s.
(define (length s)
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;; Return the nth Fibonacci number.
(define (fib n)
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  (if (= 1 n) 0 (fib-iter 1 2)))

;; Return whether s contains v.
(define (contains s v)
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Map and Reduce
Example: Reduce
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\[
\text{(define (reduce procedure s start)}\n\]
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\]

\[
\text{(reduce \ast \ '(3 4 5) 2)}
\]
Example: Reduce

\[
(\text{define} \ (\text{reduce} \ \text{procedure} \ s \ \text{start})
\]

\[
(\text{reduce} \ * \ '(3 \ 4 \ 5) \ 2)
\]

120
Example: Reduce

\[
\begin{align*}
\text{(define (reduce procedure s start)}
\end{align*}
\]

\[
\begin{align*}
\text{(reduce \ast \ '}(3 \ 4 \ 5) \ 2) & \quad 120 \\
\text{(reduce (lambda (x y) (cons y x)) \ '}(3 \ 4 \ 5) \ '(2))
\end{align*}
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Example: Reduce

(define (reduce procedure s start))

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(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2)) (5 4 3 2)
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\[
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\[
\text{(reduce procedure}
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Recursive call is a tail call

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Recursive call is a tail call
Space depends on what procedure requires

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Example: Map with Only a Constant Number of Frames
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(define (map procedure s))
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      'null
      (procedure (map procedure (cdr s)))))
```
Example: Map with Only a Constant Number of Frames

(define (map procedure s)
  (if (null? s)
      nil
      ...))
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1 Pair 2 Pair nil
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(map (lambda (x) (- 5 x)) (list 1 2))
```
Example: Map with Only a Constant Number of Frames

(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))
)

(map (lambda (x) (- 5 x)) (list 1 2))
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
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(map (lambda (x) (- 5 x)) (list 1 2))
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(map (lambda (x) (- 5 x)) (list 1 2))
```
Example: Map with Only a Constant Number of Frames

\[
\text{(define (map procedure s)}
\begin{align*}
\text{(if (null? s)} & \text{ nil) } \\
\text{ (cons (procedure (car s)))} & \text{ (map procedure (cdr s))) igr)}
\end{align*}
\]

(map (lambda (x) (- 5 x)) (list 1 2))
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))
```

```
(map (lambda (x) (- 5 x)) (list 1 2))
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```
(define (map procedure s)
  (if (null? s)
    nil
    (cons (procedure (car s))
          (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            ((map procedure (cdr s)))))
)

(map (lambda (x) (- 5 x)) (list 1 2))
```
Example: Map with Only a Constant Number of Frames

\[
\begin{align*}
\text{(define (map procedure s)} & \quad \text{(define (map procedure s)} \\
\quad \text{(if (null? s)} & \quad \text{(define (map-reverse s m)} \\
\quad \quad \text{nil)} & \quad \text{(if (null? s)} \\
\quad \quad \text{(cons (procedure (car s)))} & \quad \text{(map procedure (cdr s))))} \\
\quad \quad \text{(map procedure (cdr s)))}) & \quad (\text{nil)} \\
\end{align*}
\]

(map (lambda (x) (- 5 x)) (list 1 2))
Example: Map with Only a Constant Number of Frames

```scheme
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```scheme
(define (map-reverse s m)
  (define (map procedure s)
    (if (null? s)
        m
        (cons (procedure (car s))
              (map procedure (cdr s)))))

(map-reverse (map (lambda (x) (- 5 x)) (list 1 2)) 17)
```
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s) m)))
```
Example: Map with Only a Constant Number of Frames

(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))

(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s)))))

17
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s)) m)))))
```
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s))
                         m))
      (reverse (map-reverse s nil))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s))
                         m))
      (reverse (map-reverse s nil))))

(map (lambda (x) (- 5 x)) (list 1 2))
```
Example: Map with Only a Constant Number of Frames

```scheme
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))
)

(map (lambda (x) (- 5 x)) (list 1 2))
```

```scheme
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))
)

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s))
                         m))
    ))

(reverse (map-reverse s nil)))
```
Example: Map with Only a Constant Number of Frames

```scheme
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```scheme
(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s))
                         m))))

(reverse (map-reverse s nil))
```
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
    nil
    (cons (procedure (car s))
      (map procedure (cdr s))))
)

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
    nil
    (cons (procedure (car s))
      (map procedure (cdr s))))
)

(define (map-reverse s m)
  (if (null? s)
    m
    (map-reverse (cdr s)
      (cons (procedure (car s))
        m)))
)

(reverse (map-reverse s nil)))
```

```
(define (reverse s)
  (define (reverse-iter s r)
    )

(reverse (map-reverse s nil)))
```
Example: Map with Only a Constant Number of Frames

\[
\text{(define (map procedure s)}
\text{  (if (null? s))}
\text{    nil}
\\text{  (cons (procedure (car s))}
\text{    (map procedure (cdr s)))))}
\text{)}
\]
Example: Map with Only a Constant Number of Frames

(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
Example: Map with Only a Constant Number of Frames

(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
Example: Map with Only a Constant Number of Frames

\[
\begin{align*}
(\text{define } (\text{map } \text{procedure } s)) \\
&\begin{cases} 
\text{if } (\text{null? } s) \\
\text{nil} \\
(\text{cons } (\text{procedure } (\text{car } s)) \\
(\text{map } \text{procedure } (\text{cdr } s))) \\
\end{cases}
\end{align*}
\]

\[
(\text{map } (\lambda (x) (- 5 x)) \text{ (list 1 2)})
\]
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s))
                         m)))

(reverse (map-reverse s nil)))
```

```
(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s)
        r
        (reverse-iter (cdr s)
                      (cons (car s) r)))

(reverse-iter s nil))
```
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s))
                         m))))

(reverse (map-reverse s nil))
```

```
(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s)
        r
        (reverse-iter (cdr s)
                      (cons (car s) r)) ) )

(reverse-iter s nil))
```
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s)) m))))

(reverse (map-reverse s nil)))
```

```
(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s)
        r
        (reverse-iter (cdr s)
                      (cons (car s) r))
            )
    )

(reverse-iter s nil))
```
Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
           (map procedure (cdr s)))))

(map (lambda (x) (- 5 x)) (list 1 2))
```

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
           (map procedure (cdr s)))))

(define (map-reverse s m)
  (if (null? s)
      m
      (map-reverse (cdr s)
                   (cons (procedure (car s))
                          m)))

(reverse (map-reverse s nil)))

(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s)
        r
        (reverse-iter (cdr s)
                      (cons (car s) r))
      ))

(reverse-iter s nil))
```
Example: Map with Only a Constant Number of Frames

\[
\text{(define (map procedure s)} \\
\text{  (if (null? s)} \\
\text{    nil} \\
\text{    (cons (procedure (car s))} \\
\text{       (map procedure (cdr s)))))} \\
\text{(map (lambda (x) (- 5 x)) (list 1 2))}
\]

\[
\text{(define (map-reverse s m)} \\
\text{  (if (null? s)} \\
\text{    m} \\
\text{    (map-reverse (cdr s)} \\
\text{      (cons (procedure (car s))} \\
\text{        m))}) \\
\text{(reverse (map-reverse s nil)))}
\]

\[
\text{(define (reverse s)} \\
\text{  (define (reverse-iter s r)} \\
\text{    (if (null? s)} \\
\text{      r} \\
\text{      (reverse-iter (cdr s)} \\
\text{        (cons (car s) r))))} \\
\text{  (reverse-iter s nil))}
\]
General Computing Machines
An Analogy: Programs Define Machines
An Analogy: Programs Define Machines

Programs specify the logic of a computational device
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

factorial
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

\[
\text{factorial} = \text{factorial} \times 1 \quad \text{with} \quad 1
\]

Diagram: A flowchart illustrating the logic of computing the factorial function.
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

5 → factorial → 1 → 1

- → factorial → 1 → * →
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

\[ 5! = 5 \times 4! \]

![Diagram of the factorial function]

\[ 5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 1 \]

\[ 5 = 4 \times 3! \]

\[ 4 = 3 \times 2! \]

\[ 3 = 2 \times 1! \]

\[ 2 = 1 \times 0! \]

\[ 1! = 1 \]

\[ 0! = 1 \]
Interpreters are General Computing Machine
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

\[
\text{(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
\]

\[
5 \text{ Scheme Interpreter} \rightarrow 120
\]
**Interpreters are General Computing Machine**

An interpreter can be parameterized to simulate any machine

\[ (\text{define} \ (\text{factorial} \ n) \ (\text{if} \ (\text{zero?} \ n) \ 1 \ (* \ n \ (\text{factorial} \ (- \ n \ 1)))))) \]

Our Scheme interpreter is a universal machine
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

```
(define factorial n)
(if (zero? n) 1 (* n (factorial (- n 1)))))
```

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself.
Interpreters are General Computing Machine

An interpreter can be parameterized to simulate any machine

```
(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1))))
```

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Internally, it is just a set of evaluation rules