# 61A Extra Lecture 11

# Announcements

# The Logic Language

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- Expressions are facts or queries, which contain relations
- Expressions and relations are Scheme lists
- For example, **(likes john dogs)** is a relation

# Simple Facts

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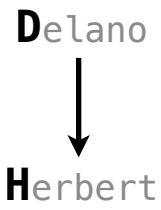

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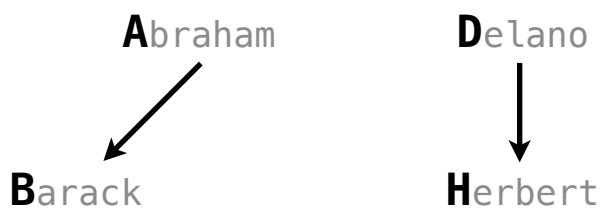

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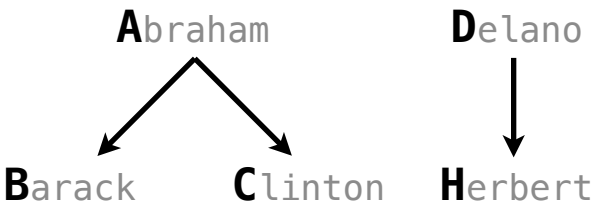

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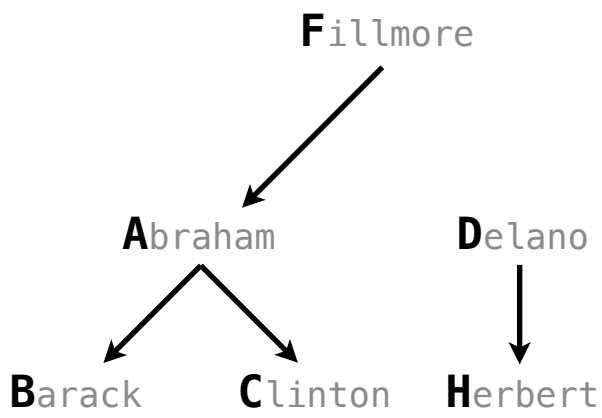

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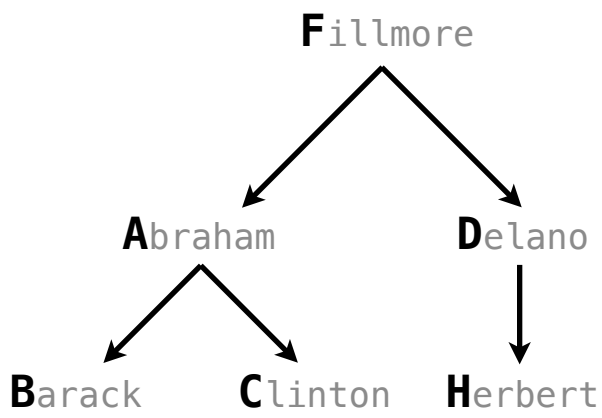

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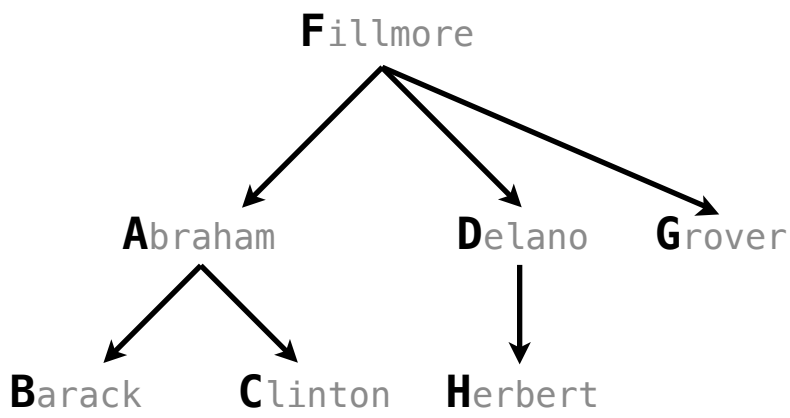

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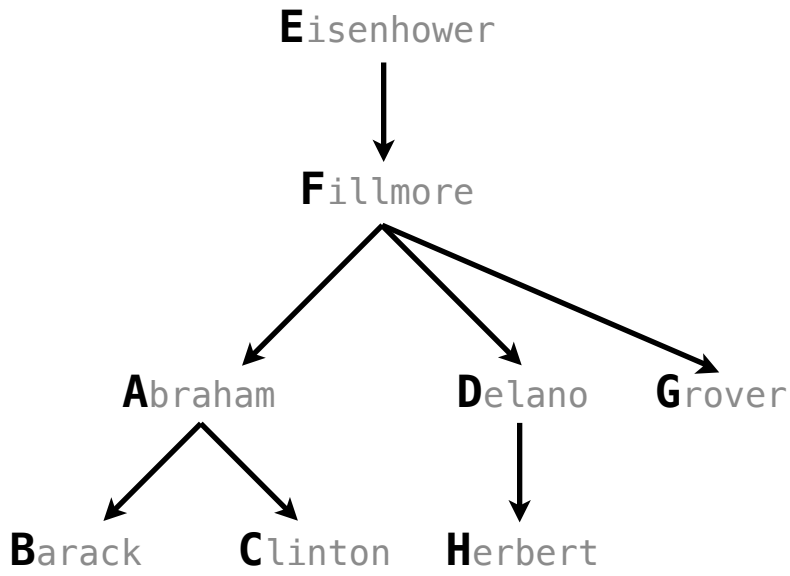

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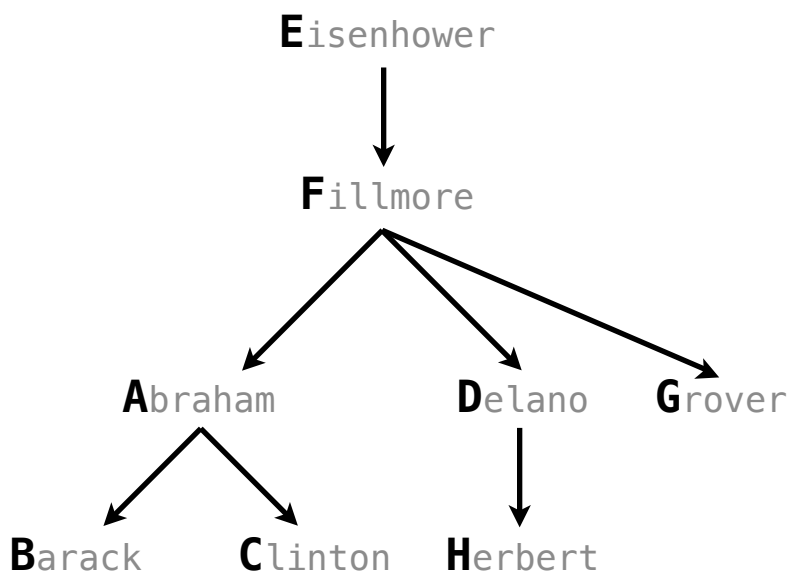

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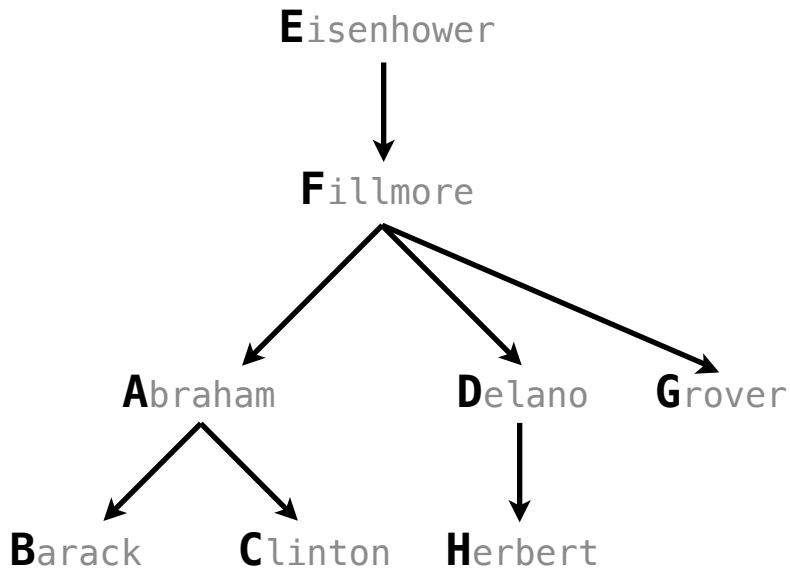

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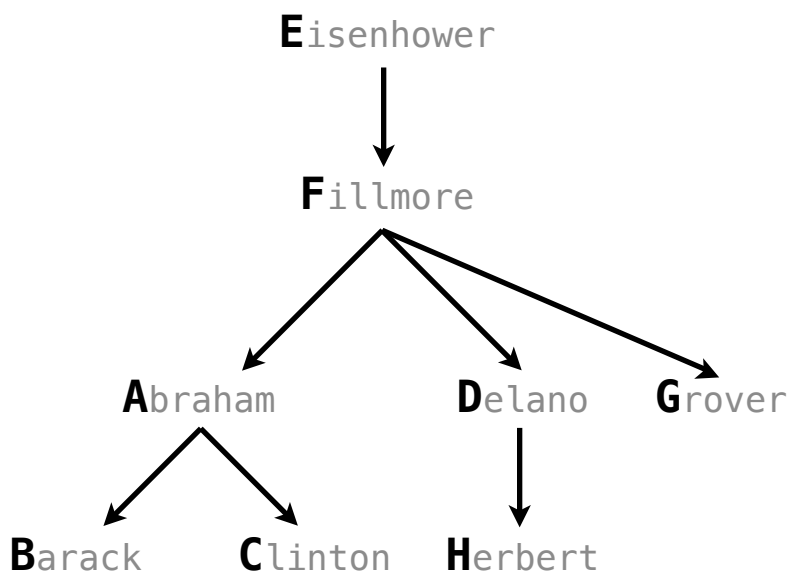

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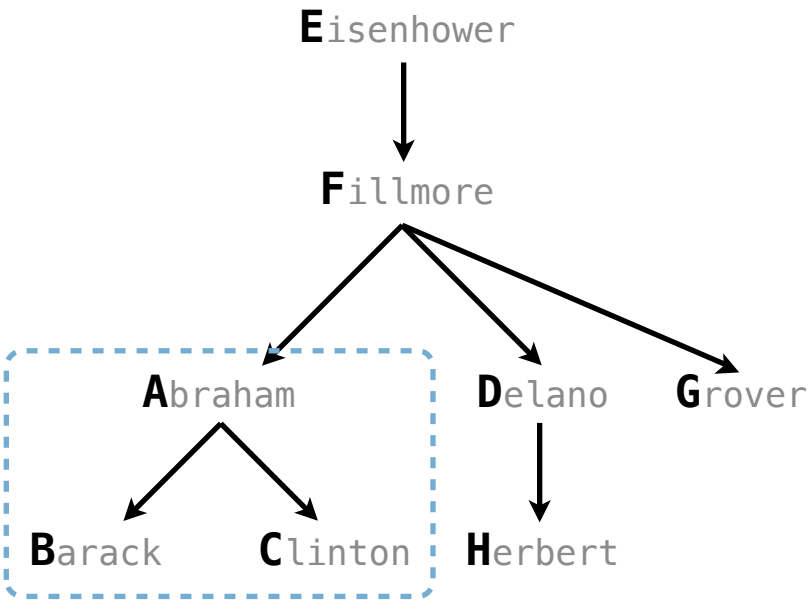

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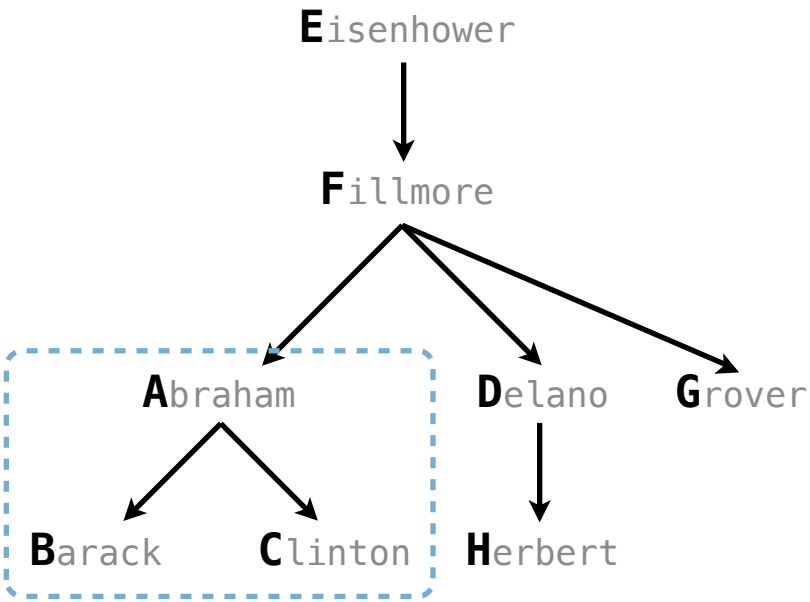

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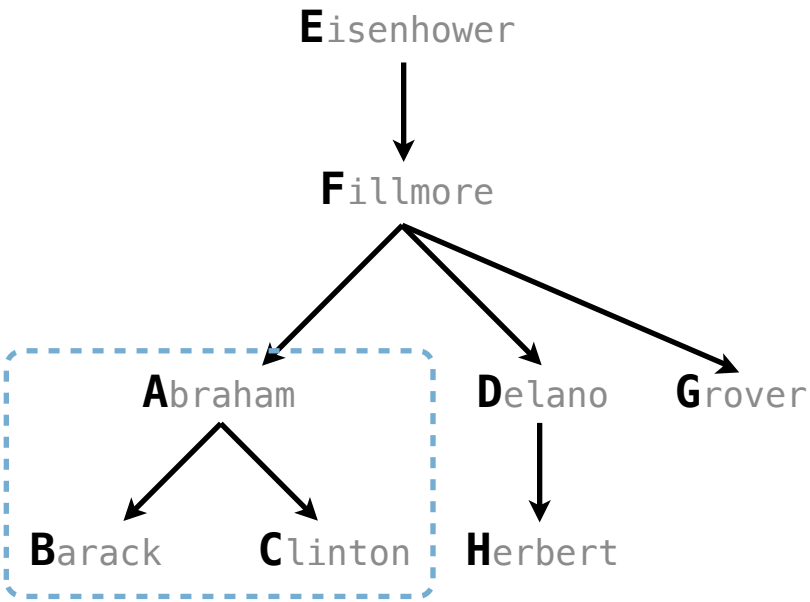

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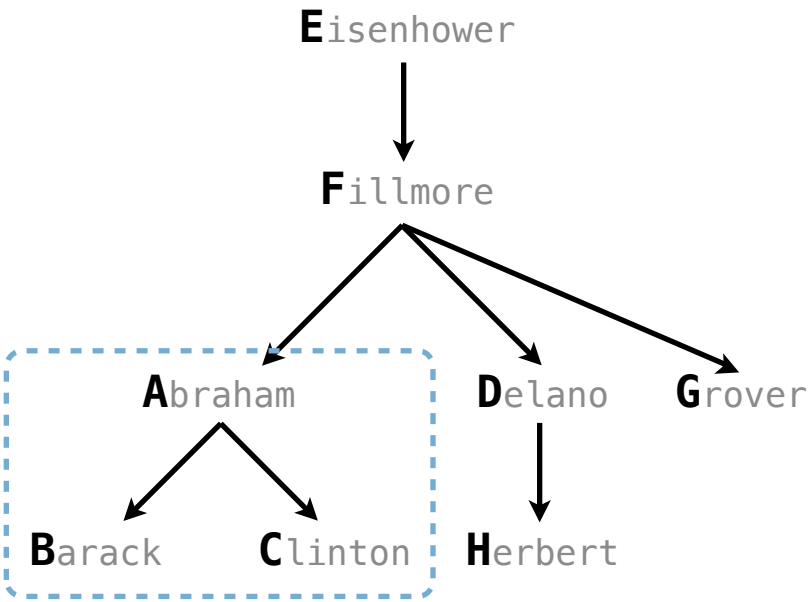

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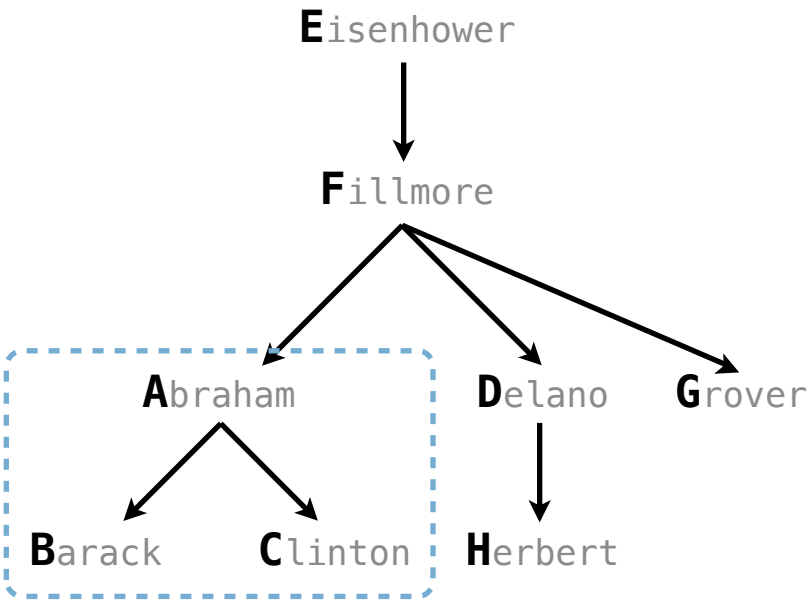

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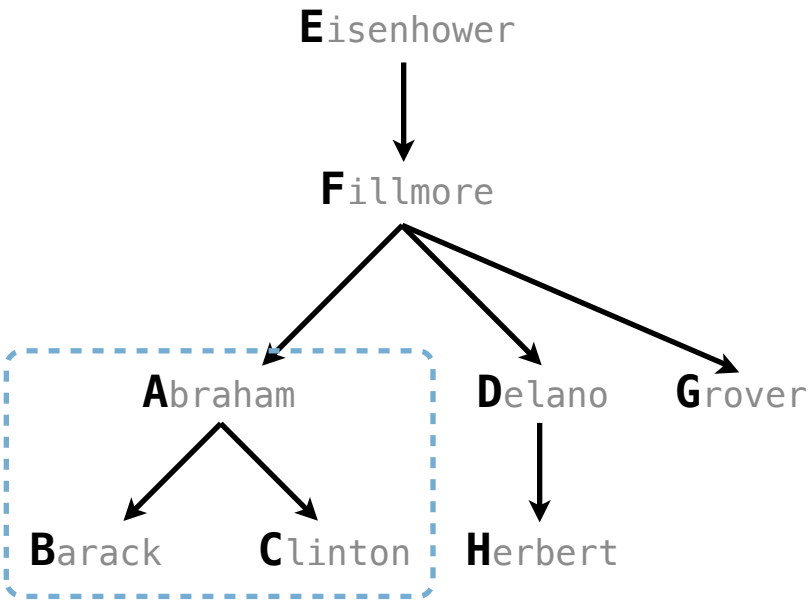


(Demo)

# Compound Facts and Queries

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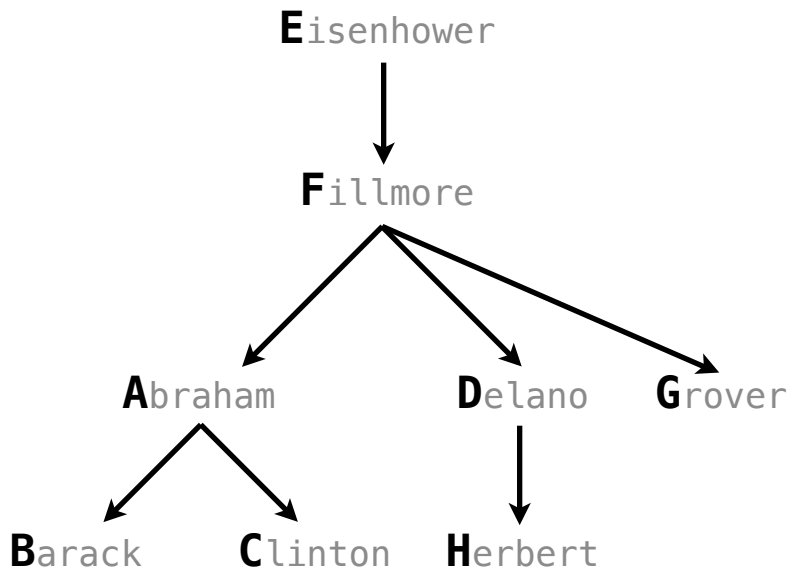

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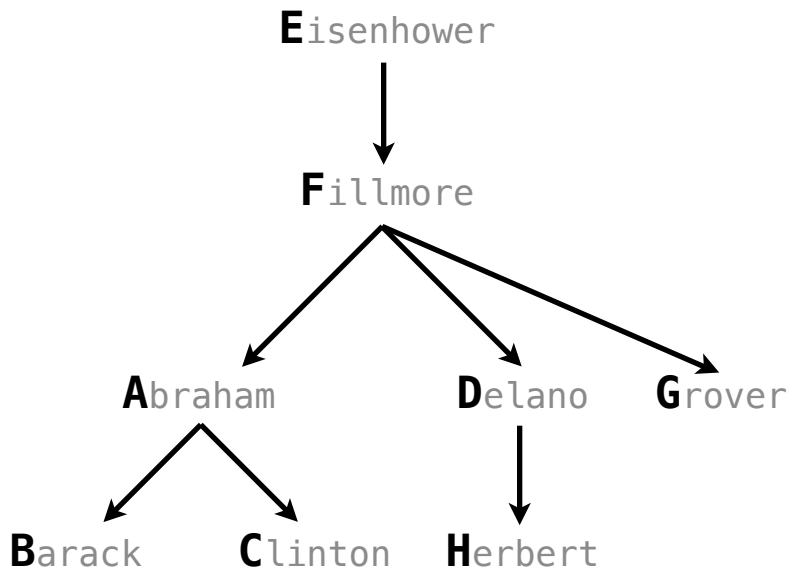

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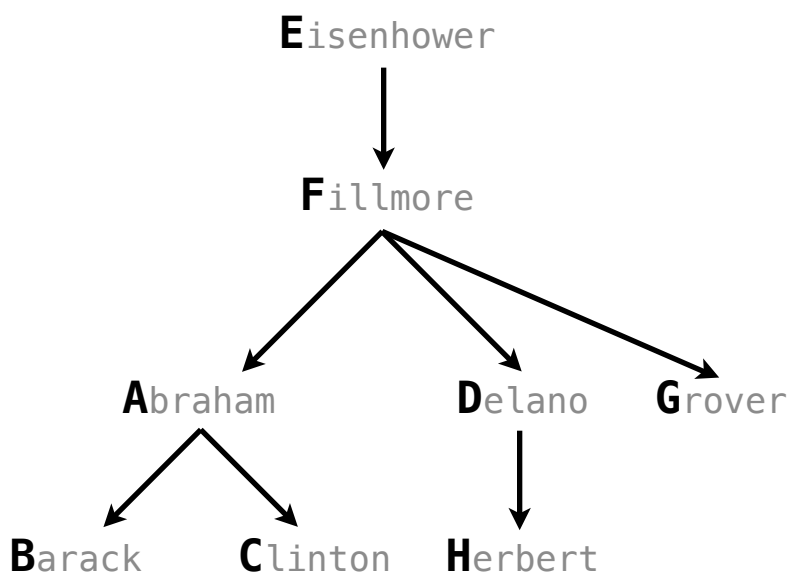

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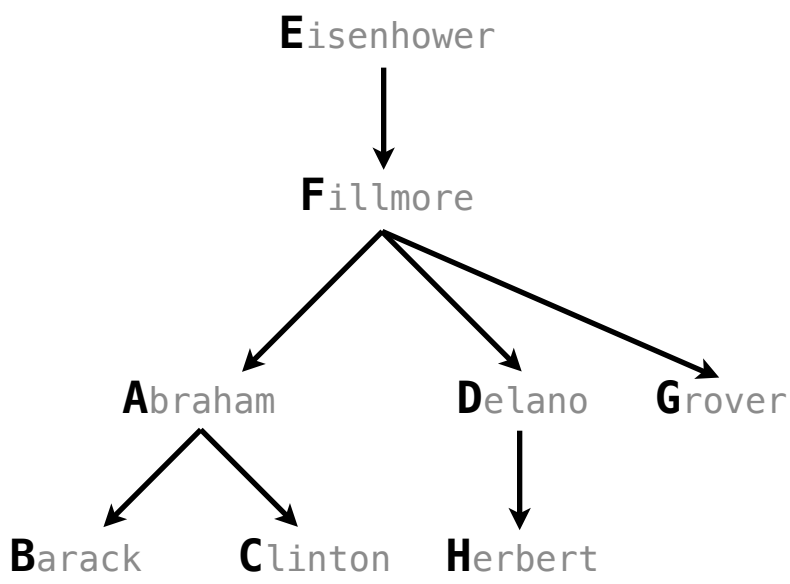

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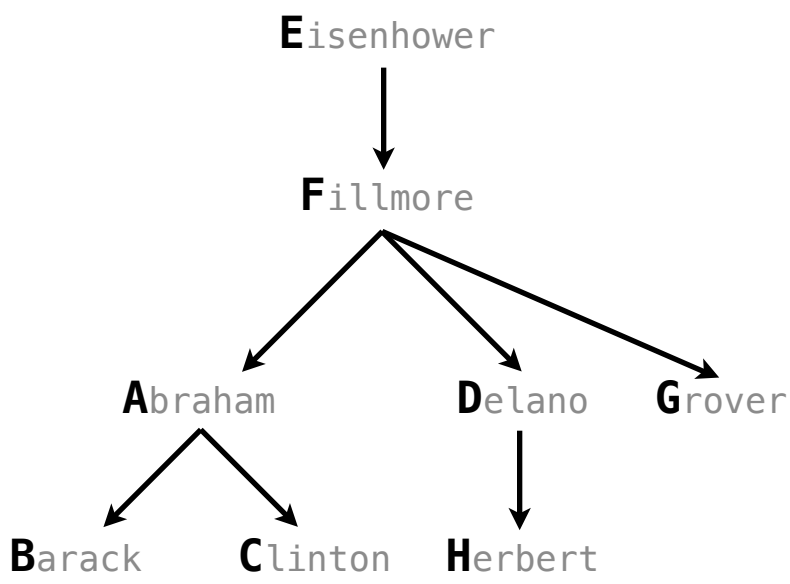

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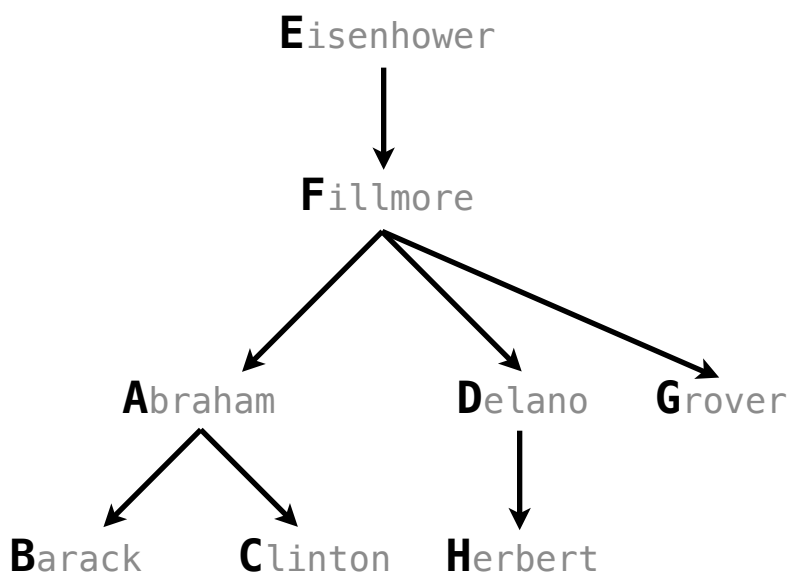

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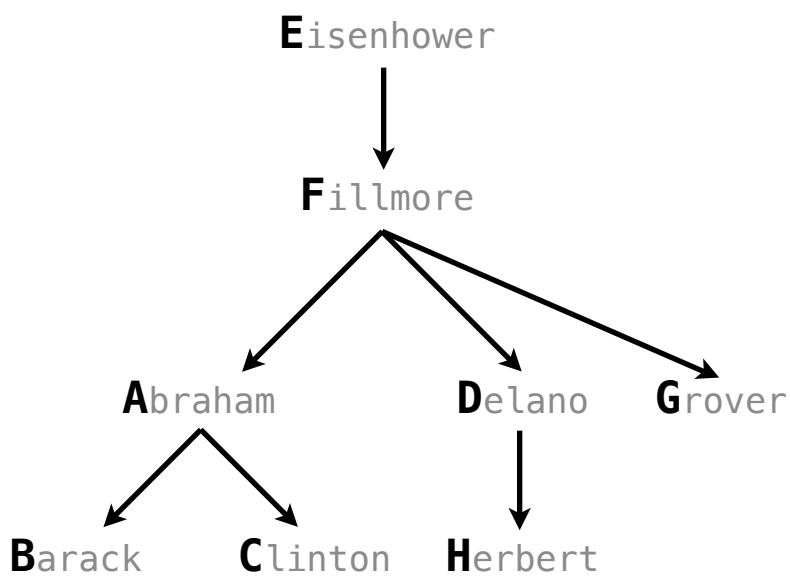

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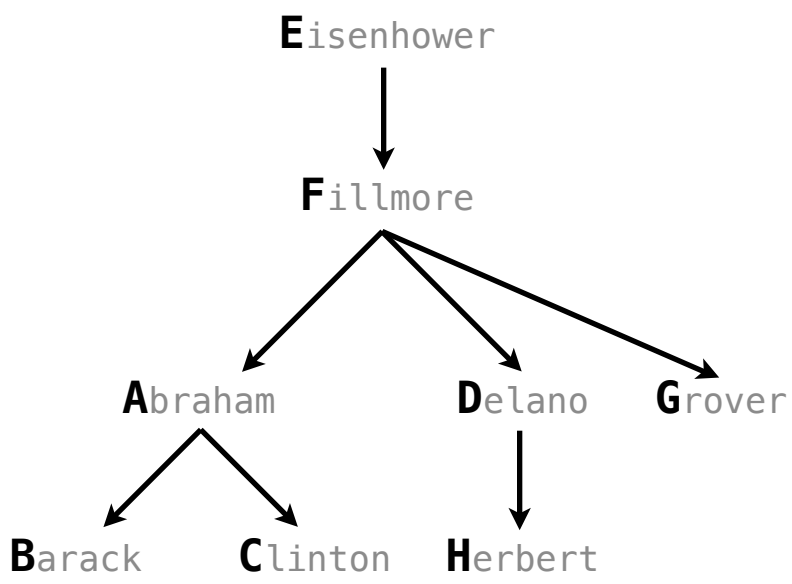

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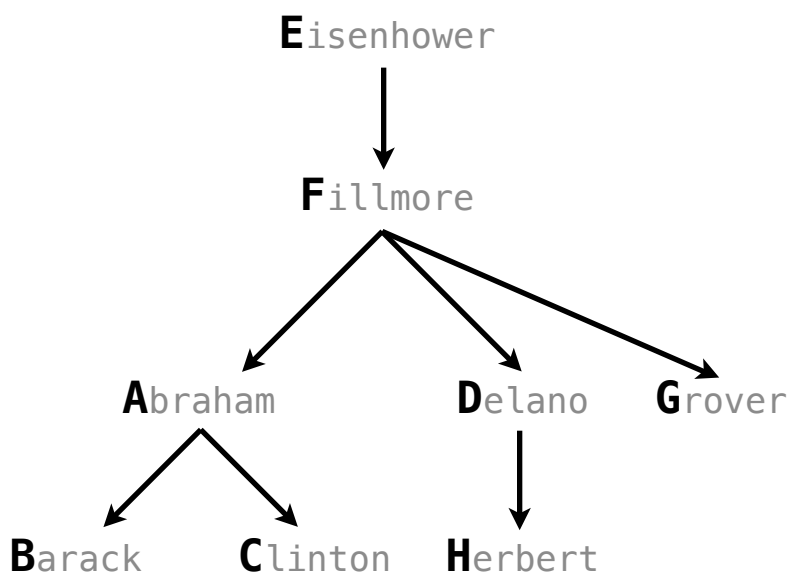

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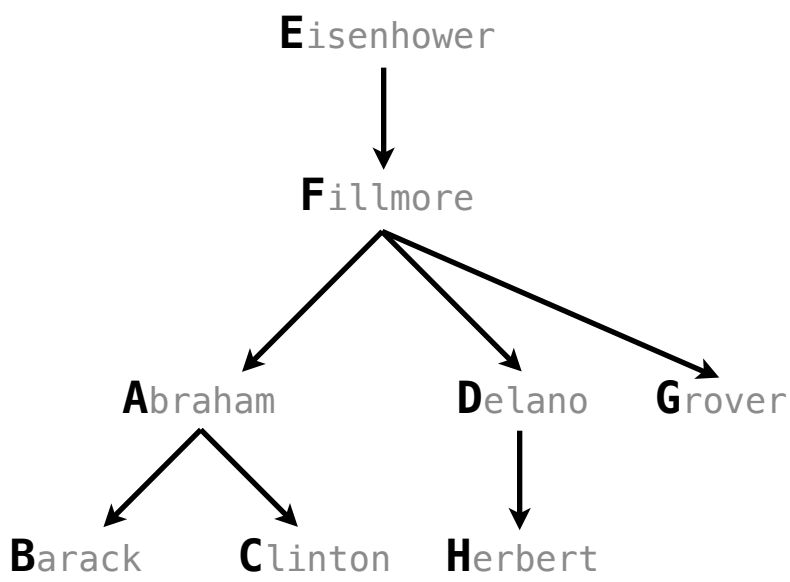

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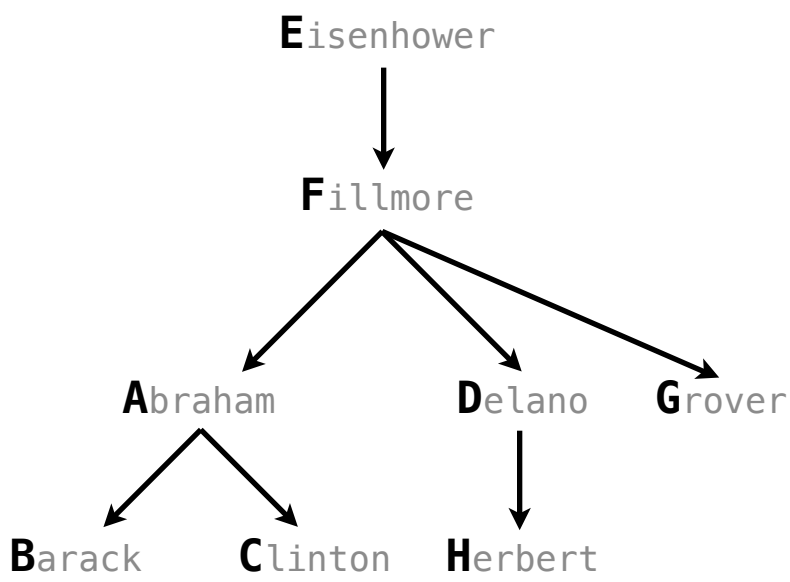

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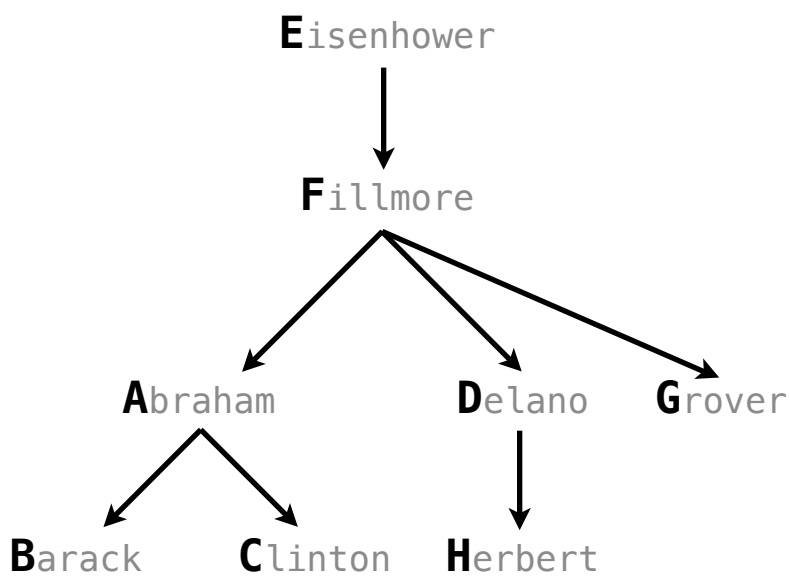

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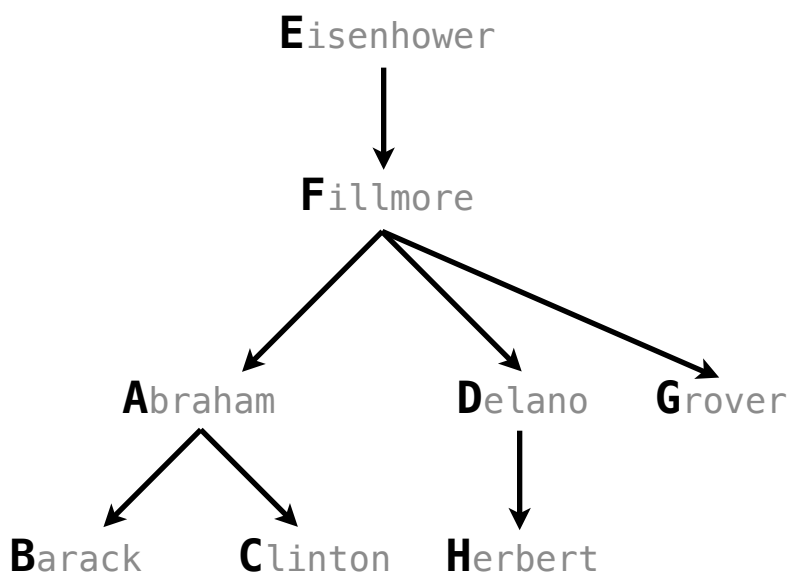

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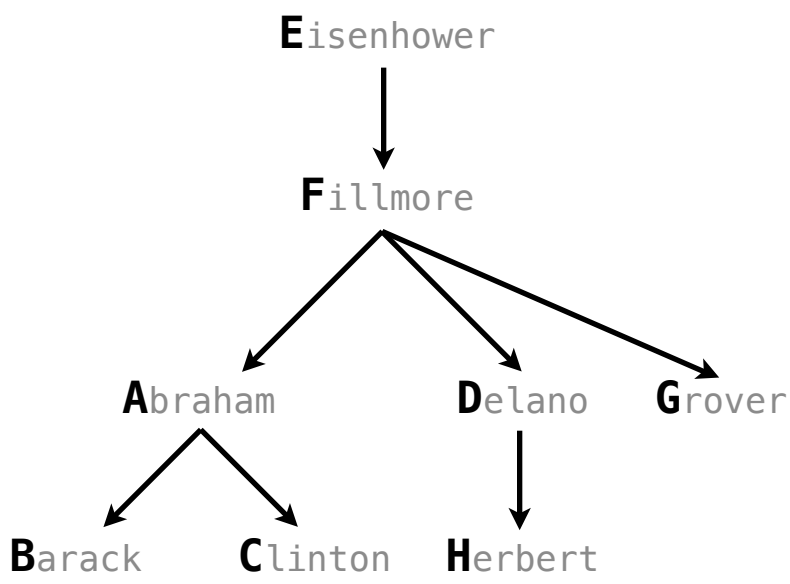

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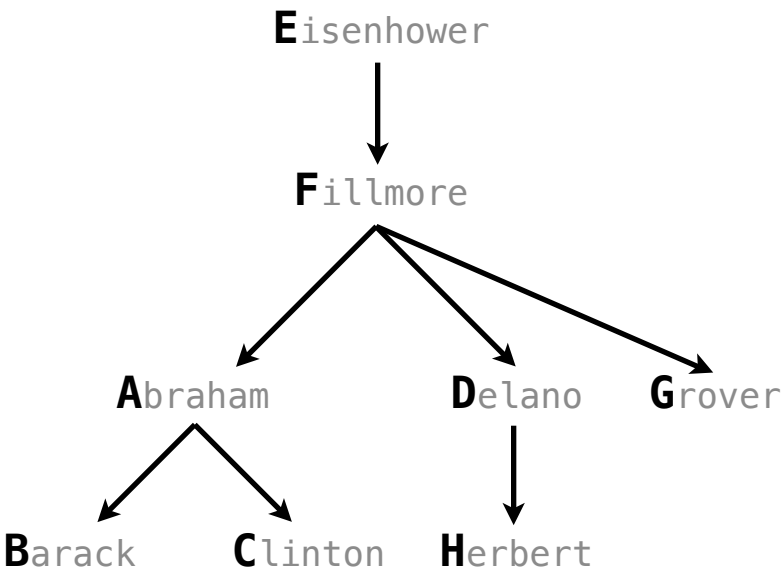

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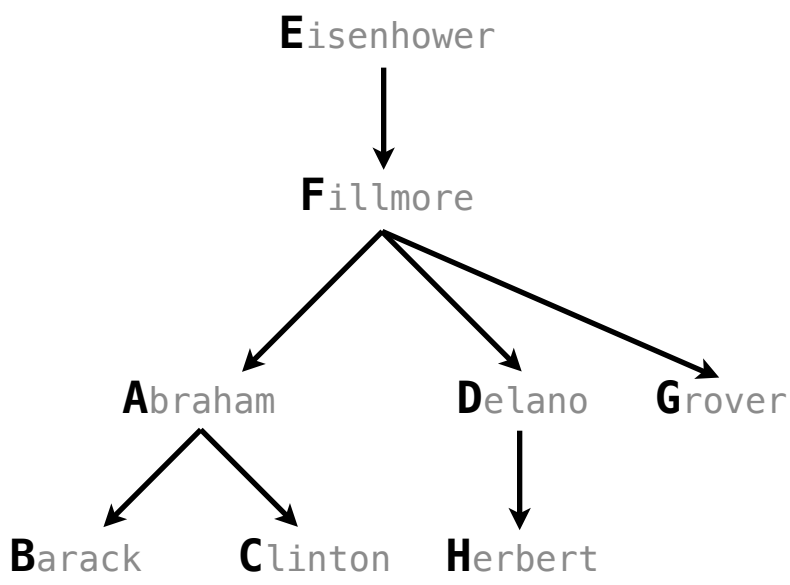

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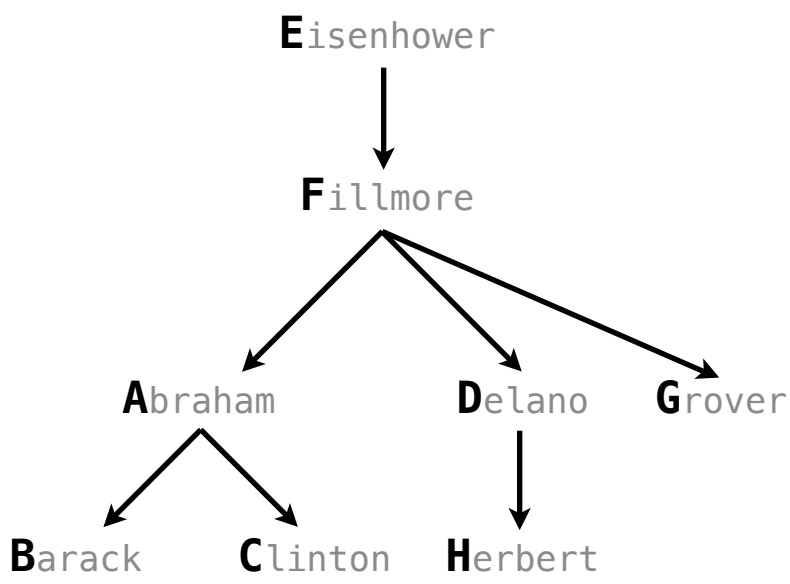

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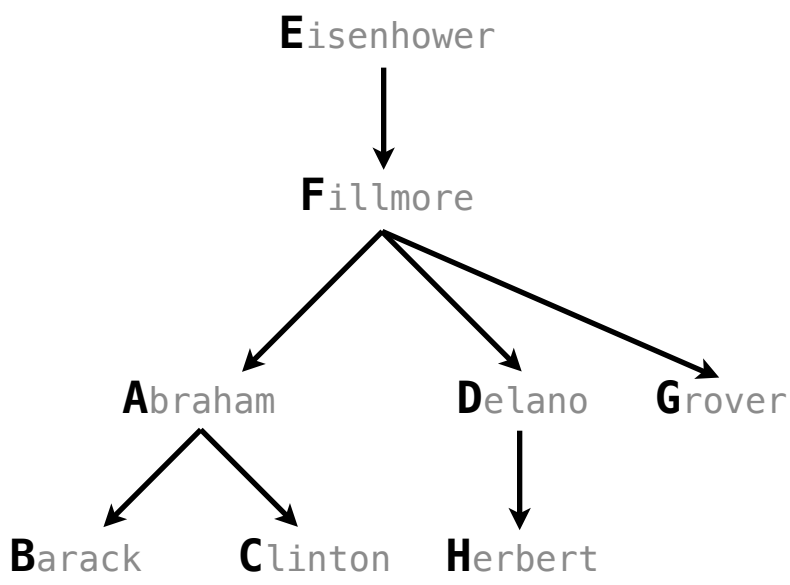

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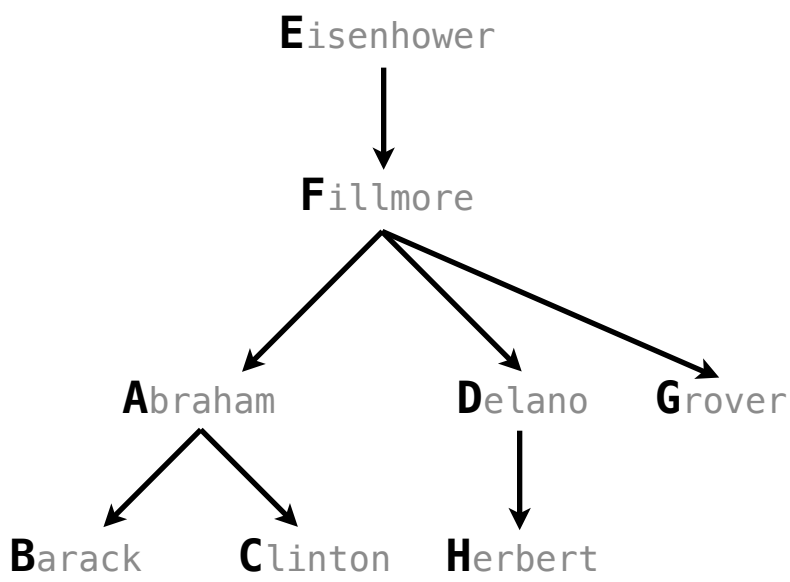

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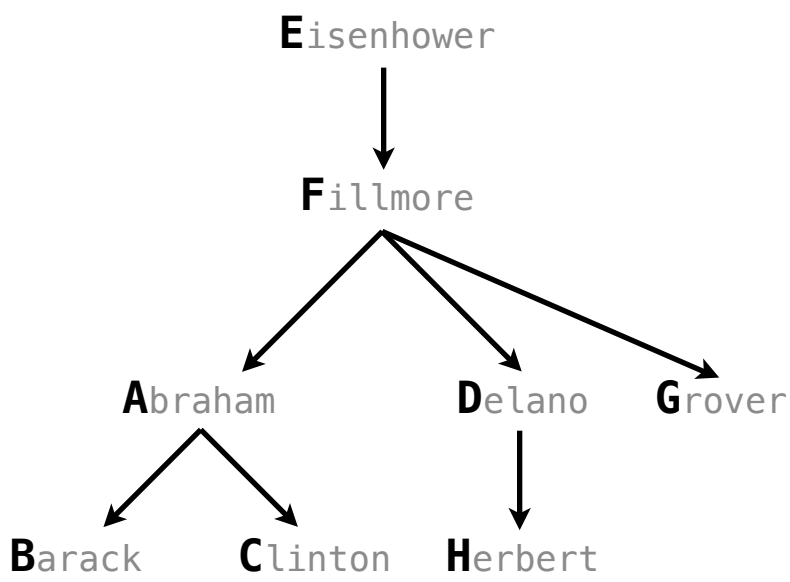

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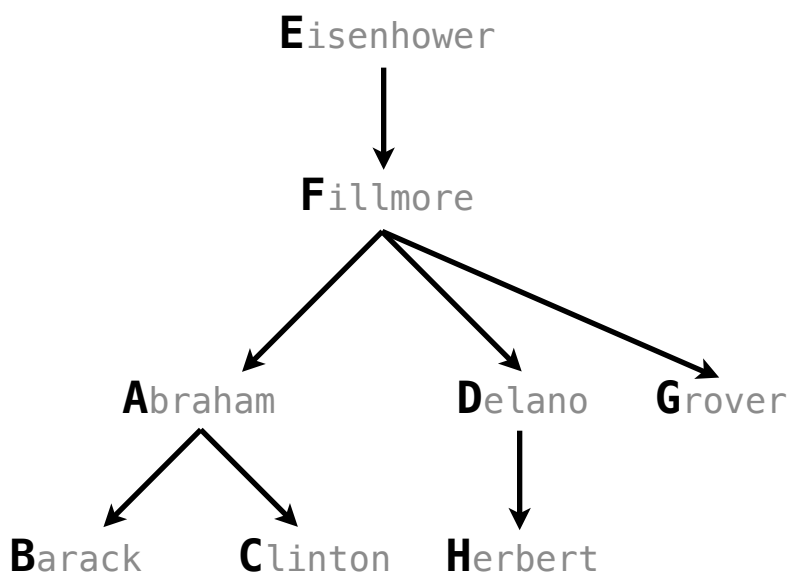

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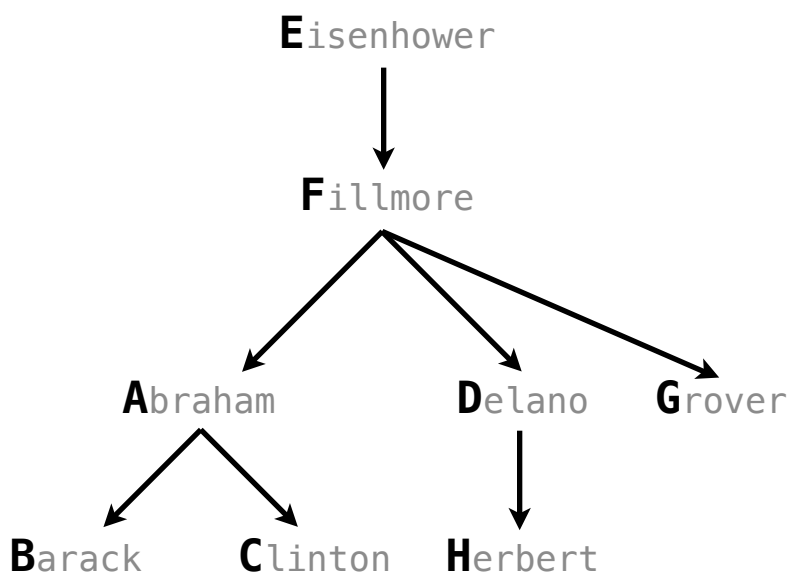

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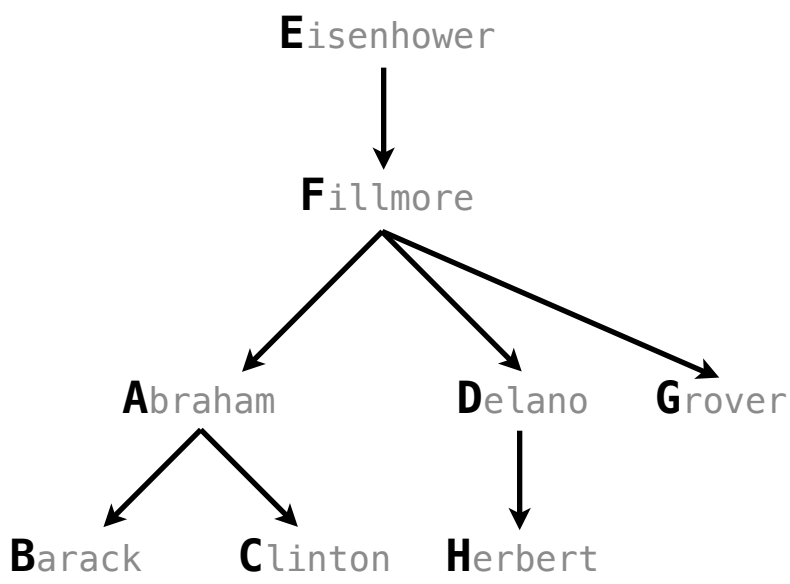

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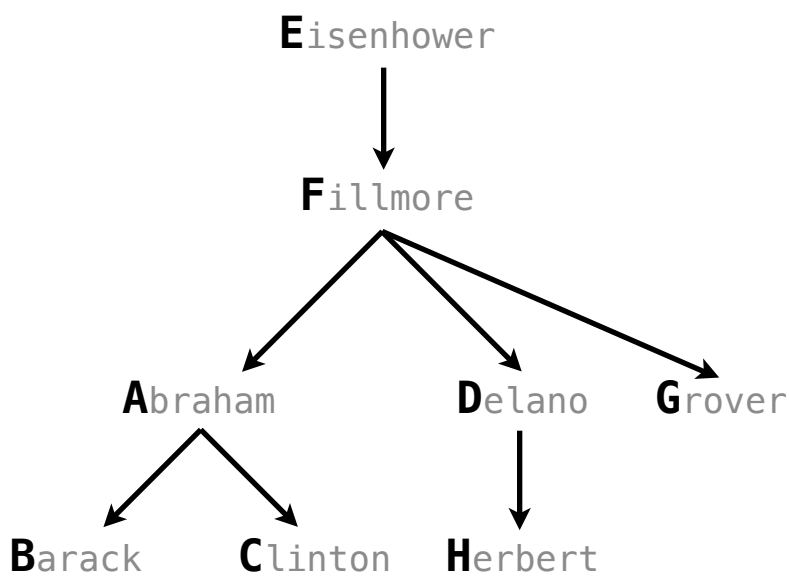

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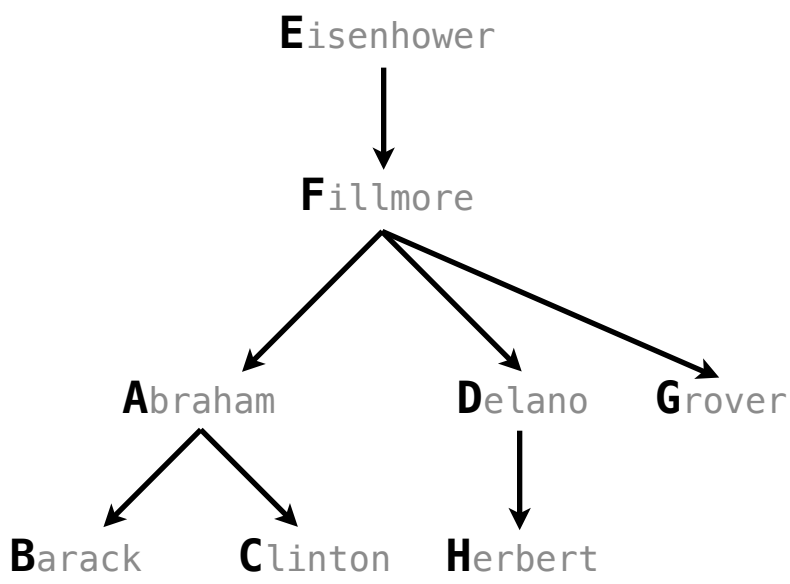

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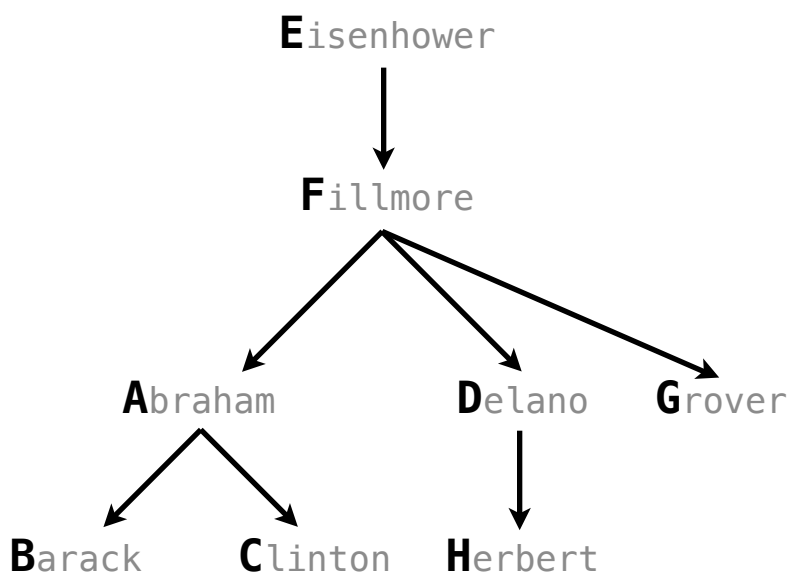

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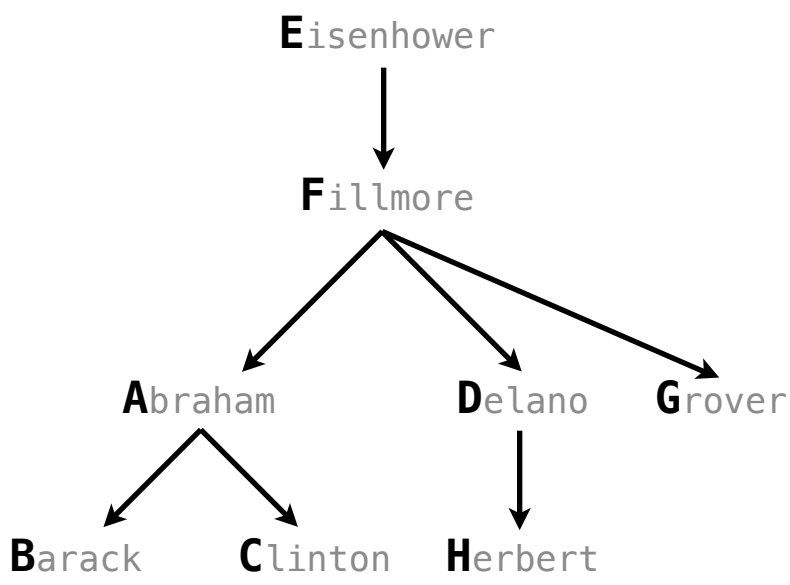

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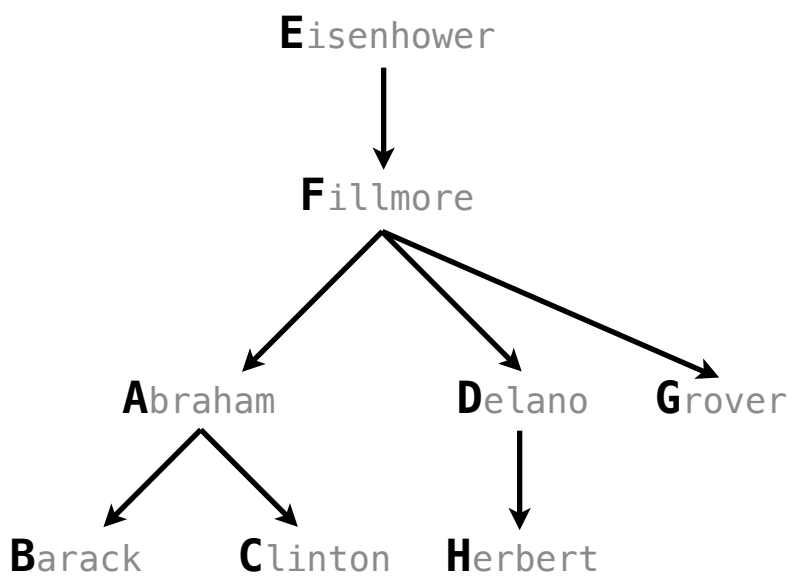

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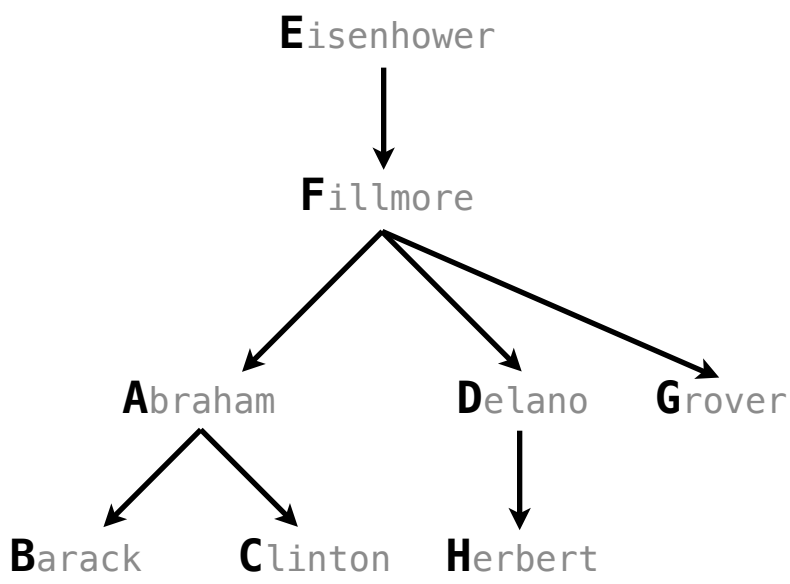

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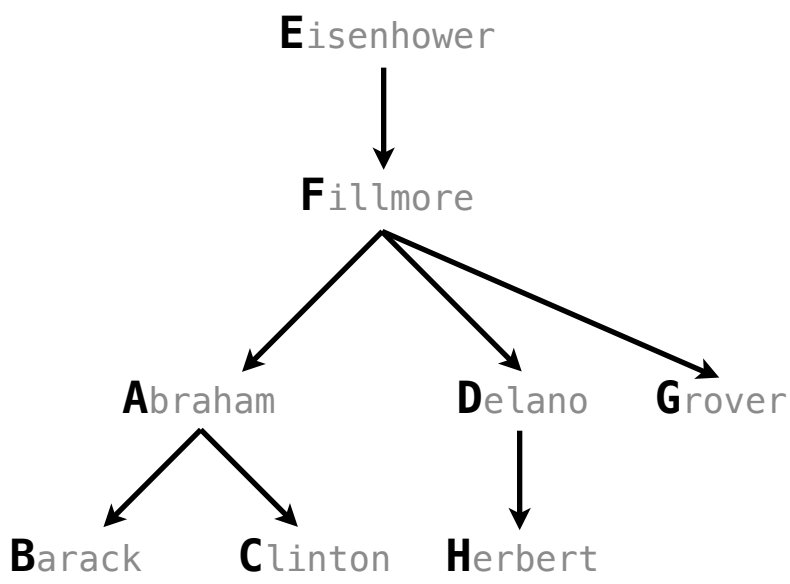

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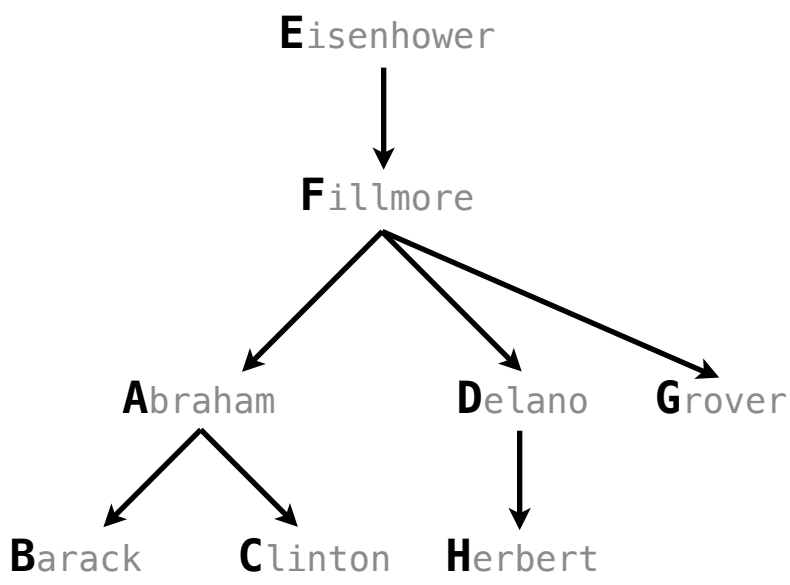

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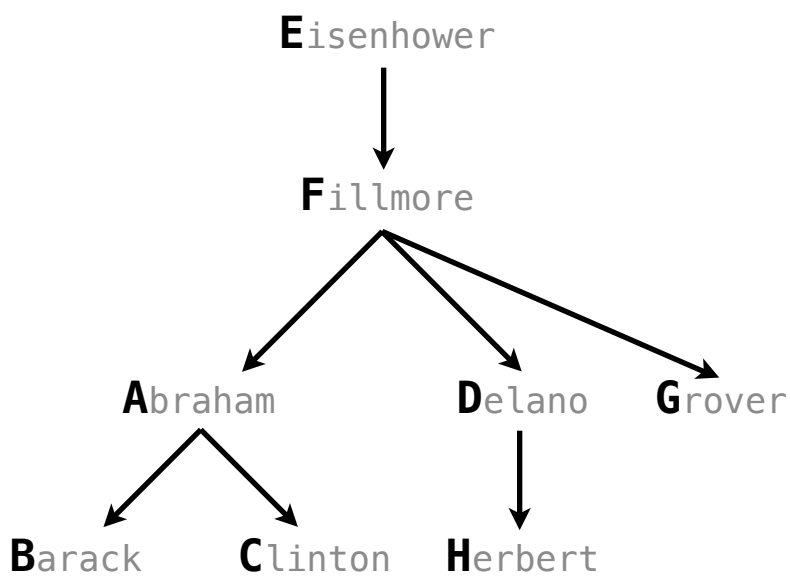

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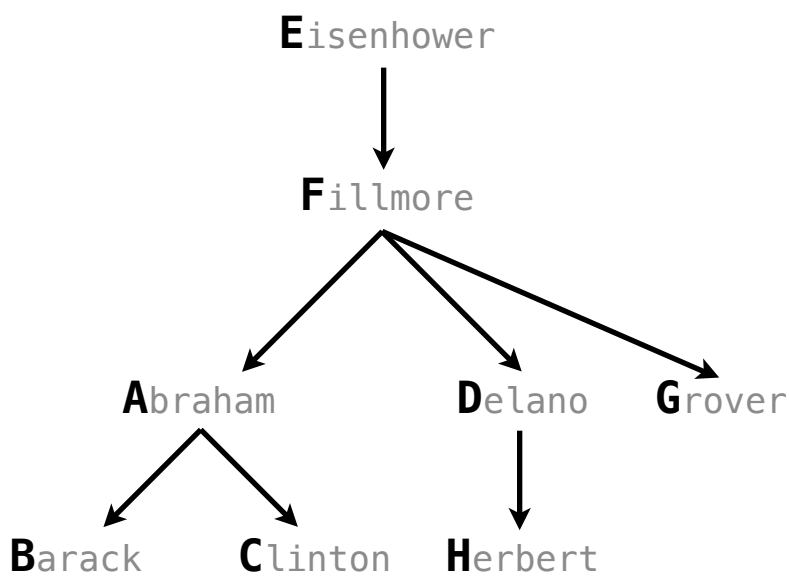

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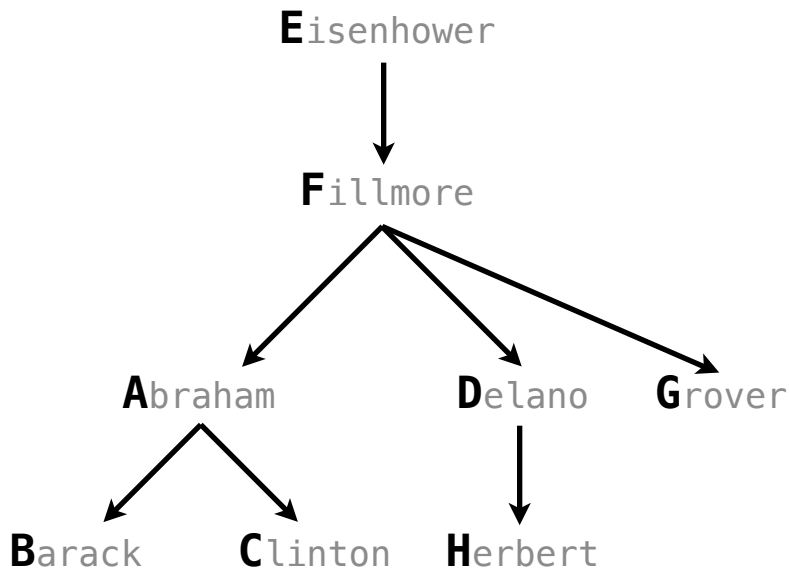

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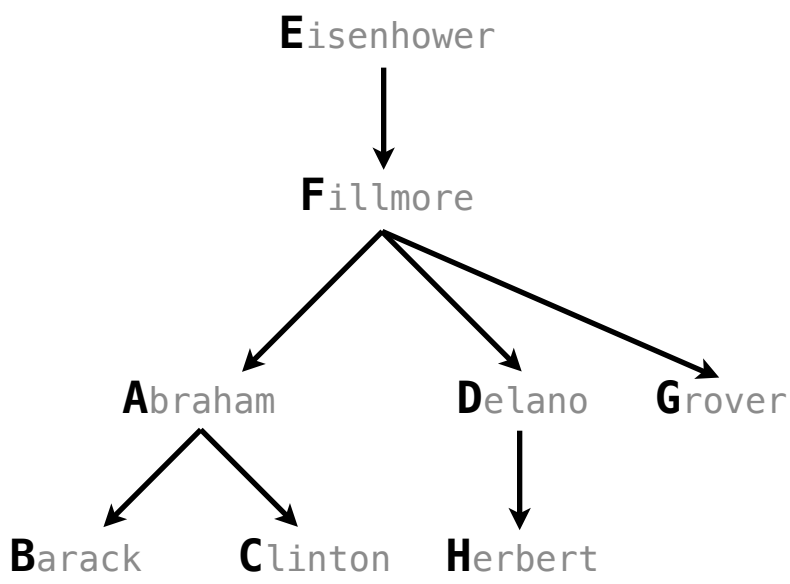

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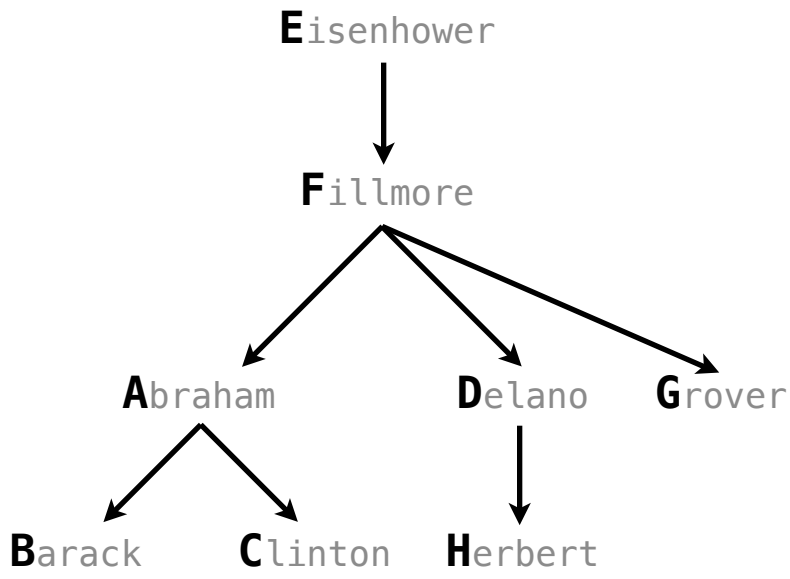

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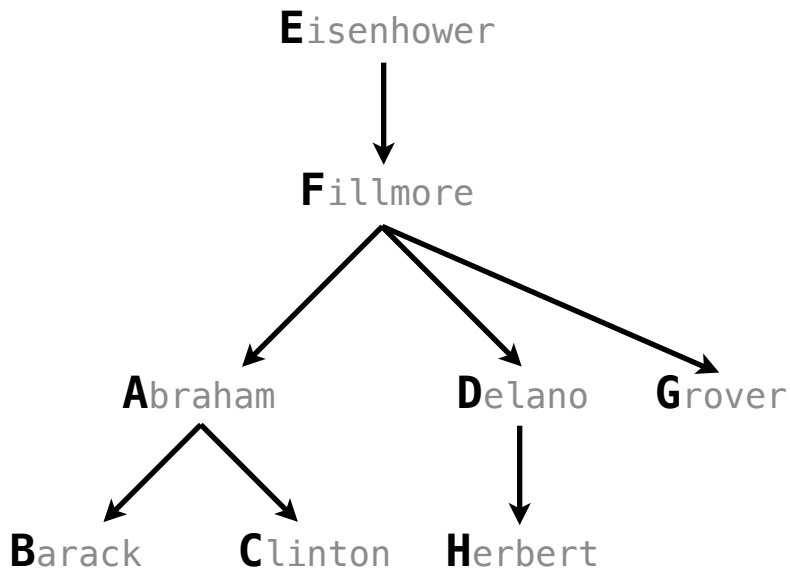

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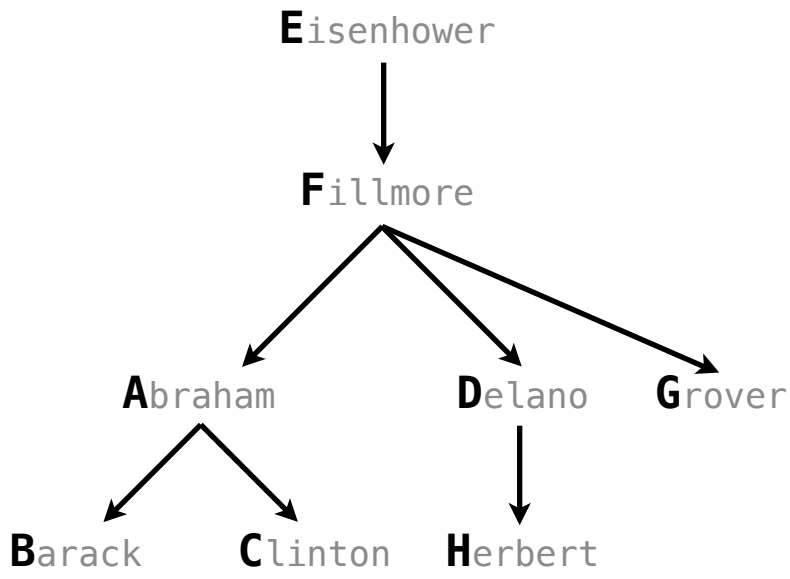

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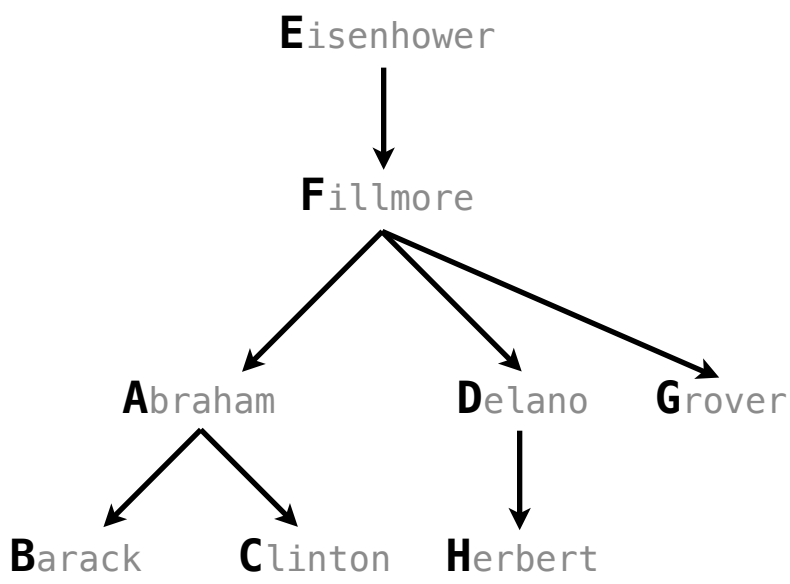

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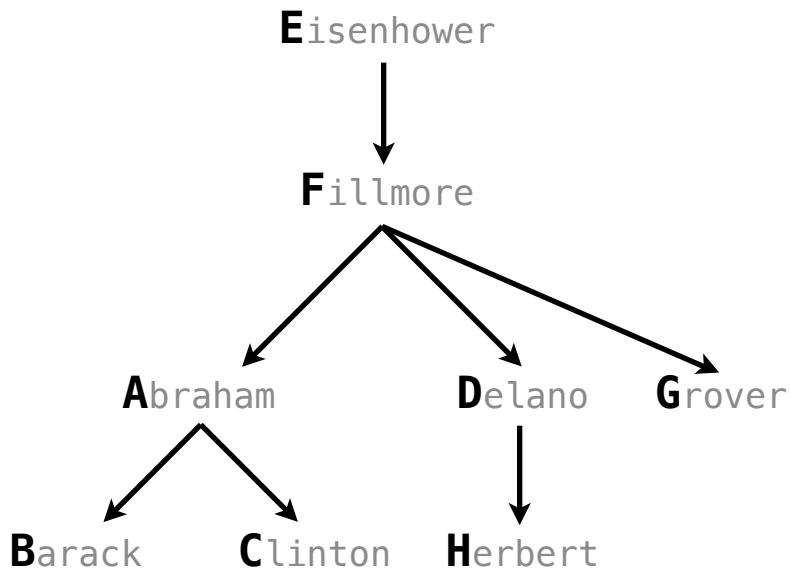

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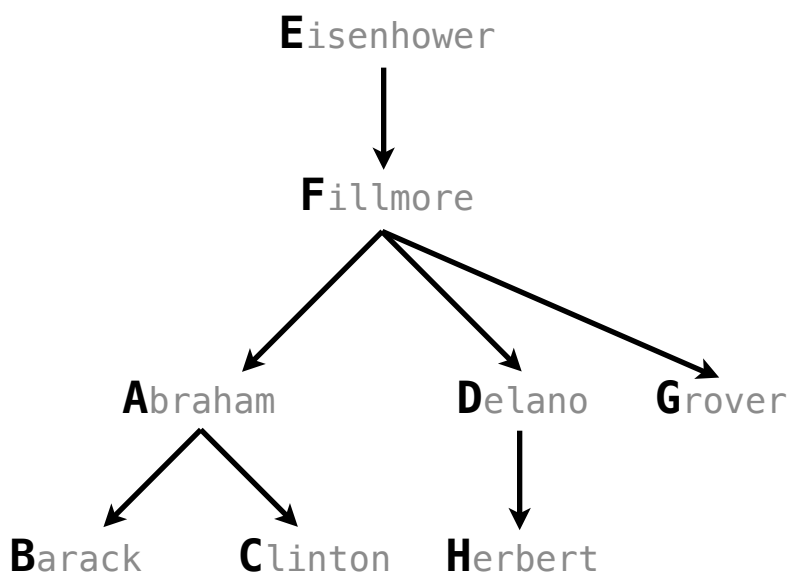

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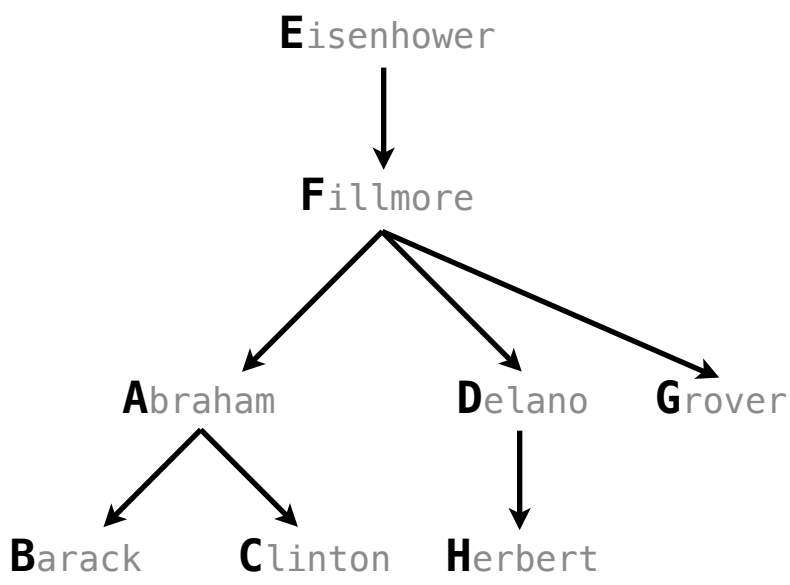

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logic> (query (ancestor ?a herbert))
Success!
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))

(parent delano herbert)     ; (1), a simple fact
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)      ; (1), a simple fact
(ancestor delano herbert)    ; (2), from (1) and the 1st ancestor fact
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)      ; (1), a simple fact
(ancestor delano herbert)    ; (2), from (1) and the 1st ancestor fact
(parent fillmore delano)     ; (3), a simple fact
```

## Searching to Satisfy Queries

The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments.

```
logic> (query (ancestor ?a herbert))
Success!
a: delano
a: fillmore ⇦
a: eisenhower
logic> (fact (parent delano herbert))
logic> (fact (parent fillmore delano))
logic> (fact (ancestor ?a ?y) (parent ?a ?y))
logic> (fact (ancestor ?a ?y) (parent ?a ?z) (ancestor ?z ?y))
```

```
(parent delano herbert)      ; (1), a simple fact
(ancestor delano herbert)    ; (2), from (1) and the 1st ancestor fact
(parent fillmore delano)     ; (3), a simple fact
(ancestor fillmore herbert) ; (4), from (2), (3), & the 2nd ancestor fact
```

# Hierarchical Facts

# Hierarchical Facts

## Hierarchical Facts

```
Relations can contain relations in addition to symbols.
```

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Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
```

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Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
```

## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
```

## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
```

## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
```

## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
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logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
```

## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
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logic> (fact (dog (name delano) (fur long)))
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logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
```

## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
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logic> (fact (dog (name herbert) (fur curly)))
```

## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
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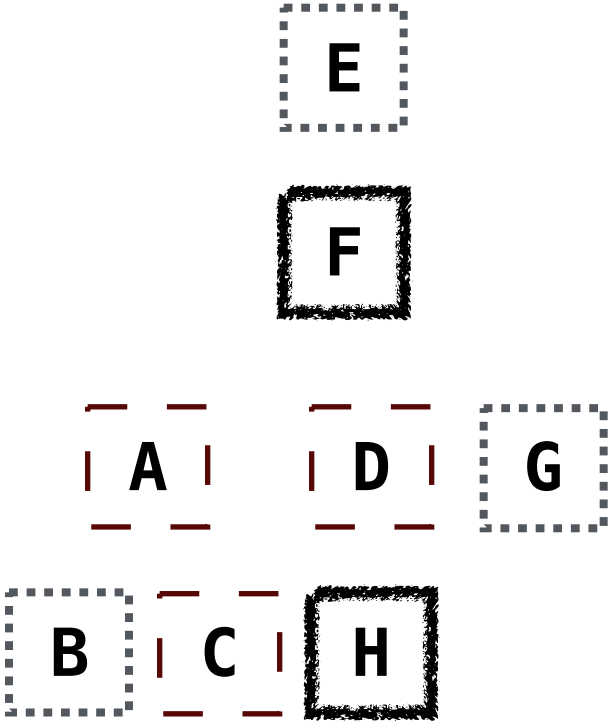
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```



## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
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logic> (fact (dog (name delano) (fur long)))
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logic> (fact (dog (name grover) (fur short)))
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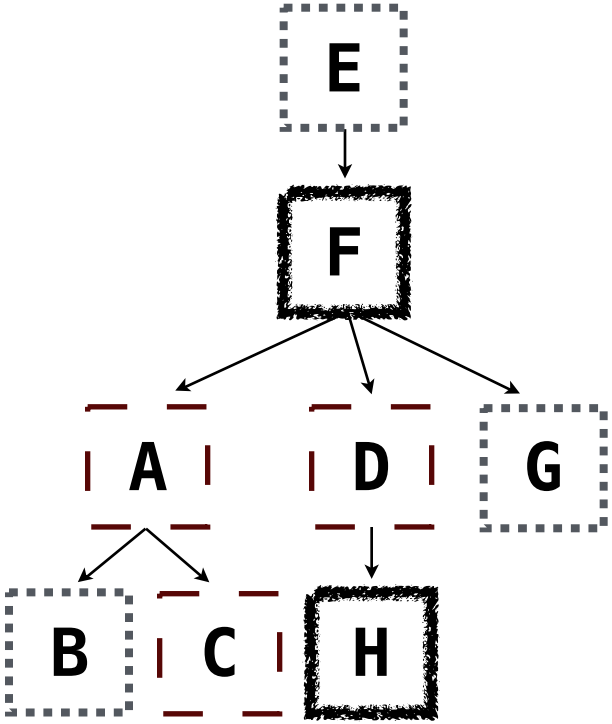
```

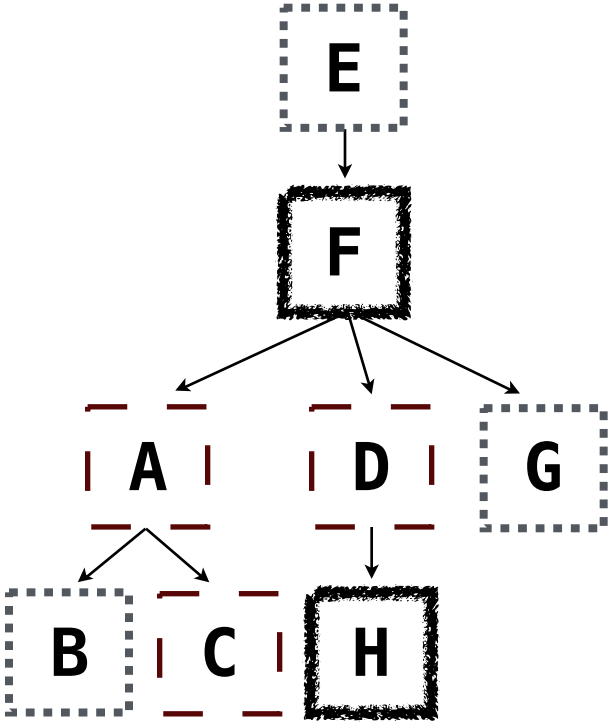


## Hierarchical Facts

Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.



## Hierarchical Facts

Relations can contain relations in addition to symbols.

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logic> (fact (dog (name abraham) (fur long)))
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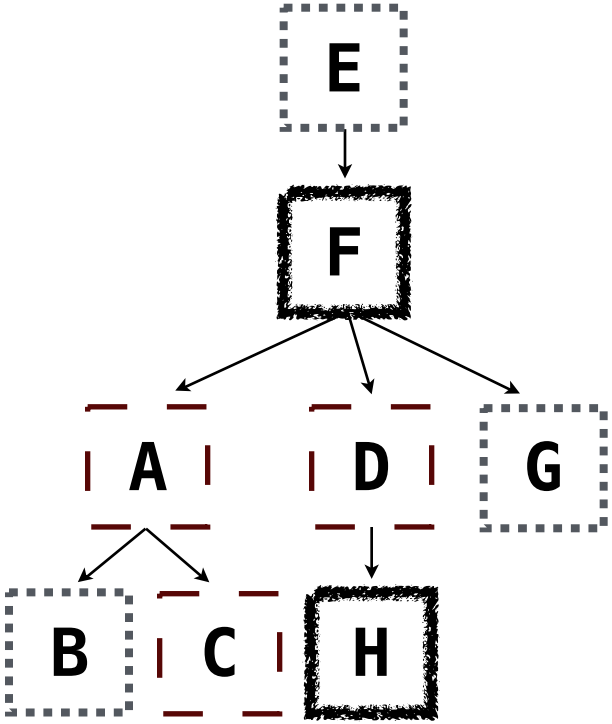
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
```



## Hierarchical Facts

Relations can contain relations in addition to symbols.

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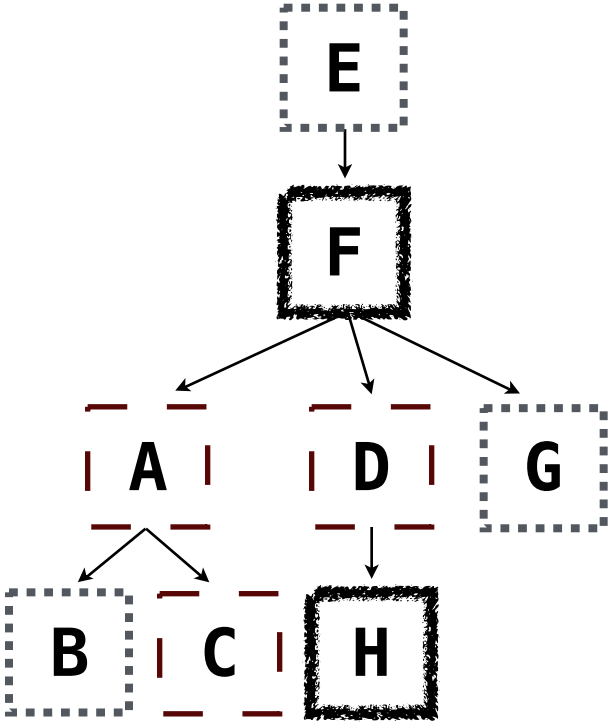
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
```



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Relations can contain relations in addition to symbols.

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logic> (fact (dog (name abraham) (fur long)))
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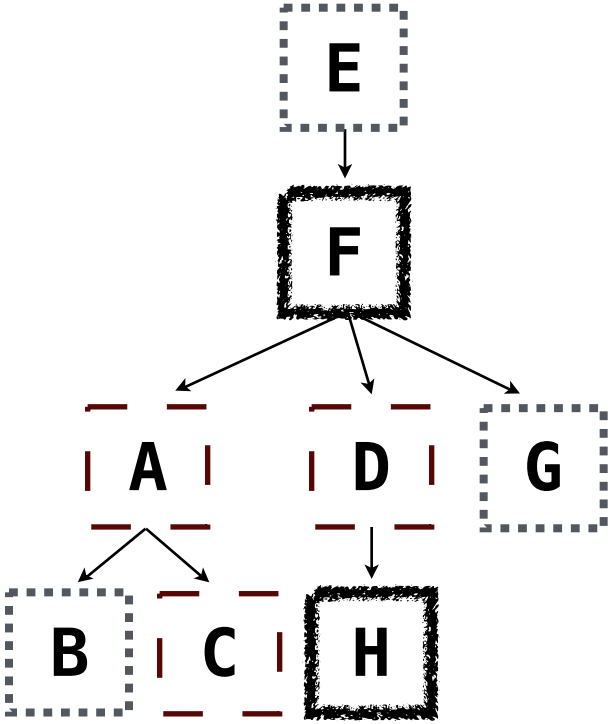
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long
```



# Hierarchical Facts

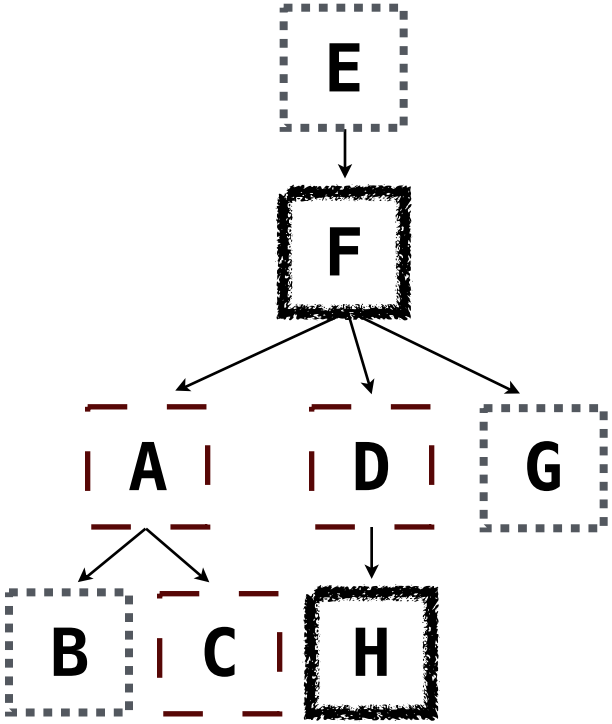
Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long

logic> (query (dog (name clinton) ?stats))
```



# Hierarchical Facts

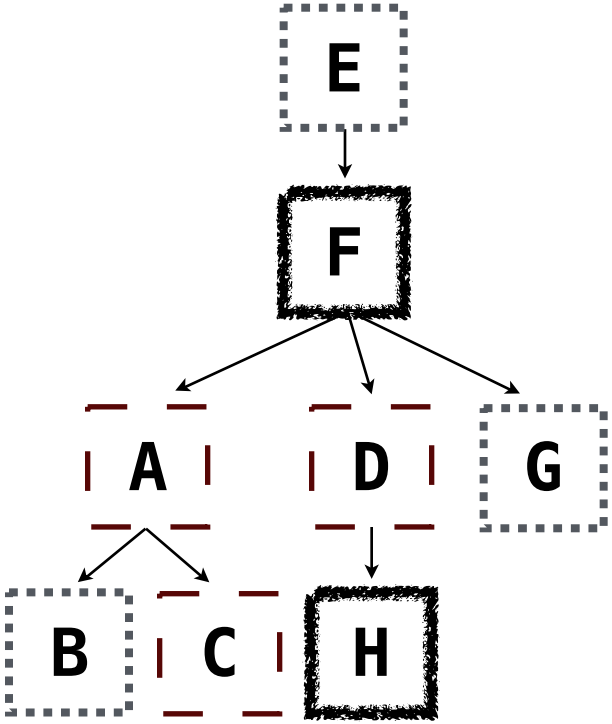
Relations can contain relations in addition to symbols.

```
logic> (fact (dog (name abraham) (fur long)))
logic> (fact (dog (name barack) (fur short)))
logic> (fact (dog (name clinton) (fur long)))
logic> (fact (dog (name delano) (fur long)))
logic> (fact (dog (name eisenhower) (fur short)))
logic> (fact (dog (name fillmore) (fur curly)))
logic> (fact (dog (name grover) (fur short)))
logic> (fact (dog (name herbert) (fur curly)))
```

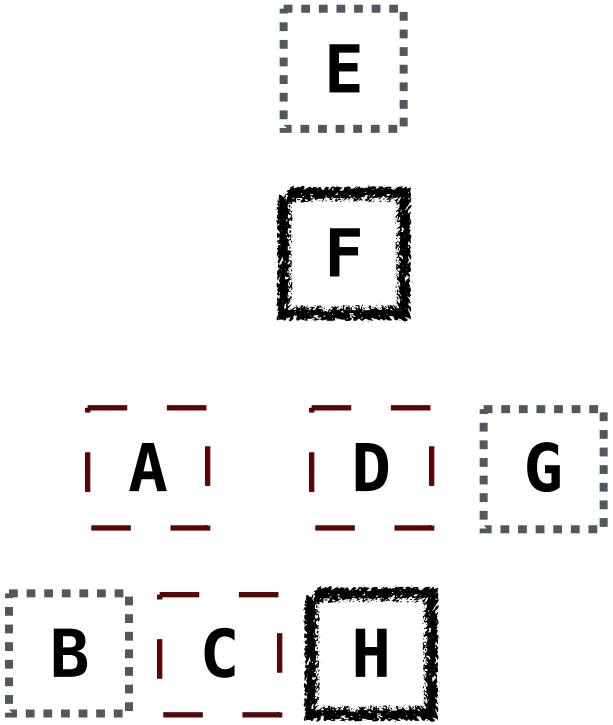
Variables can refer to symbols or whole relations.

```
logic> (query (dog (name clinton) (fur ?type)))
Success!
type: long

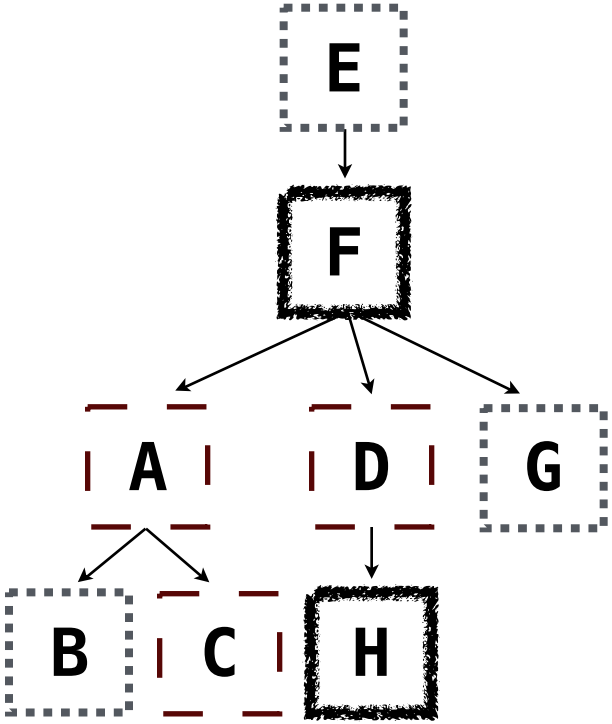
logic> (query (dog (name clinton) ?stats))
Success!
stats: (fur long)
```



# Combining Multiple Data Sources

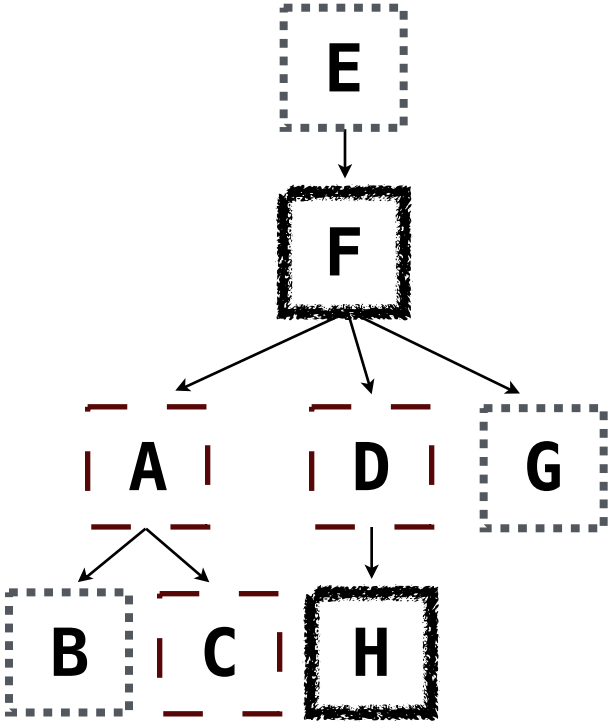


# Combining Multiple Data Sources



## Combining Multiple Data Sources

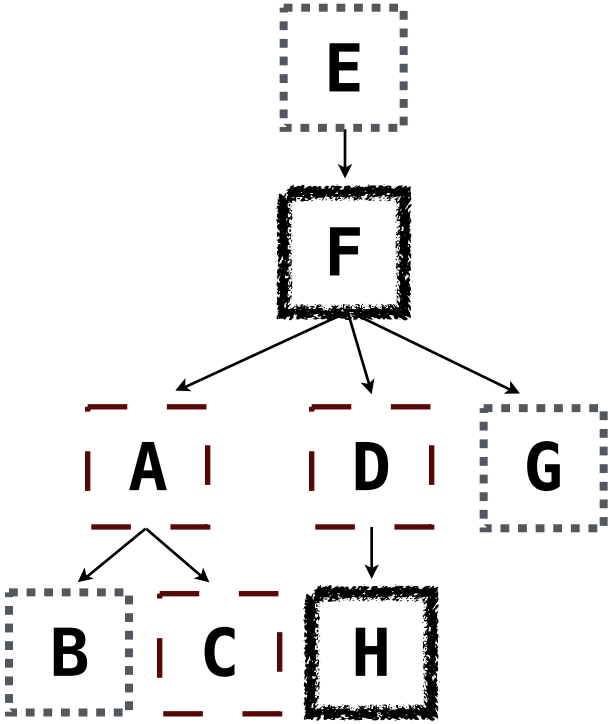
```
Which dogs have an ancestor of the same fur?
```



## Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

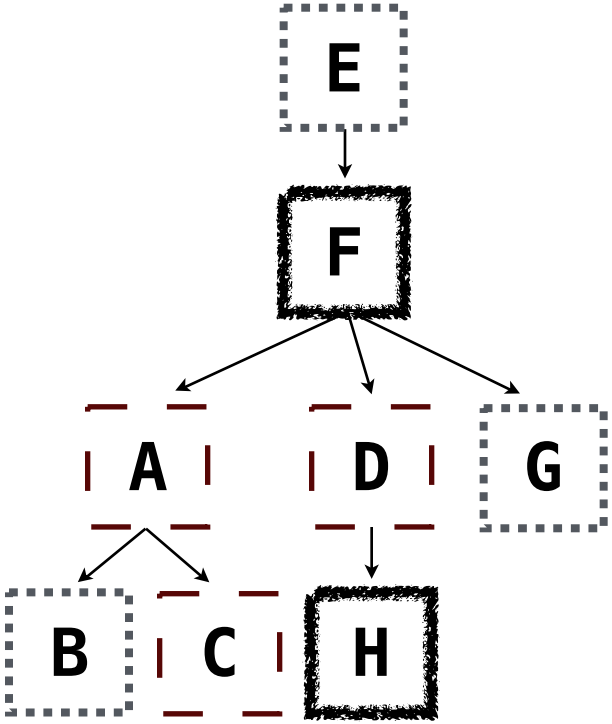
```
logic> (query (dog (name ?x) (fur ?fur))
```



## Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

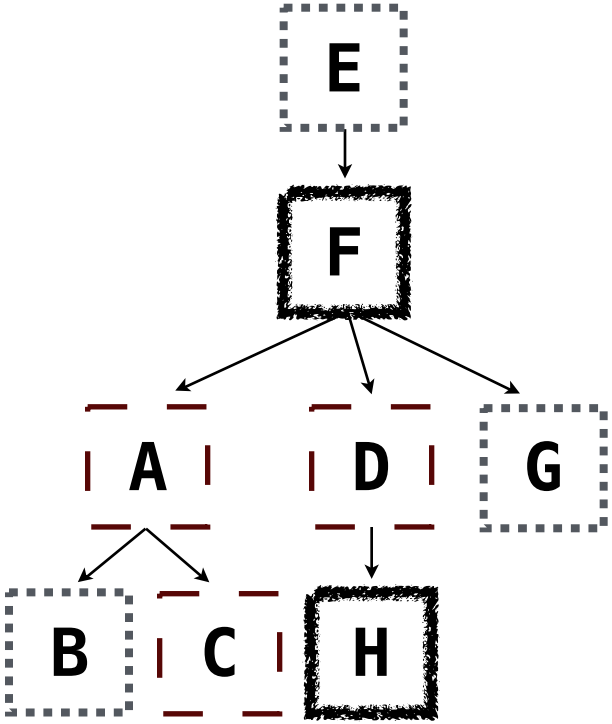
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
```



## Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

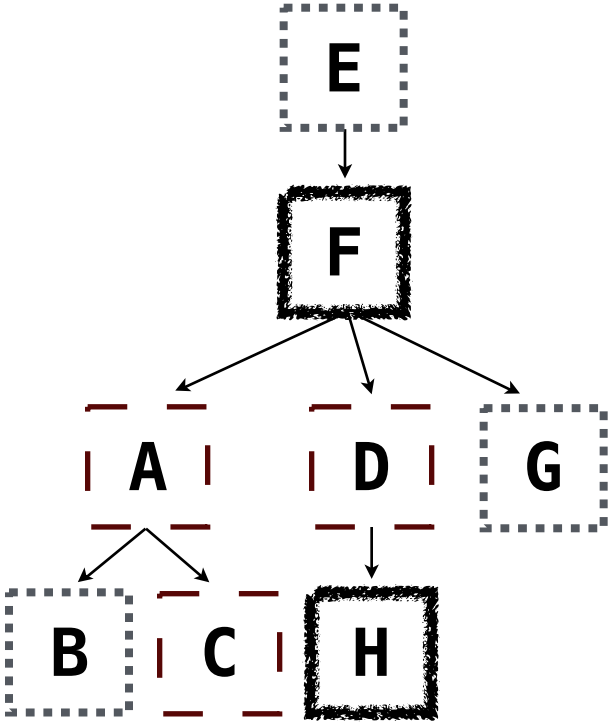
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
```



## Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

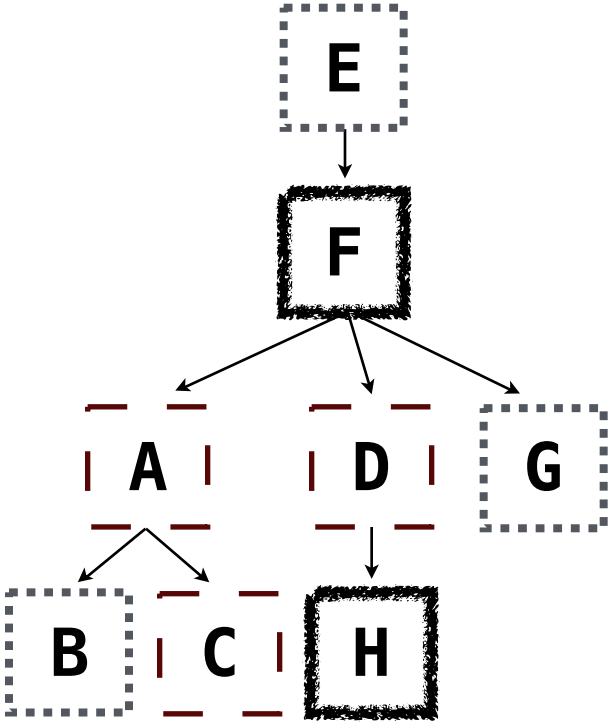
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
```



## Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

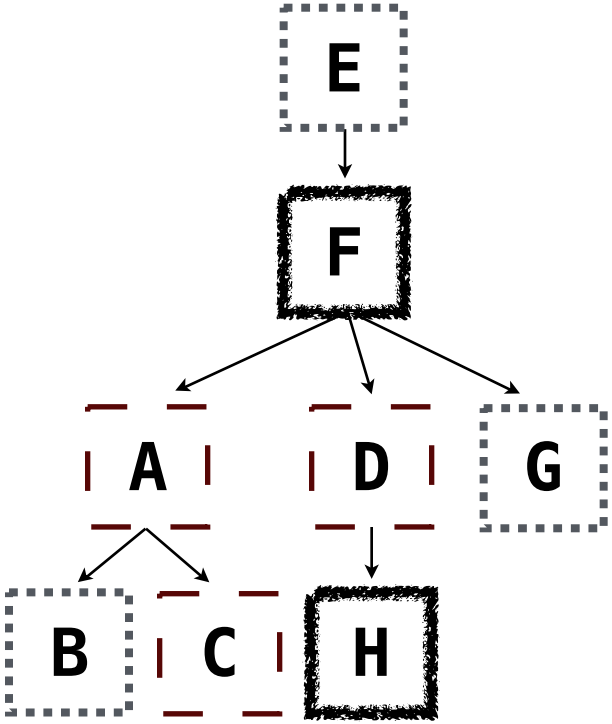
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack    fur: short   y: eisenhower
```



## Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

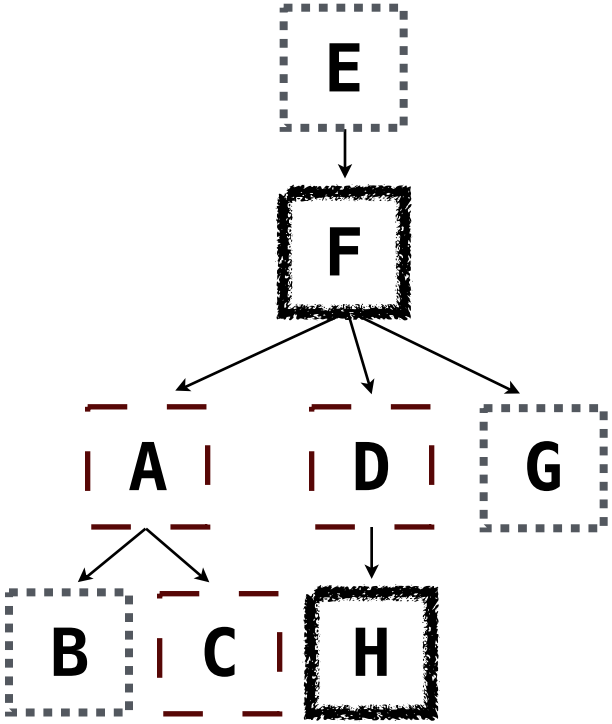
```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack    fur: short   y: eisenhower
x: clinton   fur: long    y: abraham
```



## Combining Multiple Data Sources

Which dogs have an ancestor of the same fur?

```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack     fur: short    y: eisenhower
x: clinton    fur: long     y: abraham
x: grover     fur: short    y: eisenhower
```



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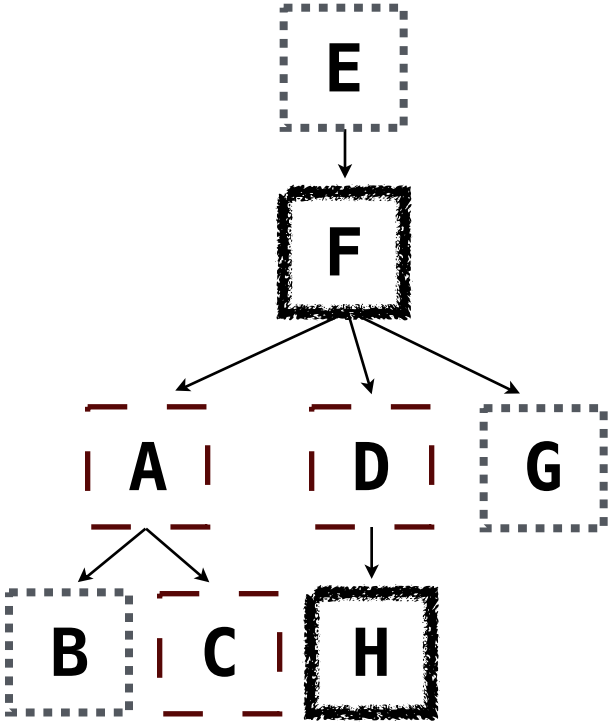
Which dogs have an ancestor of the same fur?

```
logic> (query (dog (name ?x) (fur ?fur))
              (ancestor ?y ?x)
              (dog (name ?y) (fur ?fur)))
Success!
x: barack    fur: short   y: eisenhower
x: clinton   fur: long    y: abraham
x: grover    fur: short   y: eisenhower
x: herbert   fur: curly   y: fillmore
```



# Appending Lists

(Demo)

# Lists in Logic

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Expressions begin with *query* or *fact* followed by relations.

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Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

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```
(fact (append-to-form () ?x ?x))
```

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Simple fact: Conclusion

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Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

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Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

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(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

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      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

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Expressions and their relations are Scheme lists.

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(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

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(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

**() (c d) => (c d)**

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

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(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x

```
() (c d) => (c d)
```

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

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(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

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(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x ?x

() (c d) => (c d)

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

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Simple fact: Conclusion

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(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x ?x

**() (c d) => (c d)**

**(b) (c d) => (b c d)**

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x ?x

**() (c d) => (c d)**

**(b) (c d) => (b c d)**

**(e b) (c d) => (e b c d)**

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x ?x

**() (c d) => (c d)**

**(b) (c d) => (b c d)**

**(e b) (c d) => (e b c d)**

**(e . (b)) (c d) => (e . (b c d))**

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x ?x

**() (c d) => (c d)**

**(b) (c d) => (b c d)**

**(e b) (c d) => (e b c d)**

**(e . (b)) (c d) => (e . (b c d))**

?a

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x ?x

() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)

(e . (b)) (c d) => (e . (b c d))

?a ?r

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

?x ?x

**() (c d) => (c d)**

**(b) (c d) => (b c d)**

**(e b) (c d) => (e b c d)**

**(e . (b)) (c d) => (e . (b c d))**

?a ?r

(?a . ?r)

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

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      (append-to-form       ?r  ?y       ?z ))
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Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
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```

What ?left can append with (c d) to create (e b c d)

```
                    ?x        ?x
() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)

(e . (b)) (c d) => (e . (b c d))
 ?a   ?r    ?y
(?a . ?r)
```

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

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(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
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```

What ?left can append with (c d) to create (e b c d)

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                    ?x        ?x
() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)

(e . (b)) (c d) => (e . (b c d))
 ?a   ?r    ?y      ?a
(?a . ?r)
```

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

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(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

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(query (append-to-form ?left (c d) (e b c d)))
Success!
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```

What ?left can append with (c d) to create (e b c d)

```
                    ?x        ?x
() (c d) => (c d)

(b) (c d) => (b c d)

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(e . (b)) (c d) => (e . (b c d))
 ?a   ?r   ?y      ?a    ?z
(?a . ?r)
```

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

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      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)

```
                    ?x        ?x
() (c d) => (c d)

(b) (c d) => (b c d)

(e b) (c d) => (e b c d)

(e . (b)) (c d) => (e . (b c d))
 ?a   ?r    ?y     ?a     ?z
(?a . ?r)          (?a . ?z)
```

The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

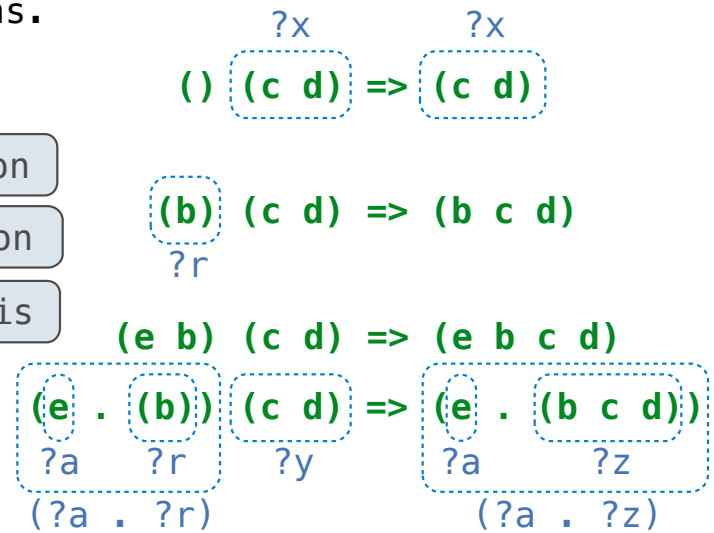
```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)




The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

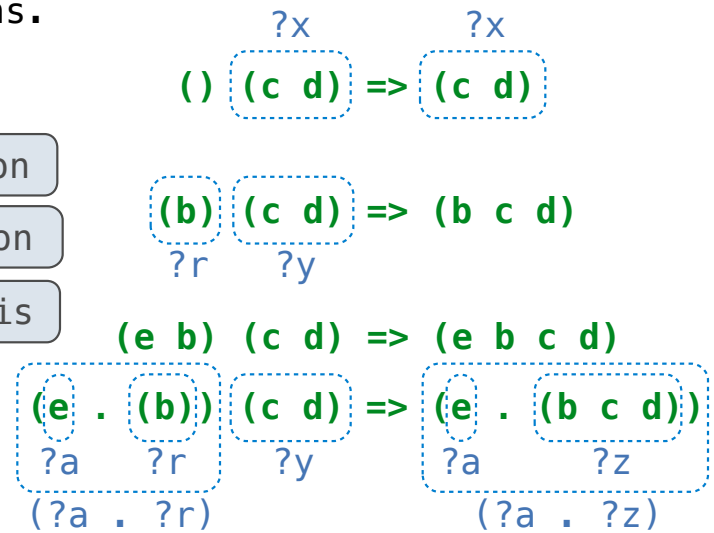
```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)




The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

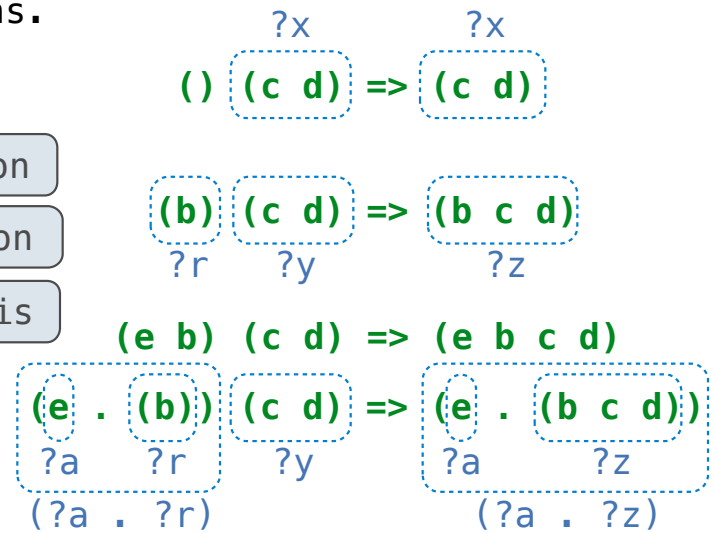
```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)




The interpreter lists all bindings of variables to values that it can find to satisfy the query.

# Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

Expressions and their relations are Scheme lists.

```
(fact (append-to-form () ?x ?x))
```

Simple fact: Conclusion

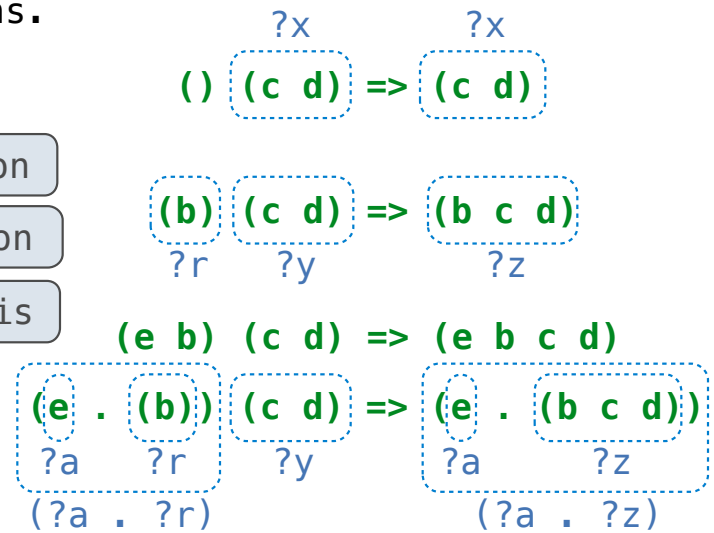
```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z ))
```

Conclusion

Hypothesis

```
(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
```

What ?left can append with (c d) to create (e b c d)




The interpreter lists all bindings of variables to values that it can find to satisfy the query.

(Demo)

# Which Hypotheses Complete append-3?

## Which Hypotheses Complete append-3?

```
I can append (a b) and (1 2) and (x y) to form (a b 1 2 x y).

(fact (append-3 ?x ?y ?z ?xyz)

  1:  (append-to-form ?x ?y ?xy)
      (append-to-form ?y ?z ?yz) )

  2:  (= (append-to-form ?x ?y) ?xy)
      (= (append-to-form ?y ?z) ?yz) )

  3:  (append-to-form ?x ?y ?xy)
      (append-to-form ?xy ?z ?xyz) )

  4:  (= (append-to-form ?x ?y) ?xy)
      (= (append-to-form ?xy ?z) ?xyz) )

  5:  None of the above
```

# Define Base Fact of append-to-form So That No Lists Can Be Empty

## Define Base Fact of append-to-form So That No Lists Can Be Empty

```
; (append-to-form () (1 2 3) (1 2 3))
; (append-to-form (1) (2 3) (1 2 3))
; (append-to-form (1 2) (3) (1 2 3))
; (append-to-form (1 2 3) () (1 2 3))
```

# Define Base Fact of append-to-form So That No Lists Can Be Empty

```
; (append-to-form () (1 2 3) (1 2 3))
; (append-to-form (1) (2 3) (1 2 3))
; (append-to-form (1 2) (3) (1 2 3))
; (append-to-form (1 2 3) () (1 2 3))
```

# Define Base Fact of append-to-form So That No Lists Can Be Empty

```
; (append-to-form () (1 2 3) (1 2 3))
; (append-to-form (1) (2 3) (1 2 3))
; (append-to-form (1 2) (3) (1 2 3))
; (append-to-form (1 2 3) () (1 2 3))
```

# Define Base Fact of append-to-form So That No Lists Can Be Empty

~~; (append-to-form () (1 2 3) (1 2 3))~~

```
; (append-to-form (1) (2 3) (1 2 3))
; (append-to-form (1 2) (3) (1 2 3))
```

~~; (append-to-form (1 2 3) () (1 2 3))~~

*Recursive fact*:

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z))
```

# Define Base Fact of append-to-form So That No Lists Can Be Empty

~~; (append-to-form () (1 2 3) (1 2 3))~~

; (append-to-form (1) (2 3) (1 2 3))

; (append-to-form (1 2) (3) (1 2 3))

~~; (append-to-form (1 2 3) () (1 2 3))~~

**1:** (fact (append-to-form () ?x ?x))

**2:** (fact (append-to-form ?a ?x (?a . ?x)))

**3:** (fact (append-to-form ?a (?b . ?x) (?a ?b . ?x)))

**4:** (fact (append-to-form (?a) ?x (?a . ?x)))

**5:** (fact (append-to-form (?a) (?b . ?x) (?a ?b . ?x)))

*Recursive fact*:

```
(fact (append-to-form (?a . ?r) ?y (?a . ?z))
      (append-to-form       ?r  ?y       ?z))
```

# Permuting Lists

# Anagrams in Logic

# Anagrams in Logic

```
A permutation (i.e., anagram) of a list is:
```

## Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

```
(fact (insert ?a ?r (?a . ?r)))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

```
(fact (insert ?a ?r (?a . ?r)))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element List

```
(fact (insert ?a ?r (?a . ?r)))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element | List | List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element | List | List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
```

List with ?a somewhere

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s)))
```

List with ?a somewhere

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s)))
```

List with ?a somewhere

```
(fact (anagram () ()))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s)))
```

List with ?a somewhere

```
(fact (anagram () ()))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s)))
```

List with ?a somewhere

```
(fact (anagram () ()))

(fact (anagram (?a . ?r) ?b)
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s)))
```

List with ?a somewhere

```
(fact (anagram () ()))

(fact (anagram (?a . ?r) ?b)
      (insert   ?a   ?s   ?b))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

a r t

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s))
```

List with ?a somewhere

```
(fact (anagram () ()))

(fact (anagram (?a . ?r) ?b)
      (insert   ?a   ?s   ?b)
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
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a | r t

Element

List

List with ?a in front

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(fact (insert ?a ?r (?a . ?r)))
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Bigger list with ?a somewhere

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(fact (insert ?a (?b . ?r) (?b . ?s)))
```

List with ?a somewhere

```
(fact (anagram () ()))

(fact (anagram (?a . ?r) ?b)
      (insert   ?a   ?s   ?b))
```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

a | r t

r t

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s)))
```

List with ?a somewhere

```
(fact (anagram () ()))

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```

# Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

a | r t

r t

ar t

Element

List

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r t
ar t
rat
r ta

t r
at r
tar
t ra

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a | r t

r t
ar t
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(Demo)

# Unification

# Pattern Matching

## Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

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```

```
True, {y: b, z: c}
```

```
( (a  b) c  (a  b) )
(   ?x  ?x    ?x   )
```

```
False
```

# Unification

# Unification

```
Unification recursively unifies each pair of corresponding elements in two relations,
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```

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```

Lookup

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```

*Success!*

Symbols/relations without variables only unify if they are the same

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( (a  b) c   (a  b) )
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Lookup

```
c
(a b)
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(a  b)
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```

Lookup

```
c
(a b)
```

```
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```

*Failure.*

# Unifying Variables

## Unifying Variables

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Two relations that contain variables can be unified as well.
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Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

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```
(    ?x        )   (    ?x        )
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Lookup

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```

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**`lookup('?x')`**

# Unifying Variables

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```
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((a ?y c))   ((a b ?z))
```

Lookup

```
(a  ?y  c)
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True, {x: (a ?y c),
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This process is called *grounding*. Two unified expressions have the same grounded form.

**lookup('?x')** ⇨ **(a ?y c)**

# Unifying Variables

Two relations that contain variables can be unified as well.

```
(    ?x       )   (    ?x       )
((a ?y c))       ((a b ?z))
```

Lookup

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(a  ?y  c)
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**lookup('?x')** ⇨ **(a ?y c)**  **lookup('?y')**

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**lookup('?x')** ⇨ **(a ?y c)**  **lookup('?y')** ⇨ **b**

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True, {x: (a ?y c),
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This process is called *grounding*. Two unified expressions have the same grounded form.

**lookup('?x')** ⇨ **(a ?y c)**  **lookup('?y')** ⇨ **b**  **ground('?x')**

# Unifying Variables

Two relations that contain variables can be unified as well.

```
(    ?x      )   (    ?x      )
((a ?y c))       (a b ?z))
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

Lookup

```
(a  ?y  c)
(a   b ?z)
```

Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

**lookup('?x')** ⇨ **(a ?y c)**  **lookup('?y')** ⇨ **b**  **ground('?x')** ⇨ **(a b c)**

## Implementing Unification

```
def unify(e, f, env):
  e = lookup(e, env)
  f = lookup(f, env)
  if e == f:
      return True
  elif isvar(e):
      env.define(e, f)
      return True
  elif isvar(f):
      env.define(f, e)
      return True
  elif scheme_atomp(e) or scheme_atomp(f):
      return False
  else:
      return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

## Implementing Unification

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def unify(e, f, env):
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```

1. Look up variables in the current environment

# Implementing Unification

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        env.define(f, e)
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    else:
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```

1. Look up variables in the current environment

2. Establish new bindings to unify elements.

# Implementing Unification

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    e = lookup(e, env)
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        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

# Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
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Recursively unify the first and rest of any lists.

# Implementing Unification

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(   ?x   c   ?x   )
```

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```
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(    ?x    c    ?x    )

env: {                }
```

# Implementing Unification

```
def unify(e, f, env):
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        env.define(e, f)
        return True
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        env.define(f, e)
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( ?x     c   ?x   )
```

env: { x: (a b) }

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    f = lookup(f, env)
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        return True
    elif isvar(e):
        env.define(e, f)
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        env.define(e, f)
        return True
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        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
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        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
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```
( (a  b) c (a  b) )
(   ?x   c   ?x   )
```

Lookup

```
(a  b)
(a  b)
```

env: { x: (a b) }

# Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
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    if e == f:
        return True
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        return True
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( (a  b) c (a  b) )
(   ?x   c   ?x   )
```

Lookup

```
(a  b)
(a  b)
```

env: { x: (a b) }

# Search

# Searching for Proofs

# Searching for Proofs

```
The Logic interpreter searches
the space of facts to find
unifying facts and an env that
prove the query to be true.
```

# Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

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```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
```

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```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
```

```
(app (e . ?r) (c d) (e b c d))
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(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

# Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
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```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

Variables are local to facts & queries

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The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

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(fact  (app () ?x ?x))

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```

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(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

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```
(app (e . ?r) (c d) (e b c d))
```

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(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

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```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

# Searching for Proofs

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(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
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```

```
(app ?left (c d) (e b c d))
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    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
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```

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    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

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    {r2: (), x: (c d)}
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```
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Variables are local to facts & queries

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*?left*:

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```
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```
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Variables are local to facts & queries

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    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
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(app ?r2 (c d) (c d))
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(app () ?x ?x)
```

```
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Variables are local to facts & queries

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(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
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(app (e . ?r) (c d) (e b c d))
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Variables are local to facts & queries

```
?left: (e .
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```
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# Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

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(query (app ?left (c d) (e b c d)))
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# Depth-First Search

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(Demo)

# Addition

(Demo)