61A Extra Lecture 12

Announcements

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Quines

Quine: A Program That Evaluates to Itself

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Self-evaluating expressions evaluate to themselves
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(Demo)

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Church Turing thesis (1936)**.** Turing machines can do anything that can be described by any physically harnessable process of this universe**.** (Equivalently for lambda calculus)

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Church Turing thesis (1936). Turing machines can do anything that can be described by any physically harnessable process of this universe. (Equivalently for lambda calculus)

The Church–Turing thesis is a not a theorem, but instead a claim that has withstood time

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This result was proved about functions on integers, but every function was given a number and the proof involves calling a function on itself

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The outcome hinged on self-referential mathematical statements, akin to Quine's paradox:

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The outcome hinged on self-referential mathematical statements, akin to Quine's paradox:

"Yields falsehood when preceded by its quotation" yields falsehood when preceded by its quotation.

Halting Problem

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`(define (length exp) ...)` that computes the length of an expression?

`(define (errors? procedure arg) ...)` that returns whether `(procedure arg)` would error?

`(define (halts? procedure arg) ...)` that returns whether `(procedure arg)` would terminate?

The Halting Problem

Exercise (from SICP). Given a one-argument procedure **p** and an object **a**, **p** is said to *halt* on **a** if evaluating the expression (p a) returns a value (as opposed to terminating with an error message or running forever). Show that it is impossible to write a procedure **halts?** that correctly determines whether **p** halts on **a** for any procedure **p** and object **a.** Use the following reasoning: If you had such a procedure **halts?,** you could implement the following program:

```
(define (run-forever) (run-forever))

(define (try p)
  (if (halts? p p)
      (run-forever)
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Now consider evaluating the expression (try try).

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- (halts? p p) runs forever, which means halts? doesn't return the answer we desire

If (try try) errors, then it must be because (halts? p p) errors, which is also wrong

Decidability

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- Is a sub-expression within a procedure ever executed?
- What is the shortest program that is equivalent to some procedure?
- Are two procedures equivalent?