61A Extra Lecture 12
Announcements
Quines
Quine: A Program That Evaluates to Itself
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Self-evaluating expressions evaluate to themselves
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Self-evaluating expressions evaluate to themselves

Some expressions evaluate to themselves by constructing expressions
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(Demo)
Church-Turing Thesis

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Church Turing thesis (1936). Turing machines can do anything that can be described by any physically harnessable process of this universe. (Equivalently for lambda calculus)
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The Church–Turing thesis is a not a theorem, but instead a claim that has withstood time
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Since $F$ has a fixed point, there must be some $e$ that prints out the source code of $e$

This result was proved about functions on integers, but every function was given a number and the proof involves calling a function on itself
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The outcome hinged on self-referential mathematical statements, akin to Quine's paradox:
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The outcome hinged on self-referential mathematical statements, akin to Quine's paradox:

"Yields falsehood when preceded by its quotation" yields falsehood when preceded by its quotation.
Halting Problem
Computability
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(define (length exp) ...) that computes the length of an expression?
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We can (\texttt{define (eval exp env) ...}) and (\texttt{define (apply procedure args) ...}), so can we (\texttt{define (length exp) ...}) that computes the length of an expression? (\texttt{define (errors? procedure arg) ...}) that returns whether (procedure arg) would error?
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We can \texttt{(define (eval exp env) \ldots)} and \texttt{(define (apply procedure args) \ldots)}, so can we \texttt{(define (length exp) \ldots)} that computes the length of an expression?

\texttt{(define (errors? procedure arg) \ldots)} that returns whether \texttt{(procedure arg)} would error?

\texttt{(define (halts? procedure arg) \ldots)} that returns whether \texttt{(procedure arg)} would terminate?
The Halting Problem

**Exercise** (from SICP). Given a one-argument procedure \( p \) and an object \( a \), \( p \) is said to halt on \( a \) if evaluating the expression \( (p \ a) \) returns a value (as opposed to terminating with an error message or running forever). Show that it is impossible to write a procedure \( \text{halts?} \) that correctly determines whether \( p \) halts on \( a \) for any procedure \( p \) and object \( a \). Use the following reasoning: If you had such a procedure \( \text{halts?} \), you could implement the following program:

```
(define (run-forever) (run-forever))

(define (try p)
  (if (halts? p p)
      (run-forever)
      'halted))
```

Now consider evaluating the expression \( (\text{try \ try}) \).
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Show that it is impossible to write a procedure `halts?` that correctly determines whether `p` halts on `a` for any procedure `p` and object `a`.

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  \begin{itemize}
    \item (\texttt{halts? p p}) returns true, so (\texttt{halts? try try}) is true!
    \item (\texttt{halts? p p}) runs forever, which means \texttt{halts?} doesn't return the answer we desire
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- `(halts? p p)` returns true, so `(halts? try try)` is true!
- `(halts? p p)` runs forever, which means `halts?` doesn't return the answer we desire

If `(try try)` errors, then it must be because `(halts? p p)` errors, which is also wrong
Decidability
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Problems (such as halts?) that cannot be computed are called undecidable.
The Halting Problem is Undecidable

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This property holds for all programming languages that can compute anything.

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Problems (such as halts?) that cannot be computed are called *undecidable*.
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Undecidable problems:

- Does a procedure terminate on all inputs?
- Is a sub-expression within a procedure ever executed?
- What is the shortest program that is equivalent to some procedure?
- Are two procedures equivalent?