61A Lecture 7
Announcements
Hog Contest Rules

[cs61a.org/proj/hog_contest]
Hog Contest Rules

• Up to two people submit one entry;
  Max of one entry per person

[link to hog contest rules page]
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  against which you win more than
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Fall 2011 Winners
Kaylee Mann
Yan Duan & Ziming Li
Brian Prike & Zhenghao Qian
Parker Schuh & Robert Chatham

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**Fall 2016 Winners...**

[cs61a.org/proj/hog_contest](http://cs61a.org/proj/hog_contest)
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Fall 2016 Winners...
Order of Recursive Calls
The Cascade Function

(Demo)

Interactive Diagram
The Cascade Function

```
1 def cascade(n):
2     if n < 10:
3         print(n)
4     else:
5         print(n)
6         cascade(n//10)
7         print(n)
8     cascade(123)
```

(Demo)

Interactive Diagram
The Cascade Function

```python
1 def cascade(n):
2     if n < 10:
3         print(n)
4     else:
5         print(n)
6         cascade(n//10)
7         print(n)
8
cascade(123)
```

Program output:
123
12
1
12

Interactive Diagram
The Cascade Function

```python
1   def cascade(n):
2       if n < 10:
3           print(n)
4       else:
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6           cascade(n//10)
7       print(n)
8
9   cascade(123)
```

Program output:

```
123
12
1
12
```

(Demo)

- Each cascade frame is from a different call to `cascade`. 

Interactive Diagram
The Cascade Function

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```

(Demo)

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.

Interactive Diagram
The Cascade Function

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```

(Demo)

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram
The Cascade Function

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
    print(n)
cascade(123)
```

Program output:

```
123
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(Demo)

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The Cascade Function

def cascade(n):
    if n < 10:
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Program output:
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12
1
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(Demo)

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
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The Cascade Function

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Program output:

123
12
1
12

Interactive Diagram
The Cascade Function

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(Demo)

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Program output:
```
123
12
1
12
```
Two Definitions of Cascade

(Demo)
Two Definitions of Cascade

(Demo)

def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```python
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure
Example: Inverse Cascade
Inverse Cascade

Write a function that prints an inverse cascade:
Inverse Cascade

Write a function that prints an inverse cascade:

```
1
12
123
1234
123
12
1
1
```
**Inverse Cascade**

Write a function that prints an inverse cascade:

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```
Inverse Cascade

Write a function that prints an inverse cascade:

```python
def inverse_cascade(n):
grow(n)
print(n)
shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
g(n)
```
Inverse Cascade

Write a function that prints an inverse cascade:

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow = lambda n: f_then_g(grow, shrink, n)
shrink = lambda n: f_then_g(grow, shrink, n)
```
Inverse Cascade

Write a function that prints an inverse cascade:

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow = lambda n: f_then_g(grow, print, n//10)
shrink = lambda n: f_then_g(print, shrink, n//10)
```
Tree Recursion
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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call
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\[ n: 0, 1, 2, 3, 4, 5, 6, 7, 8, \]

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

n: 0, 1, 2, 3, 4, 5, 6, 7, 8,

fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21,
Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

\[
\begin{align*}
  n &: \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
  \text{fib}(n) &: \quad 0, 1, 1, 2, 3, 5, 8, 13, 21,
\end{align*}
\]

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Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

\[
\begin{align*}
\text{n:} & \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
\text{fib(n):} & \quad 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465
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```python
def fib(n):
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\end{align*}
\]

```python
def fib(n):
    if n == 0:
```
Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

\[
\begin{array}{cccccccccccc}
\text{n:} & 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7, & 8, & \ldots, & 35 \\
\text{fib(n):} & 0, & 1, & 1, & 2, & 3, & 5, & 8, & 13, & 21, & \ldots, & 9,227,465 \\
\end{array}
\]

def fib(n):
    if n == 0:
        return 0

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\end{align*}
\]

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```
Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call.

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```python
def fib(n):
    if n == 0:
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    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

A Tree-Recursive Process

The computational process of fib evolves into a tree structure.
A Tree-Recursive Process

The computational process of fib evolves into a tree structure

fib(5)
A Tree-Recursive Process

The computational process of fib evolves into a tree structure

```
fib(5)
  \------->
   fib(3)
```
A Tree-Recursive Process

The computational process of fib evolves into a tree structure

```
   fib(5)
  /     \
fib(3)   fib(4)
```
A Tree-Recursive Process

The computational process of fib evolves into a tree structure
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The computational process of fib evolves into a tree structure.
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The computational process of fib evolves into a tree structure.
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The computational process of fib evolves into a tree structure

```
fib(5)
  /   \
/     \
fib(3)  fib(4)
  /   \
/     \
fib(1) fib(2)
  /   \
/     \
fib(0) fib(1)
  /   \
/     \
0     1
```

```
fib(3)
  /   \
/     \
fib(1) fib(2)
  /   \
/     \
fib(0) fib(1)
  /   \
/     \
0     1
```

```
fib(4)
  /   \
/     \
fib(2)
  /   \
/     \
fib(0) fib(1)
  /   \
/     \
0     1
```

```
fib(2)
  /   \
/     \
fib(0) fib(1)
  /   \
/     \
0     1
```

```
fib(1)
  /   \
/     \
fib(0) fib(1)
  /   \
/     \
0     1
```

```
fib(0)
  /   \
/     \
0     1
```
A Tree-Recursive Process

The computational process of fib evolves into a tree structure.
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The computational process of fib evolves into a tree structure
A Tree-Recursive Process

The computational process of fib evolves into a tree structure
A Tree-Recursive Process

The computational process of $\text{fib}$ evolves into a tree structure

![Tree Diagram](image-url)
A Tree-Recursive Process

The computational process of fib evolves into a tree structure
A Tree-Recursive Process

The computational process of fib evolves into a tree structure
A Tree-Recursive Process

The computational process of fib evolves into a tree structure
Repetition in Tree-Recursive Computation
Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times.
Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times.
Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times

(We will speed up this computation dramatically in a few weeks by remembering results)
Example: Counting Partitions
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

```python
count_partitions(6, 4)
```
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

\[
\text{count_partitions}(6, 4)
\]

\[
\begin{align*}
2 + 4 &= 6 \\
1 + 1 + 4 &= 6 \\
3 + 3 &= 6 \\
1 + 2 + 3 &= 6 \\
1 + 1 + 1 + 3 &= 6 \\
2 + 2 + 2 &= 6 \\
1 + 1 + 2 + 2 &= 6 \\
1 + 1 + 1 + 1 + 2 &= 6 \\
1 + 1 + 1 + 1 + 1 + 1 &= 6
\end{align*}
\]
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

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2 + 4 &= 6 \\
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1 + 1 + 1 + 1 + 2 &= 6 \\
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The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

\[
\text{count\_partitions}(6, 4)
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\[
2 + 4 = 6
\]
\[
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\]
\[
3 + 3 = 6
\]
\[
1 + 2 + 3 = 6
\]
\[
1 + 1 + 1 + 3 = 6
\]
\[
2 + 2 + 2 = 6
\]
\[
1 + 1 + 2 + 2 = 6
\]
\[
1 + 1 + 1 + 1 + 2 = 6
\]
\[
1 + 1 + 1 + 1 + 1 + 1 = 6
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\begin{align*}
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1 + 1 + 1 + 1 + 2 &= 6 \\
1 + 1 + 1 + 1 + 1 + 1 &= 6
\end{align*}
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Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

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\text{count_partitions}(6, 4)
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\texttt{count_partitions(6, 4)}
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```

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```python
def count_partitions(n, m):
    # Implementation goes here
```
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def count_partitions(n, m):
    if n == 0:
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        with_m = count_partitions(n-m, m)
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```python
def count_partitions(n, m):
    if n < m:
        return 1
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

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Counting Partitions

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def count_partitions(n, m):
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```python
def count_partitions(n, m):
    if m > n:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```
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(Demo)

Interactive Diagram