Announcements
Data Abstraction
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• Compound values combine other values together
Data Abstraction

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  ▪ A date: a year, a month, and a day
Data Abstraction

- Compound values combine other values together
  - A date: a year, a month, and a day
  - A geographic position: latitude and longitude
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  ▪ A date: a year, a month, and a day
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• Data abstraction lets us manipulate compound values as units
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• Isolate two parts of any program that uses data:
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  ▪ How data are represented (as parts)
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Rational Numbers
Rational Numbers

\[ \frac{\text{numerator}}{\text{denominator}} \]
Rational Numbers

numerator

\[\frac{\text{numerator}}{\text{denominator}}\]

denominator

Exact representation of fractions
Rational Numbers

\[
\text{numerator} \quad \frac{\text{numerator}}{\text{denominator}} \quad \text{denominator}
\]

Exact representation of fractions

A pair of integers
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
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Exact representation of fractions

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As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:
Rational Numbers

\[
\frac{\text{numerator}}{\text{denominator}}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number \( x \)
Rational Numbers

\[
\begin{array}{c}
\text{numerator} \\
\hline
\text{denominator}
\end{array}
\]

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- \text{rational}(n, d) \text{ returns a rational number } x
- \text{numer}(x) \text{ returns the numerator of } x
Rational Numbers

\[
\begin{align*}
\text{numerator} & \\
\text{-------------------} & \\
\text{denominator} & \\
\end{align*}
\]

Exact representation of fractions
A pair of integers
As soon as division occurs, the exact representation may be lost! (Demo)
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Rational Numbers

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

\[
\text{rational}(n, d) \quad \text{returns a rational number } x
\]

- \( \text{numerator}(x) \) returns the numerator of \( x \)
- \( \text{denominator}(x) \) returns the denominator of \( x \)
Rational Numbers

A rational number is a pair of integers: $\frac{\text{numerator}}{\text{denominator}}$.

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- $\text{rational}(n, d)$ returns a rational number $x$
- $\text{numer}(x)$ returns the numerator of $x$
- $\text{denom}(x)$ returns the denominator of $x$
Rational Number Arithmetic

Example

General Form
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5}
\]

Example

General Form
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

General Form
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy}
\]
Rational Number Arithmetic

Example

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
\]

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]
Rational Number Arithmetic

\[
\begin{align*}
\frac{3}{2} \times \frac{3}{5} &= \frac{9}{10} \\
\end{align*}
\]

Example

\[
\begin{align*}
\frac{3}{2} + \frac{3}{5} &= \frac{15}{10} + \frac{6}{10} = \frac{21}{10}
\end{align*}
\]

General Form

\[
\begin{align*}
\frac{nx}{dx} \times \frac{ny}{dy} &= \frac{nx \times ny}{dx \times dy}
\end{align*}
\]
Rational Number Arithmetic

\[
\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}
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\[
\frac{3}{2} + \frac{3}{5} = \frac{21}{10}
\]

**Example**

**General Form**

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]
Rational Number Arithmetic

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\end{align*}
\]

Example

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

General Form
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\[
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\]

Example

General Form

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

\[
\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}
\]
Rational Number Arithmetic Implementation

- `rational(n, d)` returns a rational number \( x \)
- `numer(x)` returns the numerator of \( x \)
- `denom(x)` returns the denominator of \( x \)

\[
\frac{nx}{dx} \times \frac{ny}{dy} = \frac{nx \times ny}{dx \times dy}
\]

\[
\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx \times dy + ny \times dx}{dx \times dy}
\]
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                  denom(x) * denom(y))
```

- `rational(n, d)` returns a rational number \( x \)
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- `numer(x)` returns the numerator of \( x \)
- `denom(x)` returns the denominator of \( x \)

These functions implement an abstract representation for rational numbers.

\[
\begin{align*}
\frac{nx}{dx} \times \frac{ny}{dy} &= \frac{nx \times ny}{dx \times dy} \\
\frac{nx}{dx} + \frac{ny}{dy} &= \frac{nx \times dy + ny \times dx}{dx \times dy}
\end{align*}
\]
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

```python
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x
- denom(x) returns the denominator of x

These functions implement an abstract representation for rational numbers
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
def print_rational(x):
    print(numer(x), '/', denom(x))
```

- `rational(n, d)` returns a rational number $x$
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- `denom(x)` returns the denominator of $x$

These functions implement an abstract representation for rational numbers.
Rational Number Arithmetic Implementation

```python
def mul_rational(x, y):
    return rational(numer(x) * numer(y), denom(x) * denom(y))

def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

- `rational(n, d)` returns a rational number $x$
- `numer(x)` returns the numerator of $x$
- `denom(x)` returns the denominator of $x$
Pairs
Representing Pairs Using Lists
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```python
>>> pair = [1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
```

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal: Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator

```python
>>> from operator import getitem
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

A list literal:
Comma-separated expressions in brackets

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Element selection using the selection operator
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```python
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Comma-separated expressions in brackets

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>>> pair
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"Unpacking" a list

```python
>>> x, y = pair
```

```python
>>> x
1
```

```python
>>> y
2
```

Element selection using the selection operator

```python
>>> pair[0]
1
```

```python
>>> pair[1]
2
```

Element selection function

```python
>>> from operator import getitem
```

```python
>>> getitem(pair, 0)
1
```

```python
>>> getitem(pair, 1)
2
```
Representing Pairs Using Lists

```python
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator

Element selection function

More lists next lecture
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
Representing Rational Numbers

```python
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
```

Construct a list
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
Representing Rational Numbers

def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
Representing Rational Numbers

```python
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]

def numer(x):
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    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

Construct a list

Select item from a list
Representing Rational Numbers

```python
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
    Construct a list

def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
    Select item from a list

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
    Select item from a list

(Demo)
```
Reducing to Lowest Terms

Example:
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} * \frac{5}{3} = \frac{5}{2} + \frac{1}{10}
\]

\[
\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \\
\frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2}
\]
Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1/3}{1/3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1/25}{1/25} = \frac{1}{2}
\]
Reducing to Lowest Terms

Example:

\[
\begin{align*}
\frac{3}{2} \times \frac{5}{3} &= \frac{5}{2} \\
\frac{2}{5} + \frac{1}{10} &= \frac{1}{2}
\end{align*}
\]

from fractions import gcd
from fractions import gcd

def rational(n, d):

Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
\]

\[
\frac{15}{6} \times \frac{1}{3} = \frac{5}{2} \quad \frac{25}{50} \times \frac{1}{25} = \frac{1}{2}
\]
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    Reduced to Lowest Terms

Example:

\[
\begin{array}{ccc}
\frac{3}{2} \times \frac{5}{3} &=& \frac{5}{2} \\
\frac{2}{5} + \frac{1}{10} &=& \frac{1}{2}
\end{array}
\]

\[
\begin{array}{ccc}
\frac{15}{6} \times \frac{1}{3} &=& \frac{5}{2} \\
\frac{25}{50} \times \frac{1}{25} &=& \frac{1}{2}
\end{array}
\]
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)
    return n // g, d // g
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
    g = gcd(n, d)
    return [n//g, d//g]
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Reducing to Lowest Terms

Example:

\[
\frac{3}{2} \times \frac{5}{3} = \frac{5}{2} \quad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}
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(Demo)
Abstraction Barriers
Abstraction Barriers
### Abstraction Barriers

<table>
<thead>
<tr>
<th>Parts of the program that...</th>
<th>Treat rationals as...</th>
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## Abstraction Barriers

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Use rational numbers to perform computation whole data values

Create rationals or implement rational operations
# Abstraction Barriers

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**Implementation of lists**
add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
Violating Abstraction Barriers

Does not use constructors

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Twice!

No selectors!
Violating Abstraction Barriers

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```

- Does not use constructors
- Twice!
- No selectors!
- And no constructor!
Violating Abstraction Barriers
Data Representations
What is Data?
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- We need to guarantee that constructor and selector functions work together to specify the right behavior.
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- Behavior condition: If we construct rational number $x$ from numerator $n$ and denominator $d$, then $\text{numerator}(x)/\text{denominator}(x)$ must equal $n/d$
What is Data?

- We need to guarantee that constructor and selector functions work together to specify the right behavior.

- Behavior condition: If we construct rational number x from numerator n and denominator d, then numer(x)/denom(x) must equal n/d.

- Data abstraction uses selectors and constructors to define behavior.
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You can recognize an abstract data representation by its behavior.

(Demo)
Rationals Implemented as Functions
Rationals Implemented as Functions

def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select

def numer(x):
    return x('n')

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Rationals Implemented as Functions

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Selector calls x.
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This function represents a rational number

Constructor is a higher-order function

Selector calls x

x = rational(3, 8)
numer(x)
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
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def numer(x):
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Interactive Diagram