61A Lecture 27
Announcements
Interpreting Scheme
The Structure of an Interpreter
The Structure of an Interpreter

Eval

Apply
The Structure of an Interpreter

Base cases: Eval

Apply
The Structure of an Interpreter

Base cases:
• Primitive values (numbers)
The Structure of an Interpreter

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Recursive calls:
The Structure of an Interpreter

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• Primitive values (numbers)

Recursive calls:
• Eval(operator, operands) of call expressions
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- Built-in primitive procedures
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**Apply**

Base cases:
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Recursive calls:
- Eval(body) of user-defined procedures

Requires an environment for symbol lookup

Creates a new environment each time a user-defined procedure is applied
Special Forms
Scheme Evaluation
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The scheme_eval function choose behavior based on expression form:
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- Self-evaluating expressions are returned as values
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The `scheme_eval` function choose behavior based on expression form:

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\[
\text{(if } \text{<predicate>} \text{ <consequent>} \text{ <alternative>)}
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\text{(lambda (}<\text{formal-parameters}>\text{) <body>)}
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(define <name> <expression>)
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(\text{lambda} \ (<\text{formal-parameters}>)) \ <\text{body}>
\]
\[
(\text{define} \ <\text{name}> \ <\text{expression}>)
\]
\[
(<\text{operator}> \ <\text{operand 0}> ... \ <\text{operand k}>)
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**Scheme Evaluation**

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(if <predicate> <consequent> <alternative>)
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Any combination that is not a known special form is a call expression.
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\text{(if <predicate> <consequent> <alternative>)} \\
\text{(lambda (<formal-parameters>) <body>)} \\
\text{(define <name> <expression>)} \\
\text{(<operator> <operand 0> ... <operand k>)}
\]

Special forms are identified by the first list element.

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\[
\text{(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))) )}
\]
Scheme Evaluation

The scheme_eval function choose behavior based on expression form:
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\[
\begin{align*}
\text{if} & \quad \langle \text{predicate} \rangle \quad \langle \text{consequent} \rangle \quad \langle \text{alternative} \rangle \\
\text{lambda} & \quad \langle \text{formal-parameters} \rangle \quad \langle \text{body} \rangle \\
\text{define} & \quad \langle \text{name} \rangle \quad \langle \text{expression} \rangle \\
\text{operator} & \quad \langle \text{operand 0} \rangle \; \ldots \; \langle \text{operand k} \rangle
\end{align*}
\]

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Any combination that is not a known special form is a call expression

\[
\begin{align*}
\text{define} & \quad \text{demo} \quad s \\
& \quad \text{if} \quad \text{null?} \quad s \quad '(3) \quad \text{cons} \quad \text{car} \quad s \quad \text{demo} \quad \text{cdr} \quad s)
\end{align*}
\]

(demo (list 1 2))
Logical Forms
Logical Special Forms
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Logical forms may only evaluate some sub-expressions
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- **If** expression: \( (if \ <predicate> \ <consequent> \ <alternative> \) \)
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- **And** and **or**: \((\text{and} \ <e_1> \ ... \ <e_n>), \ (\text{or} \ <e_1> \ ... \ <e_n>)\)
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- **If** expression: \((\text{if } \text{<predicate>} \text{<consequent>} \text{<alternative>})\)
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- **Cond** expression: \((\text{cond } (\text{<p1> <e1>}) ... (\text{<pn> <en>}) \text{ (else } \text{<e>}))\)
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The value of an if expression is the value of a sub-expression:
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- **Cond expression:** \((\text{cond} \ (<p_1> \ <e_1>) \ ... \ (<p_n> \ <e_n>) \ (\text{else} \ <e>))\)

The value of an if expression is the value of a sub-expression:

- Evaluate the predicate
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The value of an if expression is the value of a sub-expression:

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• **Cond** expression: \((\text{cond} \ (<p_1> \ <e_1>) \ ... \ (<p_n> \ <e_n>) \ (\text{else} \ <e>))\)

The value of an if expression is the value of a sub-expression:

• Evaluate the predicate

• Choose a sub-expression: <consequent> or <alternative>

• Evaluate that sub-expression to get the value of the whole expression
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(Demo)
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The quote special form evaluates to the quoted expression, which is not evaluated
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(quote <expression>)  (quote (+ 1 2))  evaluates to the three-element Scheme list  (+ 1 2)
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\begin{align*}
\text{(quote } <\text{expression}> \text{)} & \quad \text{(quote } (+ 1 2) \text{)} \quad \text{evaluates to the three-element Scheme list } (+ 1 2) \\
\end{align*}

The <expression> itself is the value of the whole quote expression
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The quote special form evaluates to the quoted expression, which is not evaluated

\[(quote \langle expression\rangle) \quad (quote (+ 1 2))\]  

\[\text{evaluates to the three-element Scheme list} \quad (+ 1 2)\]

The \langle expression\rangle itself is the value of the whole quote expression

`\langle expression\rangle` is shorthand for `(quote \langle expression\rangle)`
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The scheme_read parser converts shorthand ' to a combination that starts with quote
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(Demo)
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\[(\text{lambda} (<\text{formal-parameters}>)) \text{ <body>}\]
Lambda Expressions

Lambda expressions evaluate to user-defined procedures

\[
\text{lambda} \ (<\text{formal-parameters}> \ ) \ <\text{body}> \\
\text{lambda} \ (x) \ (* \ x \ x)
\]
Lambda Expressions

Lambda expressions evaluate to user-defined procedures

```
(lambda (<formal-parameters>) <body>)

(lambda (x) (* x x))
```

class LambdaProcedure:
    def __init__(self, formals, body, env):
        self.formals = formals
        self.body = body
        self.env = env
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Lambda expressions evaluate to user-defined procedures

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        self.formals = formals                      # A scheme list of symbols
        self.body = body                            # A scheme list of expressions
        self.env = env
```

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Lambda Expressions

Lambda expressions evaluate to user-defined procedures

\[
\text{lambda} \ (\text{<formal-parameters>}) \ \text{<body>}
\]

\[
\text{lambda} \ (x) \ (* \ x \ x)
\]

class LambdaProcedure:

def __init__(self, formals, body, env):
    self.formals = formals  # A scheme list of symbols
    self.body = body        # A scheme list of expressions
    self.env = env          # A Frame instance
Frames and Environments
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A frame represents an environment by having a parent frame
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Frames are Python instances with methods *lookup* and *define*
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In Project 4, Frames do not hold return values.
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<table>
<thead>
<tr>
<th>g: Global frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
</tr>
<tr>
<td>z</td>
</tr>
</tbody>
</table>
**Frames and Environments**

A frame represents an environment by having a parent frame.

Frames are Python instances with methods `lookup` and `define`.

In Project 4, Frames do not hold return values.

```
g: Global frame
    y | 3
    z | 5

f1: [parent=g]
    x | 2
    z | 4
```
Frames and Environments

A frame represents an environment by having a parent frame.

Frames are Python instances with methods `lookup` and `define`.

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<thead>
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(Demo)
Define Expressions
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(define <name> <expression>)
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\[(\text{define} \ <\text{name}> \ <\text{expression}>\)]

1. Evaluate the \(<\text{expression}>\)
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Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

\[(\text{define } \langle\text{name}\rangle \ \langle\text{expression}\rangle)\]

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2. Bind \langle name \rangle to its value in the current frame

\[(\text{define } x (+ 1 2))\]
Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

\[
\text{(define } \text{name} \text{ expression})
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\[
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\]

Procedure definition is shorthand of define with a lambda expression
Define Expressions

Define binds a symbol to a value in the first frame of the current environment.

\[
\text{(define } \text{<name> } \text{<expression>)}
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\[
\text{(define } (\text{<name> } \text{<formal parameters>}) \text{<body>)}
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Define binds a symbol to a value in the first frame of the current environment.

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(\text{define } x \ (\ + \ 1 \ 2))
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Applying User-Defined Procedures
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(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))))
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\]

- **g**: Global frame
- **demo**: LambdaProcedure instance [parent=g]
Applying User-Defined Procedures

To apply a user-defined procedure, create a new frame in which formal parameters are bound to argument values, whose parent is the \texttt{env} attribute of the procedure.

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\begin{verbatim}
(define (demo s) (if (null? s) '(3) (cons (car s) (demo (cdr s)))))

(demo (list 1 2))
\end{verbatim}
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\[
\text{(demo (list 1 2))}
\]
Eval/Apply in Lisp 1.5
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apply[fn;x;a] =
    [atom[fn] \rightarrow [eq[fn;CAR] \rightarrow caar[x];
        eq[fn;CDR] \rightarrow cdar[x];
        eq[fn;CONS] \rightarrow cons[car[x];cadr[x]];
        eq[fn;ATOM] \rightarrow atom[car[x]];
        eq[fn;EQ] \rightarrow eq[car[x];cadr[x]];
        T \rightarrow apply[eval[fn;a];x;a]];
    eq[car[fn];LAMBDA] \rightarrow eval[caddr[fn];pairlis[cadr[fn];x;a]];
    eq[car[fn];LABEL] \rightarrow apply[caddr[fn];x;cons[cons[cadr[fn];
        caddr[fn]]];a]]]

eval[e;a] = [atom[e] \rightarrow cdr[assoc[e;a]];
    atom[car[e]] \rightarrow
        [eq[car[e];QUOTE] \rightarrow cdr[e];
        eq[car[e];COND] \rightarrow evcon[cdr[e];a];
        T \rightarrow apply[car[e];evlis[cdr[e];a];a];
        T \rightarrow apply[car[e];evlis[cdr[e];a];a]]]