Announcements

☐ HW11 due next Wednesday

☐ Scheme project out
Dynamic Scope

The way in which names are looked up in Scheme and Python is called *lexical scope* (or *static scope*)

**Lexical scope**: The parent of a frame is the environment in which a procedure was *defined*

**Dynamic scope**: The parent of a frame is the environment in which a procedure was *called*

```scheme
(define f (lambda (x) (+ x y)))
(define g (lambda (x y) (f (+ x x))))
(g 3 7)
```

**Lexical scope**: The parent for `f`'s frame is the global frame

*Error: unknown identifier: y*

**Dynamic scope**: The parent for `f`'s frame is `g`'s frame

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Functional Programming

All functions are pure functions

No re-assignment and no mutable data types

Name-value bindings are permanent

Advantages of functional programming:

• The value of an expression is independent of the order in which sub-expressions are evaluated

• Sub-expressions can safely be evaluated in parallel or lazily

• Referential transparency: The value of an expression does not change when we substitute one of its sub-expression with the value of that sub-expression

*But... Can we make basic loops efficient?*

Yes!
Iteration Versus Recursion in Python

In Python, recursive calls always create new active frames

```python
def factorial(n):
    if n == 0:
        return 1
    return n * factorial(n - 1)
```

```python
def factorial(n):
    total = 1
    while n > 0:
        n, total = n - 1, total * n
    return total
```

<table>
<thead>
<tr>
<th>Time</th>
<th>Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta(n)$</td>
<td>$\Theta(n)$</td>
</tr>
<tr>
<td>$\Theta(n)$</td>
<td>$\Theta(1)$</td>
</tr>
</tbody>
</table>
Reminder: Iteration is a special case of recursion

Idea: The state of iteration can be passed as parameters

```python
def factorial(n):
    total = 1
    while n > 0:
        n, total = n - 1, total * n
    return total

def factorial(n, total):
    if n == 0:
        return total
    return factorial(n - 1, total * n)
```

Local names become...
Parameters in a recursive function

But this converted version still uses linear space in Python
Tail Recursion

From the Revised\textsuperscript{7} Report on the Algorithmic Language Scheme:

"Implementations of Scheme are required to be properly tail-recursive. This allows the execution of an iterative computation in constant space, even if the iterative computation is described by a syntactically recursive procedure."

\begin{verbatim}
(define (factorial n total)
  (if (= n 0) total
    (factorial (- n 1) (* total n))))

def factorial(n, total):
  if n == 0:
    return total
  return factorial(n - 1, total * n)
\end{verbatim}
Tail Calls

A procedure call that has not yet returned is *active*. Some procedure calls are *tail calls*. A Scheme interpreter should support an unbounded number of active tail calls.

A tail call is a call expression in a *tail context*:

- The last body sub-expression in a `lambda` expression
- Sub-expressions 2 & 3 in a tail context `if` expression
- All non-predicate sub-expressions in a tail context `cond`
- The last sub-expression in a tail context `and` or `or`
- The last sub-expression in a tail context `begin`

```scheme
(define (factorial n total)
  (if (= n 0) total
    (factorial (- n 1) (* total n)) ))
```
Example: Length of a List

```
(define (length s)
    (if (null? s) 0
        (+ 1 (length (cdr s))))) )
```

A call expression is not a tail call if more computation is still required in the calling procedure.

Linear recursions can often be rewritten to use tail calls.

```
(define (length-tail s)
    (define (length-iter s n)
        (if (null? s) n
            (length-iter (cdr s) (+ 1 n))
        )
    )
    (length-iter s 0)
)
```
Eval with Tail Call Optimization

The return value of the tail call is the return value of the current procedure call.

Therefore, tail calls shouldn't increase the environment size.

In the interpreter, recursive calls to `scheme_eval` for tail calls must instead be expressed iteratively.
Logical forms may only evaluate some sub-expressions.

- **If** expression: `(if <predicate> <consequent> <alternative>)`
- **And** and **or**: `(and <e₁> ... <eₙ>),    (or <e₁> ... <eₙ>)`
- **Cond** expr’n: `(cond (<p₁> <e₁>) ... (<pₙ> <eₙ>) (else <e>))`

The value of an **if** expression is the value of a sub-expression.

- Evaluate the predicate.
- Choose a sub-expression: `<consequent>` or `<alternative>`
- Evaluate that sub-expression in place of the whole expression.

Evaluation of the tail context does not require a recursive call.

E.g., replace `(if false 1 (+ 2 3))` with `(+ 2 3)` and iterate.
Example: Reduce

\[
\text{(define (reduce procedure s start)}
\]

\[
\text{(if (null? s) start}
\]

\[
\text{(reduce procedure}
\]

\[
\text{(cdr s)}
\]

\[
\text{(procedure start (car s))}
\]

\[
\text{)}
\]

\[
\text{)}
\]

Recursive call is a tail call.

Other calls are not; constant space depends on \text{procedure}.

\[
\text{(reduce * '(3 4 5) 2)}
\]

\[
\text{120}
\]

\[
\text{(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))}
\]

\[
\text{(5 4 3 2)}
\]
Example: Map

(define (map procedure s)
  (define (map-iter procedure s m)
    (if (null? s) m
      (map-iter procedure
        (car s)
      (cons (procedure (car s)) m)))))
  (reverse (map-iter procedure s nil)))

(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s) r
      (reverse-iter (cdr s)
        (cons (car s) r)))))
  (reverse-iter s nil))
An Analogy: Programs Define Machines

Programs specify the logic of a computational device

\[
\text{factorial} = 5 \cdot (\text{factorial} - 1) + 1
\]
Interpreters are General Computing Machines

An interpreter can be parameterized to simulate any machine

\[
(\text{define (factorial } n) \\
(\text{if (zero? } n) 1 (* n (\text{factorial } (- n 1)))))
\]

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Internally, it is just a set of manipulation rules
**Interpretation in Python**

**eval**: Evaluates an expression in the current environment and returns the result. Doing so may affect the environment.

**exec**: Executes a statement in the current environment. Doing so may affect the environment.

```python
eval('2 + 2')
```

```python
exec('def square(x): return x * x')
```

**os.system('python <file>')**: Directs the operating system to invoke a new instance of the Python interpreter.