Lecture #22: Search Trees and Sets, Part II
Adding (Adjoining) a Value

- Must add values to a search tree in the right place: the place `tree_find` would try to find them.
- For example, if we add 17 to the search tree on left, we get the one on the right:

```
    7
   / \
  3   19
 /     / \
2  5   11  29
```

```
    7
   / \
  3   19
 /     / \
2  5   11  29
```

- Simplest always to add at the bottom (leaves) of the tree.
Non-destructive Add

• Broadly, there are two styles for dealing with structures that change over time:
  
  - **Non-destructive** operations preserve the prior state of the structure and create a new one.
  
  - **Destructive** operations, as a side effect, *may* modify the previous structure, losing information about its previous contents.

```python
def tree_add(T, x):
    """Assuming T is a binary search tree, a new binary search tree that contains all previous values in T, plus X (if not previously present).""
    if T.is_empty:
        return ________
    elif x == T.label:
        return _
    elif x < T.label:
        return ____________________________
    else:
        return ____________________________
```
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def tree_add(T, x):
    """Assuming T is a binary search tree, a new binary search tree that contains all previous values in T, plus X (if not previously present).""
    if T.is_empty:
        return BinTree(x)
elif x == T.label:
    return _
elif x < T.label:
    return _____________________________
else:
    return _____________________________
```

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        return BinTree(x)
    elif x == T.label:
        return T
    elif x < T.label:
        return ...
    else:
        return ...
```

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Non-destructive Add

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  - **Non-destructive** operations preserve the prior state of the structure and create a new one.
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```python
def tree_add(T, x):
    """Assuming T is a binary search tree, a new binary search tree
    that contains all previous values in T, plus X
    (if not previously present).""
    if T.is_empty:
        return BinTree(x)
    elif x == T.label:
        return T
    elif x < T.label:
        return BinTree(T.label, tree_add(T.left, x), T.right)
    else:
        return BinTree(T.label, T.left, tree_add(T.right, x))
```
Destructive Operations

- Destructive operations can be appropriate in circumstances where
  - We want speed: avoid the work of creating new structures.
  - The same data structure is referenced from multiple places, and we want all of them to be updated.

- First requires that we add capabilities to our class:

```python
class BinTree(Tree):
    def set_left(self, newval):
        """Assuming NEWVAL is a BinTree, sets SELF.left to NEWVAL.""
        ...

    def set_right(self, newval):
        """Assuming NEWVAL is a BinTree, sets SELF.right to NEWVAL.""
        ...
```
Destructive Add

• Destructive add looks very much like the non-destructive variety.

```python
def dtree_add(T, x):
    """Assuming T is a binary search tree, a binary search tree that contains all previous values in T, plus X (if not previously present). May destroy the initial contents of T.""
    if T.is_empty:
        return __________
    elif x == T.label:
        return _
    elif x < T.label:
        _________________
        return _
    else:
        _________________
        return _
```
Destructive Add

- Destructive add looks very much like the non-destructive variety.

```python
def dtree_add(T, x):
    """Assuming T is a binary search tree, a binary search tree that contains all previous values in T, plus X (if not previously present). May destroy the initial contents of T.""
    if T.is_empty:
        return BinTree(x)
    elif x == T.label:
        return T
    elif x < T.label:
        return ________________
    else:
        return ________________
```
Destructive Add

• Destructive add looks very much like the non-destructive variety.

```python
def dtree_add(T, x):
    """Assuming T is a binary search tree, a binary search tree
    that contains all previous values in T, plus X
    (if not previously present). May destroy the initial contents
    of T.""
    if T.is_empty:
        return BinTree(x)
    elif x == T.label:
        return T
    elif x < T.label:
        T.set_left(dtree_add(T.left, x))
        return T
    else:
        T.set_right(dtree_add(T.right, x))
        return T
```
Binary Search Trees as Sets

- For data that has a well-behaved ordering relation (a total ordering), BinTree provides a possible implementation of Python’s set type.
- \( x \) in \( S \) corresponds to \( \text{tree\_find}(S, x) \)
- \( S.\text{union}({x}) \) or \( S + \{x\} \) correspond to \( \text{tree\_add}(S, x) \)
- \( S.\text{add}(x) \) or \( S += \{x\} \) correspond to \( \text{dtree\_add}(S, x) \)
- Actually, Python uses hash tables for its sets, which you’ll see in CS61B (plug).
Problem: Make a Balanced Tree

- I have a sorted list, and would like to turn it into the best (shallow-est) binary search tree that contains the same values.
- Hint: Getting a shallow tree requires making the two child subtrees of each node have equal numbers of values ($\pm 1$).

```python
def list_to_tree(L):
    """Assuming L is a sorted list, a (nearly) balanced search tree containing exactly the values in L.""
    if ___________
        return ___________
    else:
        root_index = ___________
    return
```
Problem: Make a Balanced Tree

- I have a sorted list, and would like to turn it into the best (shallowest) binary search tree that contains the same values.
- Hint: Getting a shallow tree requires making the two child subtrees of each node have equal numbers of values ($\pm 1$).

```python
def list_to_tree(L):
    """Assuming L is a sorted list, a (nearly) balanced search tree containing exactly the values in L.""
    if len(L) == 0:
        return Tree.empty_tree
    else:
        root_index = ________ # fill in
        return ________
```
Problem: Make a Balanced Tree

- I have a sorted list, and would like to turn it into the best (shallowest) binary search tree that contains the same values.

- Hint: Getting a shallow tree requires making the two child subtrees of each node have equal numbers of values (±1).

def list_to_tree(L):
    """Assuming L is a sorted list, a (nearly) balanced search tree containing exactly the values in L."""
    if len(L) == 0:
        return Tree.empty_tree
    else:
        root_index = len(L) // 2
        return
Problem: Make a Balanced Tree

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- Hint: Getting a shallow tree requires making the two child subtrees of each node have equal numbers of values ($\pm 1$).

```python
def list_to_tree(L):
    """Assuming L is a sorted list, a (nearly) balanced search tree containing exactly the values in L.""
    if len(L) == 0:
        return Tree.empty_tree
    else:
        root_index = len(L) // 2
        return BinaryTree(L[root_index],
                          list_to_tree(L[:root_index]),
                          list_to_tree(L[root_index+1:]))
```

Problem: Iterating Through All Values

• Iterating over a tree gives us only the children, at present.
• Could we get all the nodes or labels in a tree,
• ... and for binary search trees, could we get them in sorted order?
• All it takes is a method that returns an appropriate iterator or iterable, and we can write, e.g.,

  for val in T.preorder_values():
    ...

• How would we do that?

  class Tree:
    ...
    def preorder_values(self):
      return ?

• Here, ? could be a list of all values in the tree, which we've done already. What else?
Creating an Iterator (Review)

- As we’ve seen (Lecture 17), an iterator is an object that implements a method `__next__` on itself.
- When called, it should either return a value or raise `StopException`.
- An iterable is an object that either
  - Implements a method `__iter__(self)` that returns an iterator, or
  - Implements a method `__getitem__(self, k)` that returns item number `k` (or raises an exception).
- Many methods and constructs take iterables, including `for` clauses, `map`, `reduce`, `zip`, and many others.
- When given an iterable, these create a new iterator from it (using `__iter__`), which allows one pass over the data.
Iterating Over a Binary Tree: Strategy

- To create an iterator on a tree, consider this reimplementation of `tree_to_list_preorder` from Lecture 21 (for binary trees):

```python
def tree_to_list_preorder(T):
    """The list of all labels in T, listing the labels of trees before those of their children, and listing their children left to right (preorder)."""
    if T.is_empty:
        return ()
    else:
        return (T.label,) + tree_to_list_preorder(T.left) \ 
                + tree_to_list_preorder(T.right)
```

- Suppose that we wanted to return just the first item (T’s label). What work would be left to do?

- Clearly, returning (iterating through) all the values in the left child and then on the right.

- To get the next value (after T’s label), we’ll need to start iterating through the left child, leaving its children to be processed.

- When the next tree in the queue is empty, discard it.
Iterating Over a Binary Tree: Data Structure

- So, to iterate over a tree, let's have our iterator consist of a list of subtrees that still need iterating over.

```python
class BinTree(Tree):
    ...
    def __iter__(self): return tree_iter(self)
class tree_iter:
    def __init__(self, the_tree):
        self._work_queue = [ the_tree ]
    ...
    def __next__(self): ?

    # Standard hack: by making iterators implement __iter__, they
    # are themselves iterable, so you can use them in
    # for statements, etc.
    def __iter__(self): return self
```

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Iterating Over a Binary Tree: Example

- Suppose that we create `iter = T.__iter__()` where `T` is

- Initially, `iter._work_queue` would contain just the tree rooted at the node labeled 10 (let's just say 'Tree 10' from now on).
- After the first call to `iter.__next__()`, which returns 10, `iter._work_queue` would contain [Tree 5, Tree 15]
- After the second call to `iter.__next__()`, which returns 5, `iter._work_queue` would contain [Tree 2, Tree 6, Tree 15]
- Then [Empty, Empty, Tree 6, Tree 15]