

Import statement

```
1 from math import pi
2 tau = 2 * pi
```

Assignment statement

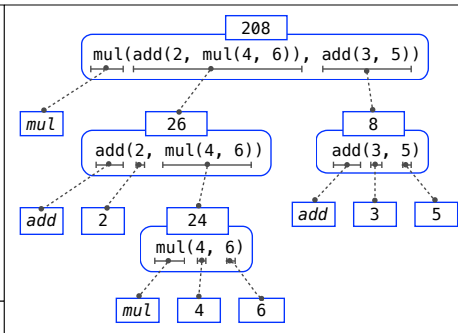
Global frame

| Name | Value |
|------|--------|
| pi | 3.1416 |

Binding

Code (left): Statements and expressions
Red arrow points to next line. Gray arrow points to the line just executed

Frames (right): A name is bound to a value
In a frame, there is at most one binding per name



Pure Functions

```
-2 >> abs(number): 2
2, 10 >> pow(x, y): 1024
```

Non-Pure Functions

```
-2 >> print(...): None
```

display "-2"

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(-2)
```

Global frame

| Name | Value |
|--------|----------------|
| mul | func mul(...) |
| square | func square(x) |

Intrinsic name of function called

Local frame

| Name | Value |
|--------------|-------|
| x | -2 |
| Return value | 4 |

Formal parameter bound to argument

Return value is not a binding!

Defining:

```
>>> def square(x):
    return mul(x, x)
```

Def statement

Formal parameter: x

Return expression: mul(x, x)

Body (return statement): return mul(x, x)

Call expression: square(2+2)

operator: square
function: func square(x)

operand: 2+2
argument: 4

Compound statement

```
<header>:
<statement>
<statement>
...
<separating header>:
<statement>
<statement>
...
def abs_value(x):
    if x > 0:
        return x
    elif x == 0:
        return 0
    else:
        return -x
```

Clause

Suite

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(square(3))
```

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

Global frame

| Name | Value |
|--------|----------------|
| mul | func mul(...) |
| square | func square(x) |

f1: square [parent=Global]

| Name | Value |
|--------------|-------|
| x | 3 |
| Return value | 9 |

f2: square [parent=Global]

| Name | Value |
|--------------|-------|
| x | 9 |
| Return value | 81 |

Calling/Applying:

```
4 >>> square(x):
    return mul(x, x)
16
```

Argument: x

Intrinsic name: square

Return value: 16

def abs_value(x):

```
1 statement,
3 clauses,
3 headers,
3 suites,
2 boolean contexts
```

Evaluation rule for call expressions:

1. Evaluate the operator and operand subexpressions.
2. Apply the function that is the value of the operator subexpression to the arguments that are the values of the operand subexpressions.

Applying user-defined functions:

1. Create a new local frame with the same parent as the function that was applied.
2. Bind the arguments to the function's formal parameter names in that frame.
3. Execute the body of the function in the environment beginning at that frame.

```
1 def f(x, y):
2     return g(x)
3
4 def g(a):
5     return a + y
6
7 result = f(1, 2)
```

Global frame

| Name | Value |
|------|--------------|
| f | func f(x, y) |
| g | func g(a) |

f1: f [parent=Global]

| Name | Value |
|------|-------|
| x | 1 |
| y | 2 |

f2: g [parent=Global]

| Name | Value |
|------|-------|
| a | 1 |

Error: "y" is not found

- An environment is a sequence of frames
- An environment for a non-nested function (no def within def) consists of one local frame, followed by the global frame

Execution rule for def statements:

1. Create a new function value with the specified name, formal parameters, and function body.
2. Its parent is the first frame of the current environment.
3. Bind the name of the function to the function value in the first frame of the current environment.

Execution rule for assignment statements:

1. Evaluate the expression(s) on the right of the equal sign.
2. Simultaneously bind the names on the left to those values, in the first frame of the current environment.

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(4)
```

Global frame

| Name | Value |
|--------|----------------|
| mul | func mul(...) |
| square | func square(x) |

f1: square [parent=Global]

| Name | Value |
|--------------|-------|
| x | 4 |
| Return value | 16 |

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Nested def statements: Functions defined within other function bodies are bound to names in the local frame

Execution rule for conditional statements: Each clause is considered in order.

1. Evaluate the header's expression.
2. If it is a true value, execute the suite, then skip the remaining clauses in the statement.

Evaluation rule for or expressions:

1. Evaluate the subexpression <left>.
2. If the result is a true value v, then the expression evaluates to v.
3. Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for and expressions:

1. Evaluate the subexpression <left>.
2. If the result is a false value v, then the expression evaluates to v.
3. Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for not expressions:

1. Evaluate <exp>; The value is True if the result is a false value, and False otherwise.

Execution rule for while statements:

1. Evaluate the header's expression.
2. If it is a true value, execute the (whole) suite, then return to step 1.

```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1 # Zeroth and first Fibonacci numbers
    k = 1 # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

def cube(k):
return pow(k, 3)

def summation(n, term):
"""Sum the first n terms of a sequence.

```
>>> summation(5, cube)
225
```

total, k = 0, 1
while k <= n:
total, k = total + term(k), k + 1
return total

0 + 1³ + 2³ + 3³ + 4³ + 5³

Function of a single argument (not called term)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

```
square = lambda x,y: x * y
```

Evaluates to a function. No "return" keyword!

A function with formal parameters x and y that returns the value of "x * y"

Must be a single expression

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.
    """
    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

A function that returns a function

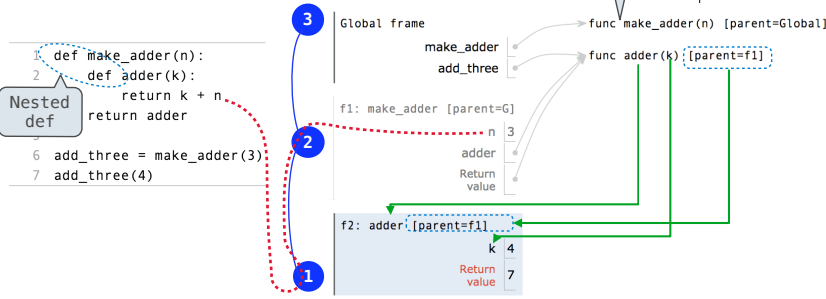
The name add_three is bound to a function

A local def statement

Can refer to names in the enclosing function

- Every user-defined function has a **parent frame** (often global)
- The parent of a function is the frame in which it was **defined**
- Every local frame has a **parent frame** (often global)
- The parent of a frame is the parent of the function **called**

A function's signature has all the information to create a local frame



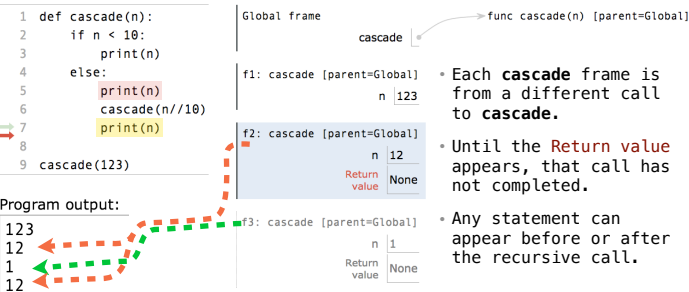
```
def curry2(f):
    """Returns a function g such that g(x)(y) returns f(x, y)."""
    def g(x):
        def h(y):
            return f(x, y)
        return h
    return g
```

Currying: Transforming a multi-argument function into a single-argument, higher-order function.

Anatomy of a recursive function:

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = n // 10, n % 10
        return sum_digits(all_but_last) + last
```



```
1 def inverse_cascade(n):
12     grow(n)
123     print(n)
1234     shrink(n)
123     def f_then_g(f, g, n):
123         if n:
12         f(n)
1         g(n)
1     grow = lambda n: f_then_g(grow, print, n//10)
1     shrink = lambda n: f_then_g(print, shrink, n//10)
```

n: 0, 1, 2, 3, 4, 5, 6, 7, 8, fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21

def fib(n): if n == 0: return 0 elif n == 1: return 1 else: return fib(n-2) + fib(n-1)

```
square = lambda x: x * x VS def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both functions have as their parent the environment in which they were defined.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name.

When a function is defined:

1. Create a **function value**: func <name>(<formal parameters>)
2. Its parent is the current frame.

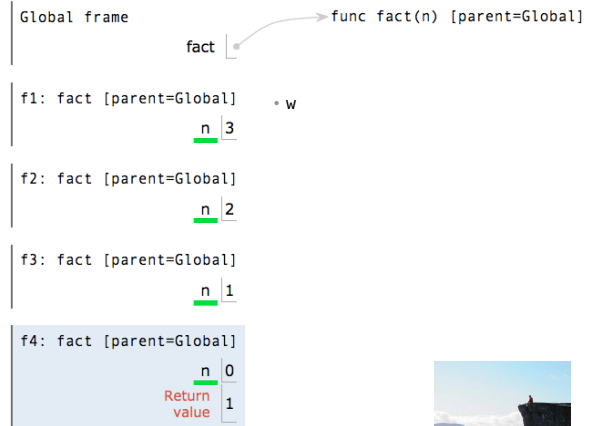
```
f1: make_adder func adder(k) [parent=f1]
```

3. Bind <name> to the **function value** in the current frame (which is the first frame of the current environment).

When a function is called:

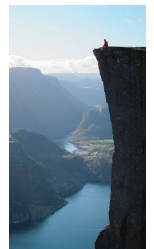
1. Add a **local frame**, titled with the <name> of the function being called.
2. Copy the parent of the function to the **local frame**: [parent=<label>]
3. Bind the <formal parameters> to the arguments in the **local frame**.
4. Execute the body of the function in the environment that starts with the **local frame**.

```
1 def fact(n):
2     if n == 0:
3         return 1
4     else:
5         return n * fact(n-1)
6
7 fact(3)
```



Is fact implemented correctly?

1. Verify the base case.
2. Treat fact as a functional abstraction!
3. Assume that fact(n-1) is correct.
4. Verify that fact(n) is correct, assuming that fact(n-1) correct.



- Recursive decomposition: finding simpler instances of a problem.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

- E.g., count_partitions(6, 4)
- Explore two possibilities:
 - Use at least one 4
 - Don't use any 4
- Solve two simpler problems:
 - count_partitions(2, 4)
 - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
from operator import floordiv, mod
def divide_exact(n, d):
    """Return the quotient and remainder of dividing N by D.

```

```
>>> q, r = divide_exact(2012, 10)
>>> q
201
>>> r
2
"""
return floordiv(n, d), mod(n, d)
```

Multiple assignment to two names

Multiple return values, separated by commas