61A Lecture 6

Monday, February 2

Announcements

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  - Conflict? Fill out the conflict form today! http://goo.gl/2P5fKq

Recursive Functions

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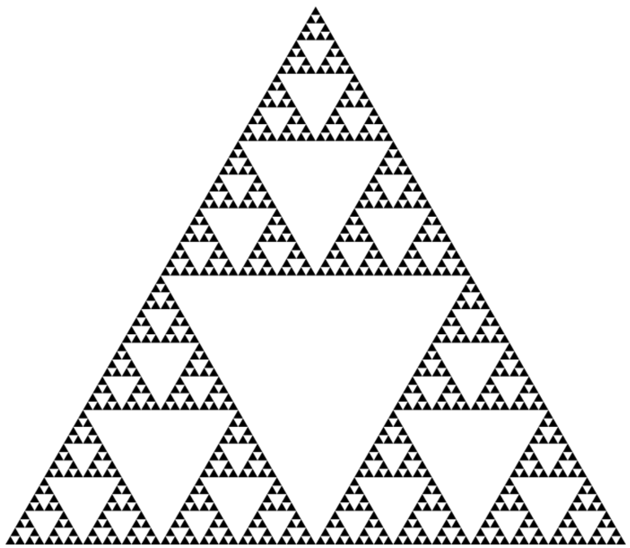

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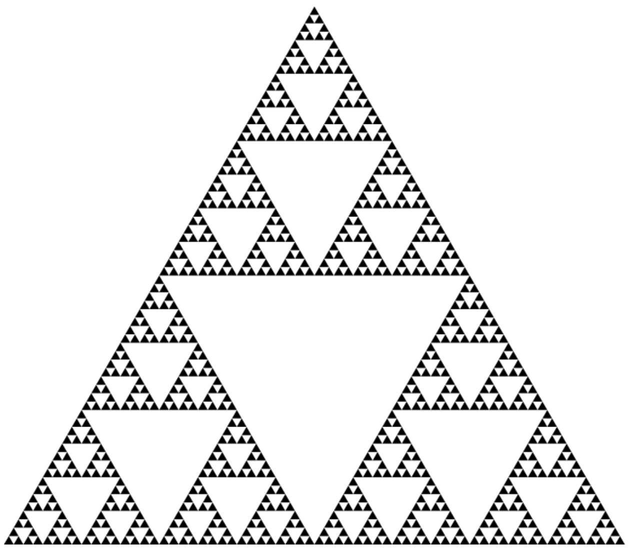

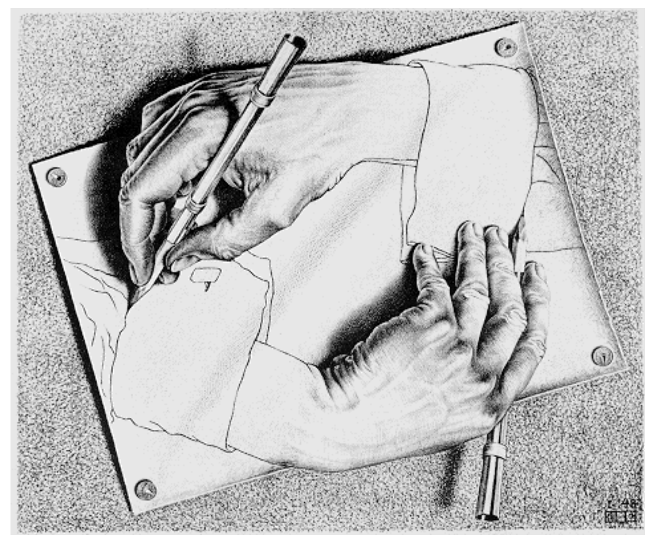

Drawing Hands, by M. C. Escher (lithograph, 1948)

Digit Sums

`2+0+1+5 = 8`

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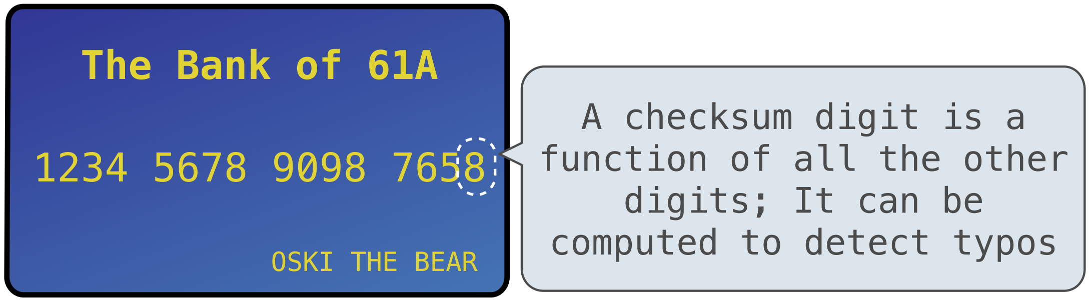

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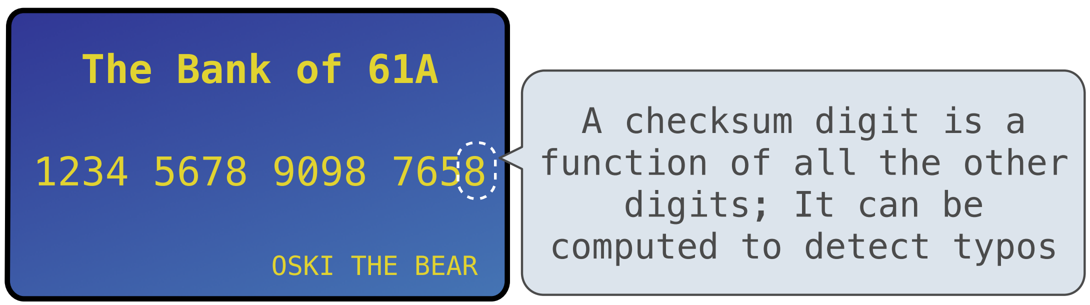

- Credit cards actually use the Luhn algorithm, which we'll implement after digit_sum.

Sum Digits Without a While Statement

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(Demo)

Recursion in Environment Diagrams

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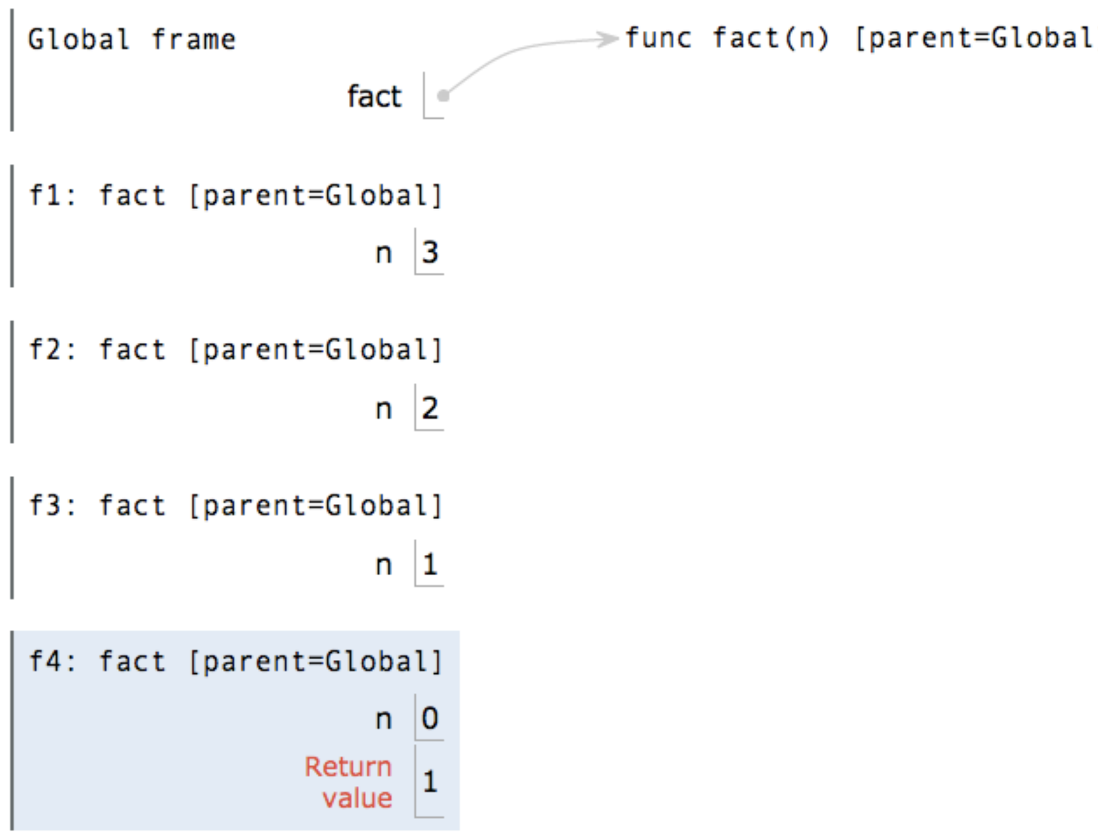

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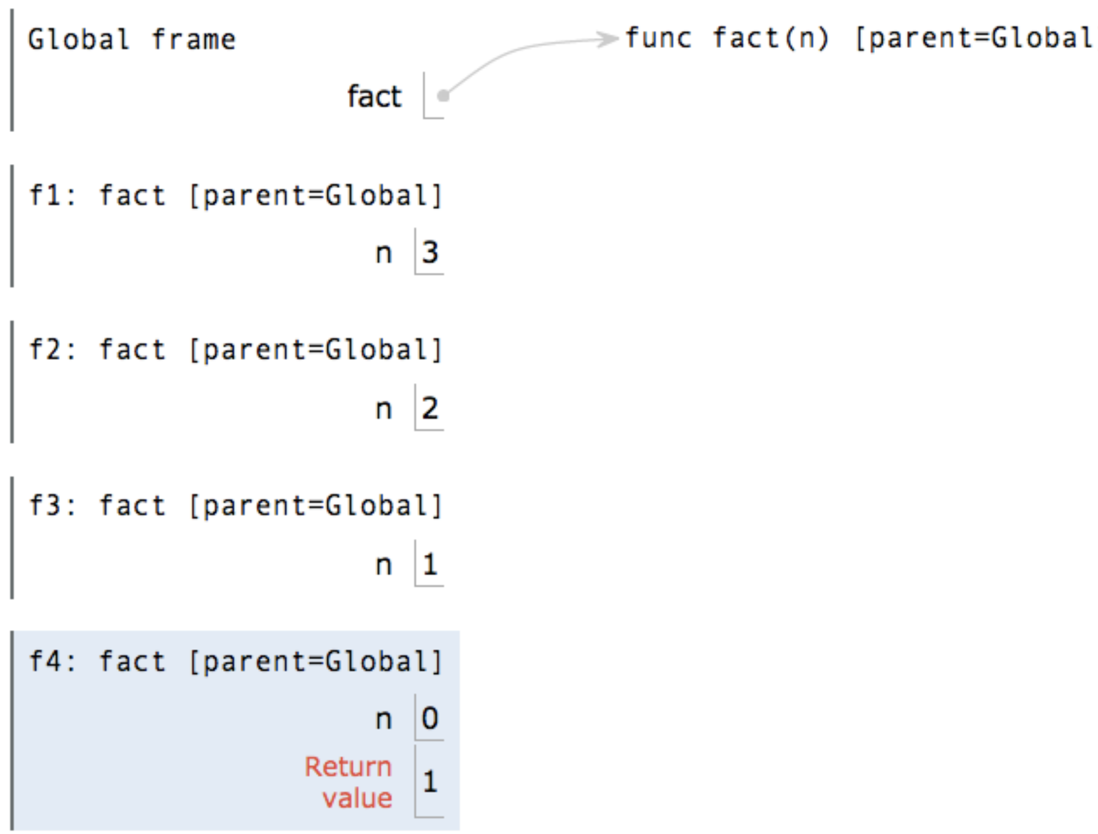

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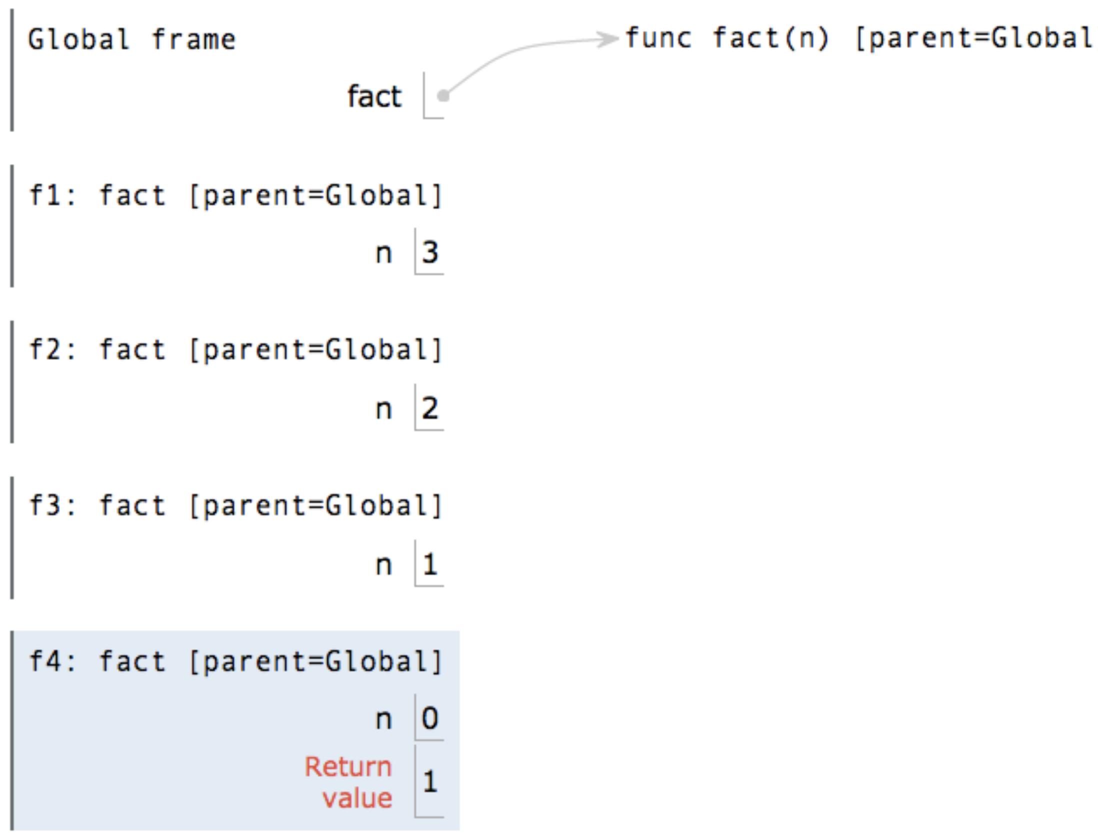

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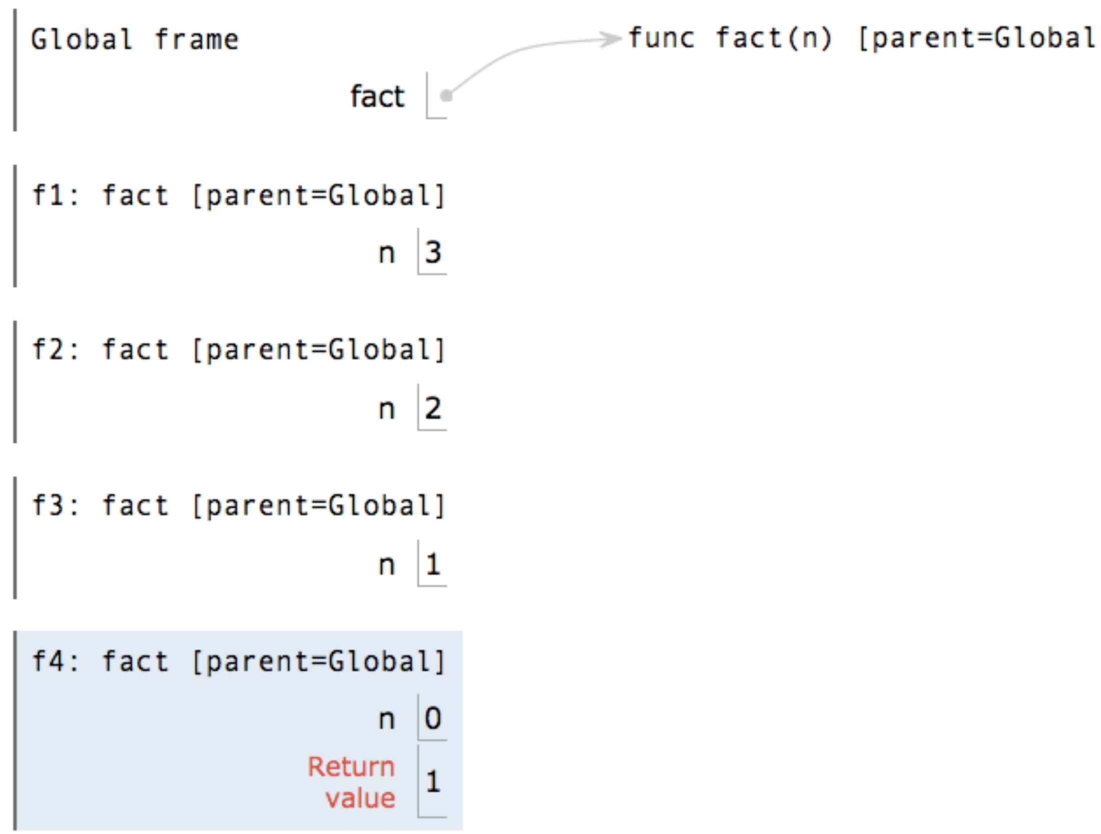

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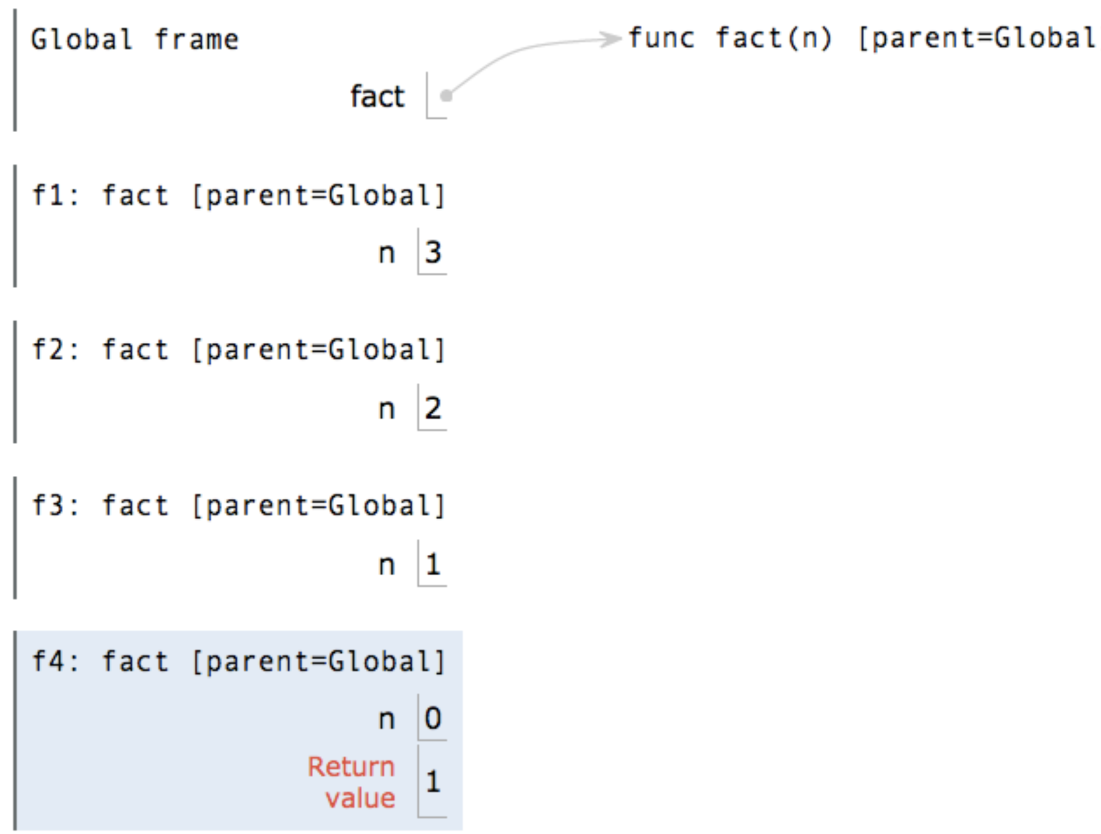

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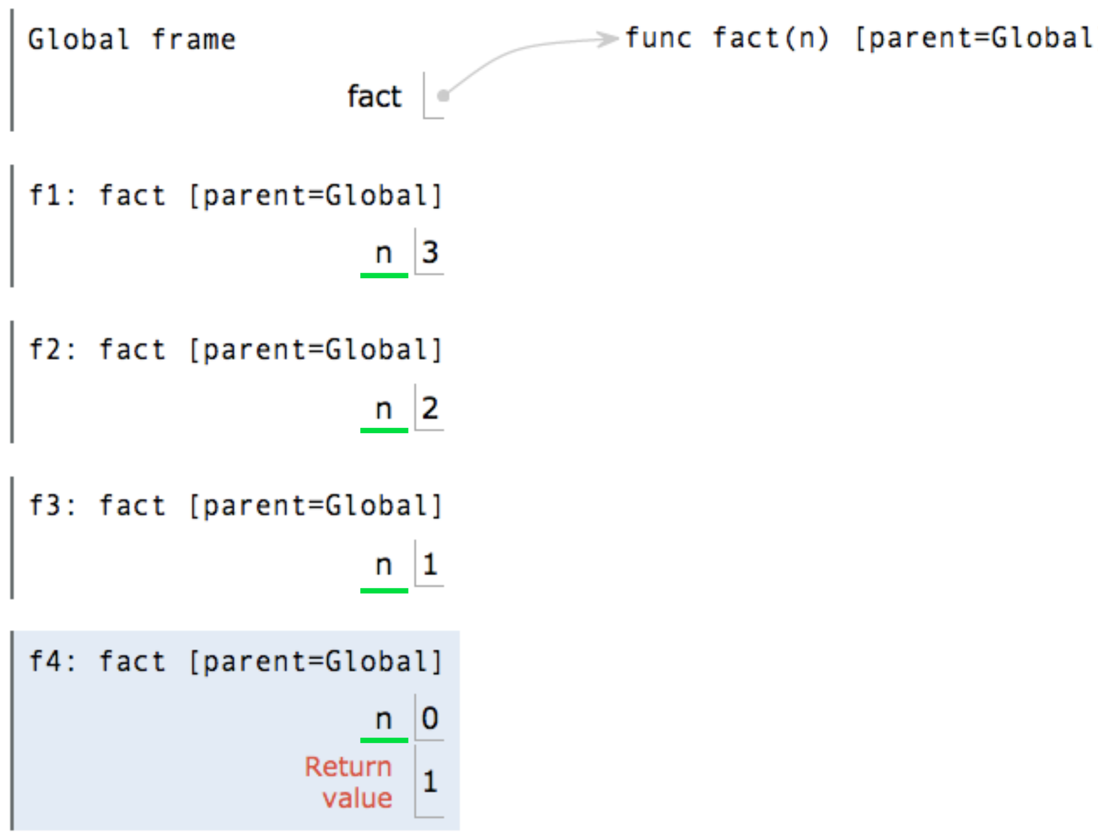

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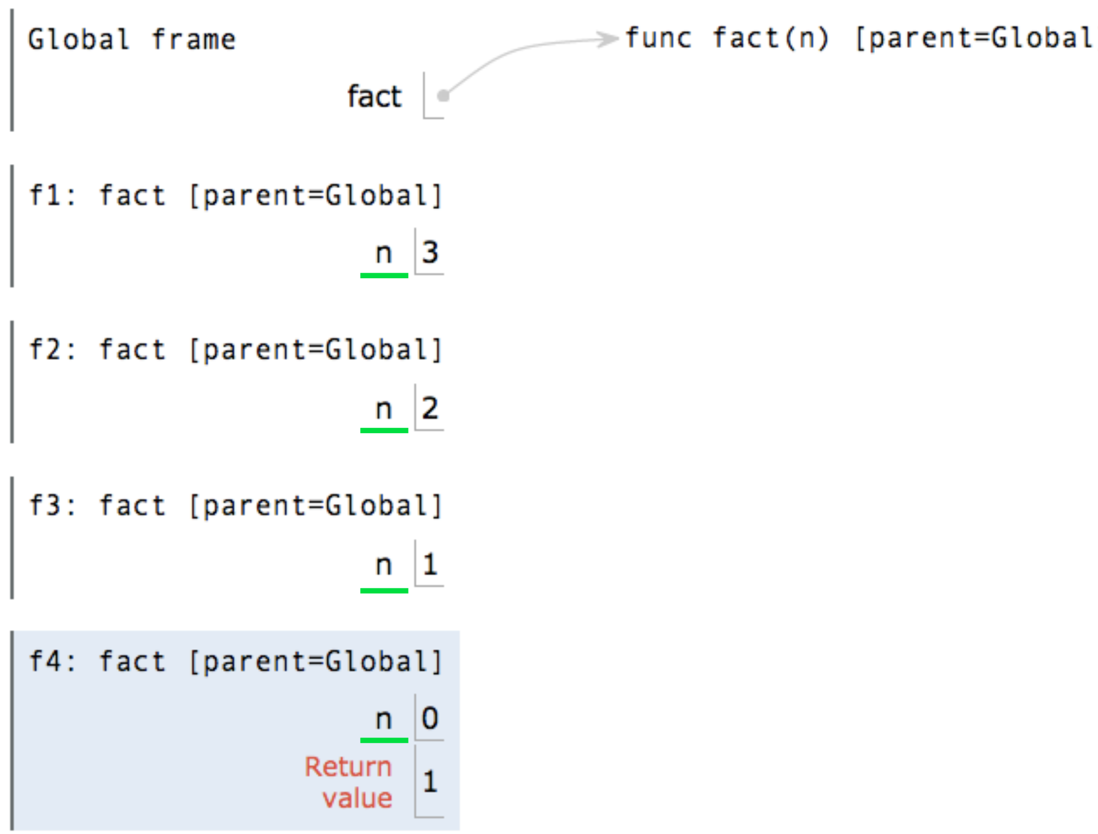

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- Each call to fact solves a simpler problem than the last: smaller n.

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Iteration vs Recursion

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`Iteration is a special case of recursion`

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Verifying Recursive Functions

The Recursive Leap of Faith

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Photo by Kevin Lee, Preikestolen, Norway

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Is fact implemented correctly?

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3. Assume that fact(n-1) is correct.

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Is fact implemented correctly?

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2. Treat **fact** as a functional abstraction!
3. Assume that fact(n-1) is correct.
4. Verify that fact(n) is correct, assuming that fact(n-1) correct.

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Verifying Digit Sum

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Mutual Recursion

The Luhn Algorithm

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Recursion and Iteration

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...arguments to a recursive call