61A Lecture 7

Wednesday, February 4
Announcements

• Project 1 is due Thursday 2/5 @ 11:59pm; Early bonus point for submitting on Wednesday!
  ▪ Extra tutor office hours on Wednesday 2/4 (See Piazza for details)
• Midterm 1 is on Monday 2/9 from 7pm to 9pm!
  ▪ Review session on Saturday 2/7
  ▪ HKN review session on Sunday 2/8
  ▪ Includes topics up to and including this lecture
  ▪ Closed book/note exam, except for one page (2 sides) of hand-written notes & study guide
  ▪ Cannot attend? Fill out the conflict form by Wednesday 2/4! http://goo.gl/2P5fKq
• Optional Hog strategy contest ends Wednesday 2/18 @ 11:59pm
Hog Contest Rules

• Up to two people submit one entry; Max of one entry per person
• Your score is the number of entries against which you win more than 50% of the time
• All strategies must be deterministic, pure functions of the current player scores
• All winning entries will receive 2 points of extra credit
• The real prize: honor and glory

Fall 2011 Winners
Kaylee Mann
Yan Duan & Ziming Li
Brian Prike & Zhenghao Qian
Parker Schuh & Robert Chatham

Fall 2012 Winners
Chenyang Yuan
Joseph Hui

Fall 2013 Winners
Paul Bramsen
Sam Kumar & Kangsik Lee
Kevin Chen

Fall 2014 Winners
Alan Tong & Elaine Zhao
Zhenyang Zhang
Adam Robert Villaflor & Joany Gao
Zhen Qin & Dian Chen
Zizheng Tai & Yihe Li

Spring 2015 Winners

YOUR NAME COULD BE HERE... FOREVER!
Order of Recursive Calls
The Cascade Function

1. def cascade(n):
   2.     if n < 10:
   3.         print(n)
   4.     else:
   5.         print(n)
   6.         cascade(n//10)
   7.         print(n)
   8. 9. cascade(123)

Program output:

123
12
1
12

(Demo)

- Each cascade frame is from a different call to cascade.
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Interactive Diagram
Two Definitions of Cascade

(Demo)

```python
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n // 10)
        print(n)
```

```python
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n // 10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (at least to me)
- When learning to write recursive functions, put the base cases first
- Both are recursive functions, even though only the first has typical structure
Example: Inverse Cascade
**Inverse Cascade**

Write a function that prints an inverse cascade:

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

```python
def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

```python
grow = lambda n: f_then_g(grow, print, n // 10)
shrink = lambda n: f_then_g(print, shrink, n // 10)
```
Tree Recursion
Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

\[
\begin{align*}
n & : \quad 0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots, 35 \\
fib(n) & : \quad 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots, 9,227,465
\end{align*}
\]

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

A Tree-Recursive Process

The computational process of fib evolves into a tree structure
Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times

(We can speed up this computation dramatically in a few weeks by remembering results)
Example: Counting Partitions
Counting Partitions

The number of partitions of a positive integer \( n \), using parts up to size \( m \), is the number of ways in which \( n \) can be expressed as the sum of positive integer parts up to \( m \) in increasing order.

\[
\text{count_partitions}(6, 4)
\]

\[
\begin{align*}
2 + 4 &= 6 \\
1 + 1 + 4 &= 6 \\
3 + 3 &= 6 \\
1 + 2 + 3 &= 6 \\
1 + 1 + 1 + 3 &= 6 \\
2 + 2 + 2 &= 6 \\
1 + 1 + 2 + 2 &= 6 \\
1 + 1 + 1 + 1 + 2 &= 6 \\
1 + 1 + 1 + 1 + 1 + 1 &= 6
\end{align*}
\]
Counting Partitions

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\[
\text{count_partitions}(6, 4)
\]

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.
Counting Partitions

The number of partitions of a positive integer $n$, using parts up to size $m$, is the number of ways in which $n$ can be expressed as the sum of positive integer parts up to $m$ in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```python
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

(Demo)

Interactive Diagram