CS 61A Lecture 11

Wednesday, February 18

Announcements

- Optional Hog Contest due Wednesday 2/18 @ 11:59pm
- Homework 3 due Thursday 2/19 @ 11:59pm
- Project 2 due Thursday 2/26 @ 11:59pm
  - Bonus point for early submission by Wednesday 2/25 @ 11:59pm!

Box-and-Pointer Notation

The Closure Property of Data Types

- A method for combining data values satisfies the *closure property* if:

  The result of combination can itself be combined using the same method

- Closure is powerful because it permits us to create hierarchical structures

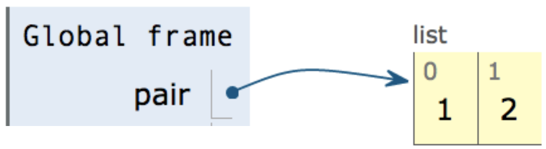
- Hierarchical structures are made up of parts, which themselves are made up of parts, and so on

Lists can contain lists as elements (in addition to anything else)

Box-and-Pointer Notation in Environment Diagrams

Lists are represented as a row of index-labeled adjacent boxes, one per element

Each box either contains a primitive value or points to a compound value




```
pair = [1, 2]
```

Interactive Diagram

Box-and-Pointer Notation in Environment Diagrams

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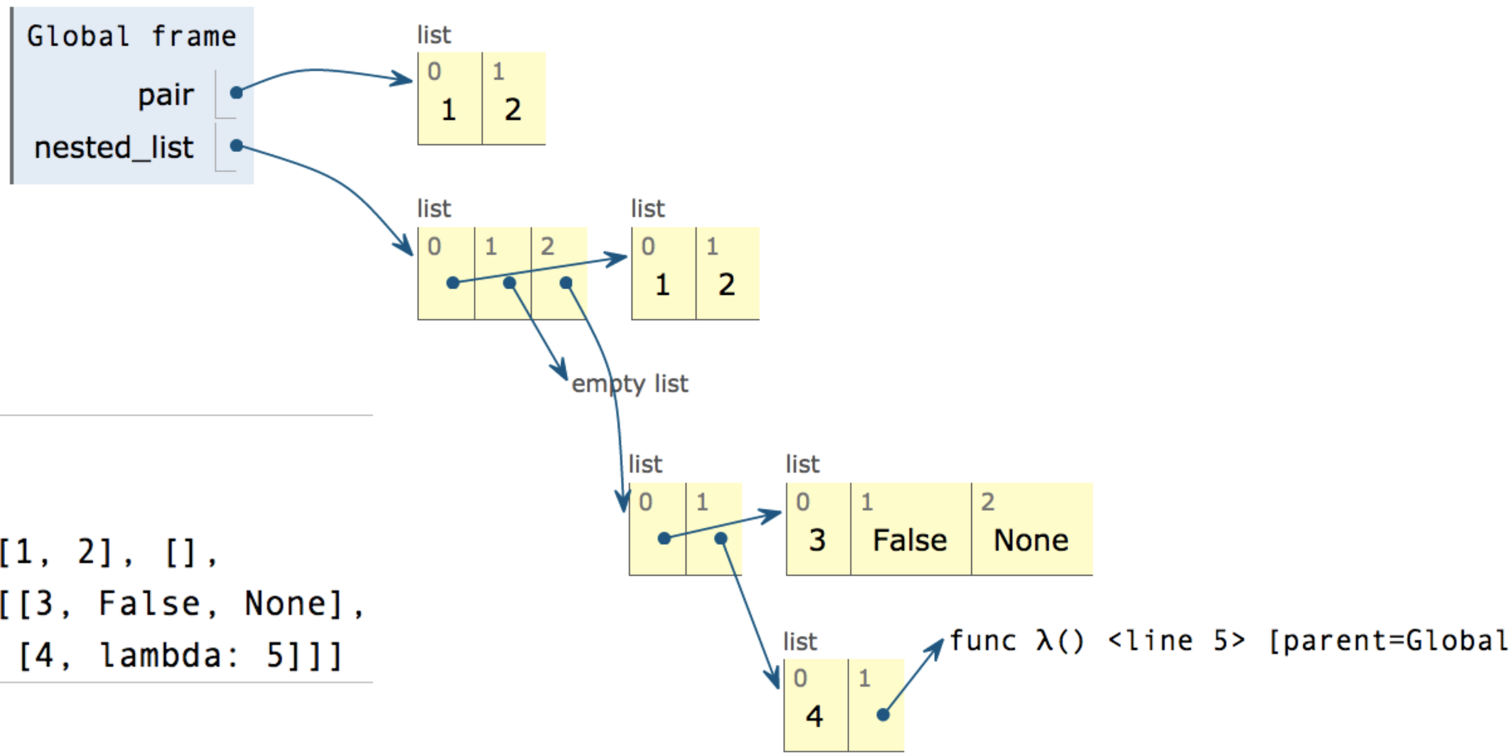



```
pair = [1, 2]

nested_list = [[1, 2], [],
               [[3, False, None],
                [4, lambda: 5]]]
```

Interactive Diagram

Sequence Operations

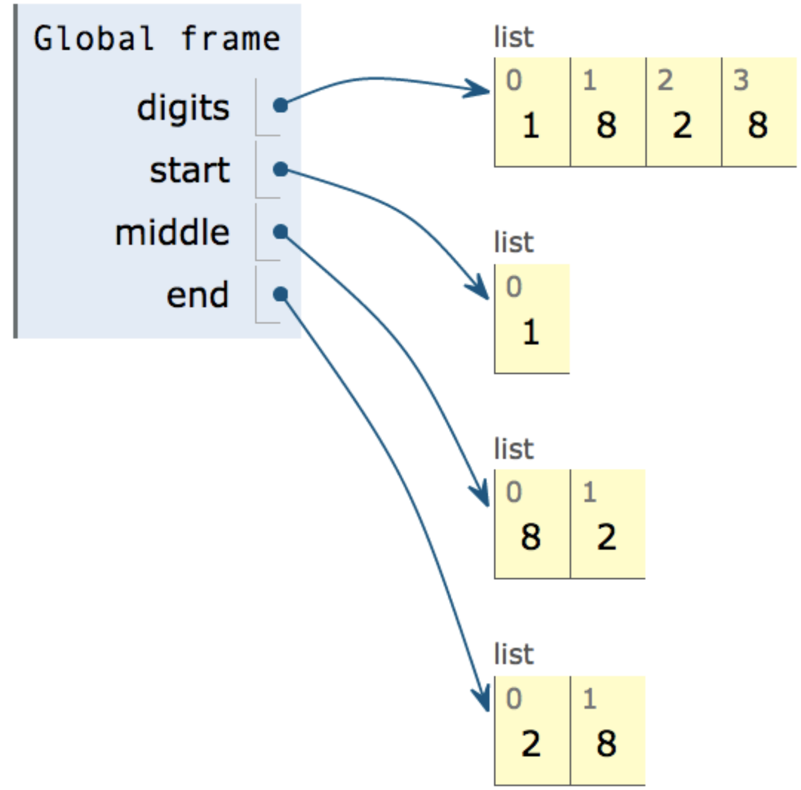
Membership & Slicing

Python sequences have operators for membership and slicing

Membership.

```
>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
True
```

```
digits = [1, 8, 2, 8]
start =  digits[:1]
middle = digits[1:3]
end =    digits[2:]
```



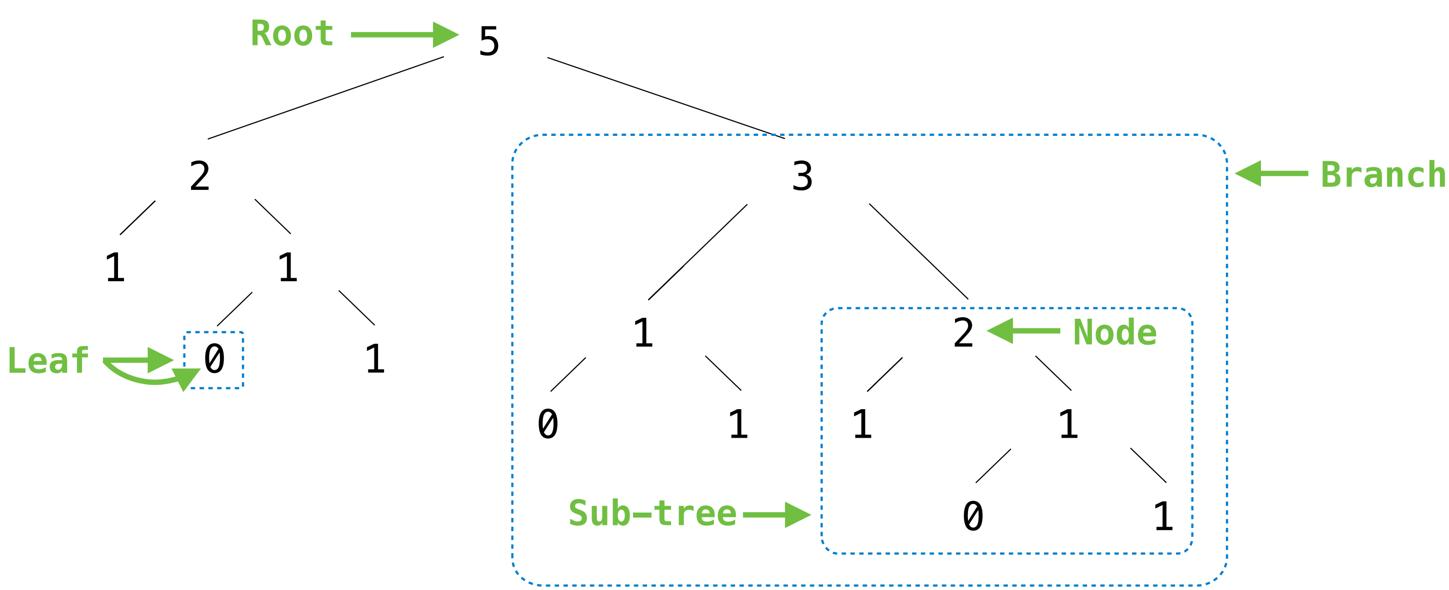
Slicing.

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

Slicing creates a new object

Trees

Tree Abstraction



A tree has a root value and a sequence of branches; each branch is a tree

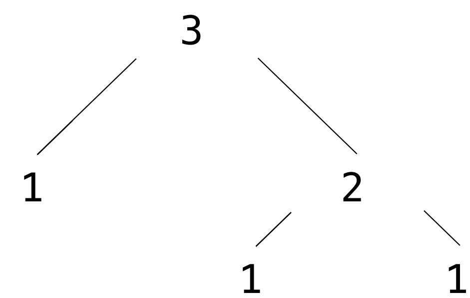
A tree with zero branches is called a leaf

The root values of sub-trees within a tree are often called node values or nodes

Implementing the Tree Abstraction

```
def tree(root, branches=[]):
   return [root] + branches

def root(tree):
    return tree[0]

def branches(tree):
    return tree[1:]
```

A tree has a root value and a sequence of branches; each branch is a tree




```
>>> tree(3, [tree(1),
...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]
```

Implementing the Tree Abstraction

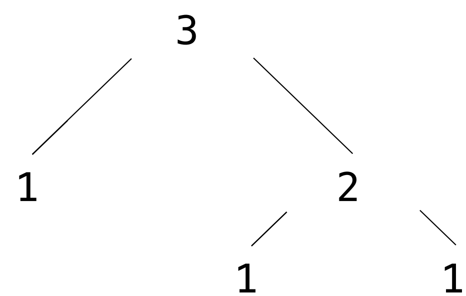
```python
def tree(root, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [root] + list(branches)

def root(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

A tree has a root value and a sequence of branches; each branch is a tree




```python
>>> tree(3, [tree(1),
...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]
```

```python
def is_leaf(tree):
    return not branches(tree)
```

(Demo)

Tree Processing Uses Recursion

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

```
def count_leaves(tree):
    """Count the leaves of a tree."""
    if is_leaf(tree):
        return 1
    else:
        branch_counts = [count_leaves(b) for b in tree]
        return sum(branch_counts)
```

(Demo)

Discussion Question

Implement leaves, which returns a list of the leaf values of a tree

Hint: If you sum a list of lists, you get a list containing the elements of those lists

```
>>> sum([ [1] ], [])
[1]
>>> sum([ [[1]], [2] ], [])
[[1], 2]
>>> sum([ [1], [2, 3], [4] ], [])
[1, 2, 3, 4]
```

```
def leaves(tree):
    """Return a list containing the leaves of tree.

    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
    if is_leaf(tree):
        return [root(tree)]
    else:
        return sum([leaves(b) for b in branches(tree)], [])
```

Example: Partition Trees

(Demo)

Interactive Diagram