61A Lecture 13

Monday, February 23
Announcements
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• Homework 4 due Monday 2/23 @ 11:59pm (small)
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• Project 2 due Thursday 2/26 @ 11:59pm (BIG!)
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  - Office hours on Monday 2/23 3pm-5pm are relocated to 310 Soda
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  ▪ Project party on Tuesday 2/24 5pm–6:30pm in 2050 VLSB
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  - Extra office hours on Wednesday 2/25 4pm–6pm in Bechtel (Garbarini Lounge)
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  ▪ Bonus point for early submission by Wednesday 2/25 @ 11:59pm!
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  ▪ Thursday office hours will be held in...
Mutable Functions
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

```python
>>> withdraw(25)
```
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100.

```python
>>> withdraw(25)
75
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A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

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>>> withdraw(25)
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Argument:
amount to withdraw
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

>>> withdraw(25)
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Return value: remaining balance
Argument: amount to withdraw
Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)
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>>> withdraw(25)
50

Argument: amount to withdraw
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

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>>> withdraw(25)
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>>> withdraw(25)
50
```

Return value: remaining balance

Argument: amount to withdraw

Second withdrawal of the same amount
Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)
75

Different return value!

>>> withdraw(25)
50

Argument: amount to withdraw

Second withdrawal of the same amount
Let's model a bank account that has a balance of $100.

**Return value:** remaining balance

- >>> withdraw(25)
- 75

**Argument:** amount to withdraw

- >>> withdraw(25)
- 50

**Second withdrawal of the same amount**

- >>> withdraw(60)
Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)
75

Second withdrawal of the same amount

Argument: amount to withdraw

>>> withdraw(25)
50

Different return value!

>>> withdraw(60)
'Insufficient funds'
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

Argument: amount to withdraw

Different return value!

>>> withdraw(25)
75

Second withdrawal of the same amount

>>> withdraw(25)
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>>> withdraw(60)
'Insufficient funds'

Where's this balance stored?
A Function with Behavior That Varies Over Time

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Return value: remaining balance

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'Insufficient funds'

Argument: amount to withdraw

Second withdrawal of the same amount

Where's this balance stored?

>>> withdraw = make_withdraw(100)
A Function with Behavior That Varies Over Time

Let's model a bank account that has a balance of $100

Return value: remaining balance

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>>> withdraw = make_withdraw(100)

Within the parent frame of the function!
Let's model a bank account that has a balance of $100

Return value: remaining balance

>>> withdraw(25)

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'Insufficient funds'

Argument: amount to withdraw

Second withdrawal of the same amount

Within the parent frame of the function!

A function has a body and a parent environment

Different return value!

Where's this balance stored?

>>> withdraw = make_withdraw(100)
Persistent Local State Using Environments

Interactive Diagram
Persistent Local State Using Environments

Interactive Diagram

The parent frame contains the balance, the local state of the withdraw function.
Persistent Local State Using Environments

The parent frame contains the balance, the local state of the withdraw function.

Every call decreases the same balance by (a possibly different) amount.

Interactive Diagram
Persistent Local State Using Environments

The parent frame contains the balance, the local state of the withdraw function.

All calls to the same function have the same parent.

Every call decreases the same balance by (a possibly different) amount.
Reminder: Local Assignment

```python
def percent_difference(x, y):
    difference = abs(x-y)
    return 100 * difference / x
diff = percent_difference(40, 50)
```

Interactive Diagram

```
Global frame

percent_difference

f1: percent_difference [parent=Global]

<table>
<thead>
<tr>
<th>x</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>50</td>
</tr>
<tr>
<td>difference</td>
<td>10</td>
</tr>
</tbody>
</table>
```
Reminder: Local Assignment

\[
def \text{percent\_difference}(x, y):
    \text{difference} = \text{abs}(x-y);
    \text{return} 100 \times \frac{\text{difference}}{x}
\]

\[
diff = \text{percent\_difference}(40, 50)
\]

Assignment binds name(s) to value(s) in the first frame of the current environment

Global frame

\[
\text{percent\_difference}
\]

f1: \text{percent\_difference} [parent=Global]

\[
\begin{align*}
    x & \quad 40 \\
    y & \quad 50 \\
    \text{difference} & \quad 10
\end{align*}
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Reminder: Local Assignment

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Assignment binds name(s) to value(s) in the first frame of the current environment

Interactive Diagram
Reminder: Local Assignment

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Execution rule for assignment statements:

Interactive Diagram
Reminder: Local Assignment

Execution rule for assignment statements:

1. Evaluate all expressions right of =, from left to right

2. Bind the names on the left to the resulting values in the current frame
Non-Local Assignment & Persistent Local State
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```python
def make_withdraw(balance):
```

def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""

Non-Local Assignment & Persistent Local State
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def withdraw(amount):
Non-Local Assignment & Persistent Local State

def make_withdraw(balance):
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    def withdraw(amount):
        nonlocal balance
Non-Local Assignment & Persistent Local State

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def withdraw(amount):
    nonlocal balance
    if amount > balance:
Non-Local Assignment & Persistent Local State

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```

(Demo)
Non-Local Assignment
The Effect of Nonlocal Statements

nonlocal <name>
The Effect of Nonlocal Statements

nonlocal <name>

**Effect**: Future assignments to that name change its pre-existing binding in the *first non-local frame* of the current environment in which that name is bound.
The Effect of Nonlocal Statements

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Python Docs: an "enclosing scope"
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From the Python 3 language reference:

Names listed in a nonlocal statement must refer to pre-existing bindings in an enclosing scope.
The Effect of Nonlocal Statements

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nonlocal <name>, <name>, ...
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http://docs.python.org/release/3.1.3/reference/simple_stmts.html#the-nonlocal-statement
The Effect of Nonlocal Statements

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\text{nonlocal } \langle\text{name}\rangle, \langle\text{name}\rangle, \ldots
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http://www.python.org/dev/peps/pep-3104/
The Many Meanings of Assignment Statements

\[ x = 2 \]
The Many Meanings of Assignment Statements

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def make_withdraw(balance):
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
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wd = make_withdraw(20)
wd(5)
```
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Local assignment
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UnboundLocalError: local variable 'balance' referenced before assignment
Mutable Values & Persistent Local State

Mutable values can be changed \textit{without} a nonlocal statement.
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

```python
def make_withdraw_list(balance):
    b = [balance]
    def withdraw(amount):
        if amount > b[0]:
            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
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withdraw = make_withdraw_list(100)
withdraw(25)
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            return 'Insufficient funds'
        b[0] = b[0] - amount
        return b[0]
    return withdraw

withdraw = make_withdraw_list(100)
withdraw(25)
```
Mutable Values & Persistent Local State

Mutable values can be changed without a nonlocal statement.
Mutable Values & Persistent Local State

Mutable values can be changed *without* a nonlocal statement.

Interactive Diagram
Multiple Mutable Functions

(Demo)
Referential Transparency, Lost

Interactive Diagram
Expressions are referentially transparent if substituting an expression with its value does not change the meaning of a program.
Referential Transparency, Lost

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\text{mul}(\text{add}(2, \text{mul}(4, 6)), \text{add}(3, 5))
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\text{mul}(\text{add}(2, \text{mul}(4, 6)), \text{add}(3, 5))
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\text{mul}(\text{add}(2, 24), \text{add}(3, 5))
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Referential Transparency, Lost

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\[
\text{mul}(26, \text{add}(3, 5))
\]
Expressions are **referentially transparent** if substituting an expression with its value does not change the meaning of a program.

```
mul(add(2, mul(4, 6)), add(3, 5))
mul(add(2, 24), add(3, 5))
mul(26, add(3, 5))
```

Mutation operations violate the condition of referential transparency because they do more than just return a value; **they change the environment.**
Referential Transparency, Lost

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\end{align*}
\]

Mutation operations violate the condition of referential transparency because they do more than just return a value; they change the environment.