Monday, March 11
Announcements

- Project 3 due Thursday 3/12 @ 11:59pm
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- Project 3 due Thursday 3/12 @ 11:59pm
  - Project party on Tuesday 3/10 5pm–6:30pm in 2050 VLSB
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  ▪ Project party on Tuesday 3/10 5pm–6:30pm in 2050 VLSB
  ▪ Bonus point for early submission by Wednesday 3/11
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• Guerrilla section this weekend on recursive data (linked lists and trees)
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• Homework 6 due Monday 3/16 @ 11:59pm

• Midterm 2 is on Thursday 3/19 7pm–9pm
  ▪ Fill out conflict form if you cannot attend due to a course conflict
Time
The Consumption of Time
The Consumption of Time

Implementations of the same functional abstraction can require different amounts of time.
The Consumption of Time

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Problem: How many factors does a positive integer n have?
Implementations of the same functional abstraction can require different amounts of time.

**Problem**: How many factors does a positive integer $n$ have?

A factor $k$ of $n$ is a positive integer that evenly divides $n$. 

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```python
def factors(n):
```
The Consumption of Time

Implementations of the same functional abstraction can require different amounts of time

Problem: How many factors does a positive integer n have?

A factor k of n is a positive integer that evenly divides n

```python
def factors(n):
    
    Slow: Test each k from 1 through n
```
The Consumption of Time

Implementations of the same functional abstraction can require different amounts of time

**Problem:** How many factors does a positive integer $n$ have?

A factor $k$ of $n$ is a positive integer that evenly divides $n$

```python
def factors(n):

    Slow: Test each $k$ from 1 through $n$

    Fast: Test each $k$ from 1 to square root $n$
          For every $k$, $n/k$ is also a factor!
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def factors(n):
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\[ n \]
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def factors(n):
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    **Slow:** Test each $k$ from 1 through $n$

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$Greatest$ $integer$ $less$ $than$ $\sqrt{n}$
The Consumption of Time

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Problem: How many factors does a positive integer $n$ have?

A factor $k$ of $n$ is a positive integer that evenly divides $n$

```python
def factors(n):
    # Time (number of divisions)
    # n
    # Greatest integer less than $\sqrt{n}$

    # Slow: Test each k from 1 through n
    # Fast: Test each k from 1 to square root n
    # For every k, n/k is also a factor!
```

(Demo)
Space
The Consumption of Space
The Consumption of Space

Which environment frames do we need to keep during evaluation?
The Consumption of Space

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At any moment there is a set of active environments
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Values and frames in active environments consume memory
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Active environments:
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Values and frames in active environments consume memory.

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**Active environments:**

- Environments for any function calls currently being evaluated.
The Consumption of Space

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Active environments:

• Environments for any function calls currently being evaluated.
• Parent environments of functions named in active environments.
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Active environments:

• Environments for any function calls currently being evaluated

• Parent environments of functions named in active environments

(Demo)

Interactive Diagram
Fibonacci Space Consumption
Fibonacci Space Consumption

\[
\text{fib}(5)
\]
Fibonacci Space Consumption

\[ \text{fib(5)} \]

\[ \text{fib(3)} \]
Fibonacci Space Consumption

\[
\text{fib(5)}
\]

\[
\text{fib(3)} \quad \text{fib(4)}
\]
Fibonacci Space Consumption

fib(5)

fib(3)

fib(1)  fib(2)

1  fib(0)  fib(1)

0 1
Fibonacci Space Consumption
Fibonacci Space Consumption

Assume we have reached this step.
Fibonacci Space Consumption

Assume we have reached this step.
Fibonacci Space Consumption

Assume we have reached this step.

Has an active environment
Fibonacci Space Consumption

Assume we have reached this step

Has an active environment
Can be reclaimed
Fibonacci Space Consumption

Assume we have reached this step.

\[
\begin{align*}
\text{fib}(5) & \quad \text{Has an active environment} \\
& \quad \text{Can be reclaimed} \\
& \quad \text{Hasn't yet been created}
\end{align*}
\]
Order of Growth
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A method for bounding the resources used by a function by the "size" of a problem
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\[ n: \text{ size of the problem} \]
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\( R(n): \text{ measurement of some resource used (time or space)} \)
Order of Growth

A method for bounding the resources used by a function by the "size" of a problem

\[ R(n) = \Theta(f(n)) \]

\( n \): size of the problem

\( R(n) \): measurement of some resource used (time or space)
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A method for bounding the resources used by a function by the "size" of a problem

\[ n: \text{ size of the problem} \]

\[ R(n): \text{ measurement of some resource used (time or space)} \]

\[ R(n) = \Theta(f(n)) \]

means that there are positive constants \( k_1 \) and \( k_2 \) such that
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\[ k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n) \]
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for all \( n \) larger than some minimum \( m \)
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for all \( n \) larger than some minimum \( m \)
Counting Factors
Counting Factors

Number of operations required to count the factors of \( n \) using factors\_fast is \( \Theta(\sqrt{n}) \)
Counting Factors

Number of operations required to count the factors of $n$ using factors_fast is $\Theta(\sqrt{n})$
Counting Factors

Number of operations required to count the factors of n using factors_fast is $\Theta(\sqrt{n})$.

To check the lower bound, we choose $k_1 = 1$:

```python
def factors_fast(n):
    sqrt_n = sqrt(n)
    k, total = 1, 0
    while k < sqrt_n:
        if divides(k, n):
            total += 2
            k += 1
        if k * k == n:
            total += 1
    return total
```
Counting Factors

Number of operations required to count the factors of \( n \) using \texttt{factors\_fast} is \( \Theta(\sqrt{n}) \).

To check the lower bound, we choose \( k_1 = 1 \):

- Statements outside the \texttt{while}: 4 or 5

```python
def factors_fast(n):
    sqrt_n = sqrt(n)
    k, total = 1, 0
    while k < sqrt_n:
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            total += 2
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```
**Counting Factors**

Number of operations required to count the factors of \( n \) using `factors_fast` is \( \Theta(\sqrt{n}) \)

To check the lower bound, we choose \( k_1 = 1 \):

- Statements outside the `while`: 4 or 5
- Statements within the `while` (including header): 3 or 4

```python
def factors_fast(n):
    sqrt_n = sqrt(n)
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- Statements within the \texttt{while} (including header): 3 or 4
- \texttt{while} statement iterations: between \( \sqrt{n} - 1 \) and \( \sqrt{n} \)

```python
def factors_fast(n):
    sqrt_n = sqrt(n)
    k, total = 1, 0
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To check the lower bound, we choose \( k_1 = 1 \):

- Statements outside the while: 4 or 5
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- while statement iterations: between \( \sqrt{n} - 1 \) and \( \sqrt{n} \)
- Total number of statements executed: at least \( 4 + 3(\sqrt{n} - 1) \)

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def factors_fast(n):
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Number of operations required to count the factors of $n$ using `factors_fast` is $\Theta(\sqrt{n})$

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To check the upper bound:

```python
def factors_fast(n):
    sqrt_n = sqrt(n)
    k, total = 1, 0
    while k < sqrt_n:
        if divides(k, n):
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        k += 1
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```
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- Maximum statements executed: \( 5 + 4\sqrt{n} \)

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Number of operations required to count the factors of $n$ using factors_fast is $\Theta(\sqrt{n})$

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- while statement iterations: between $\sqrt{n} - 1$ and $\sqrt{n}$
- Total number of statements executed: at least $4 + 3(\sqrt{n} - 1)$

To check the upper bound

- Maximum statements executed: $5 + 4\sqrt{n}$
- Maximum operations required per statement: some $p$

```python
def factors_fast(n):
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def factors_fast(n):
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    return total
```

Assumption: every statement, such as addition-then-assignment using the \( += \) operator, takes some fixed number of operations to execute
Counting Factors

Number of operations required to count the factors of $n$ using `factors_fast` is $\Theta(\sqrt{n})$

To check the lower bound, we choose $k_1 = 1$:

- Statements outside the `while`: 4 or 5
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- `while` statement iterations: between $\sqrt{n} - 1$ and $\sqrt{n}$
- Total number of statements executed: at least $4 + 3(\sqrt{n} - 1)$

To check the upper bound:

- Maximum statements executed: $5 + 4\sqrt{n}$
- Maximum operations required per statement: some $p$
- We choose $k_2 = 5p$ and $m = 25$

```python
def factors_fast(n):
    sqrt_n = sqrt(n)
    k, total = 1, 0
    while k < sqrt_n:
        if divides(k, n):
            total += 2
            k += 1
        if k * k == n:
            total += 1
    return total
```
Assumption: every statement, such as addition-then-assignment using the `+=` operator, takes some fixed number of operations to execute.
Order of Growth of Counting Factors

Implementations of the same functional abstraction can require different amounts of time

Problem: How many factors does a positive integer n have?

A factor k of n is a positive integer that evenly divides n

def factors(n):

    **Slow**: Test each k from 1 through n

    **Fast**: Test each k from 1 to square root n
    For every k, n/k is also a factor!
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    Time     Space

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<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
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<td>$\Theta(n)$</td>
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Order of Growth of Counting Factors

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**Problem:** How many factors does a positive integer $n$ have?

A factor $k$ of $n$ is a positive integer that evenly divides $n$

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def factors(n):
    # Time and Space complexities
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    # Fast: Test each k from 1 to square root n
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**Order of Growth of Counting Factors**

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Assumption: integers occupy a fixed amount of space.
Exponentiation
Exponentiation
Exponentiation

**Goal:** one more multiplication lets us double the problem size
Exponentiation

**Goal:** one more multiplication lets us double the problem size

```python
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)
```
Exponentiation

**Goal:** one more multiplication lets us double the problem size

```python
def exp(b, n):
    if n == 0:
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```

\[ b^n = \begin{cases} 
1 & \text{if } n = 0 \\
 b \cdot b^{n-1} & \text{otherwise} 
\end{cases} \]
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\[ b^n = \begin{cases} 
1 & \text{if } n = 0 \\
(b^{\frac{1}{2}}n)^2 & \text{if } n \text{ is even} \\
b \cdot b^{n-1} & \text{if } n \text{ is odd} 
\end{cases} \]
Exponentiation

**Goal:** one more multiplication lets us double the problem size

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b^n = \begin{cases} 
1 & \text{if } n = 0 \\
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```python
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)

def square(x):
    return x**x

def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)
```
Exponentiation

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def exp(b, n):
    if n == 0:
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b^n = \begin{cases} 
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b^n = \begin{cases} 
1 & \text{if } n = 0 \\
 (b^{n/2})^2 & \text{if } n \text{ is even} \\
 b \cdot b^{n-1} & \text{if } n \text{ is odd} 
\end{cases}
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```python
def exp(b, n):
    if n == 0:
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def square(x):
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def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)
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def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)
```
Comparing Orders of Growth
Properties of Orders of Growth
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process \( \Theta(n) \)
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \]
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \]
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \]
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process
\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

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**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

```python
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

```python
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

*Outer: length of a*
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

```python
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

*Outer: length of a*, *Inner: length of b*
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \]

\[ \Theta(500 \cdot n) \]

\[ \Theta(\frac{1}{500} \cdot n) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \]

\[ \Theta(\log_{10} n) \]

\[ \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

\[ n \times 500 \times \log_2 n \times \log_{10} n \times \ln n \]

```python
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

If a and b are both length n, then overlap takes \( \Theta(n^2) \) steps.
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

\[
\cdot (n) \times \cdot (500 \cdot n) \times \cdot \left(\frac{1}{500} \cdot n\right) \]

\[
\cdot (\log_2 n) \times \cdot (\log_{10} n) \times \cdot (\ln n) \]

```python
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

If a and b are both length n, then overlap takes \( \Theta(n^2) \) steps

**Lower-order terms:** The fastest-growing part of the computation dominates the total
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

\[
\text{overlap}(a, b): \quad \text{count} = 0 \\
\quad \text{for item in } a: \quad \text{if item in } b: \quad \text{count} += 1 \\
\quad \text{return count}
\]


If a and b are both length \( n \), then overlap takes \( \Theta(n^2) \) steps

**Lower-order terms:** The fastest-growing part of the computation dominates the total

\[ \Theta(n^2) \]
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process

\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process

\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps

```python
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

If \( a \) and \( b \) are both length \( n \), then overlap takes \( \Theta(n^2) \) steps

**Lower-order terms:** The fastest-growing part of the computation dominates the total

\[ \Theta(n^2) \quad \Theta(n^2 + n) \]
Properties of Orders of Growth

**Constants:** Constant terms do not affect the order of growth of a process.
\[ \Theta(n) \quad \Theta(500 \cdot n) \quad \Theta\left(\frac{1}{500} \cdot n\right) \]

**Logarithms:** The base of a logarithm does not affect the order of growth of a process.
\[ \Theta(\log_2 n) \quad \Theta(\log_{10} n) \quad \Theta(\ln n) \]

**Nesting:** When an inner process is repeated for each step in an outer process, multiply the steps in the outer and inner processes to find the total number of steps.

```python
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

If \( a \) and \( b \) are both length \( n \), then overlap takes \( \Theta(n^2) \) steps.

**Lower-order terms:** The fastest-growing part of the computation dominates the total.
\[ \Theta(n^2) \quad \Theta(n^2 + n) \quad \Theta(n^2 + 500 \cdot n + \log_2 n + 1000) \]
Comparing orders of growth (n is the problem size)
Comparing orders of growth (n is the problem size)

$\Theta(b^n)$
Comparing orders of growth (n is the problem size)

$\Theta(b^n)$ Exponential growth. Recursive `fib` takes

$\Theta(\phi^n)$ steps, where $\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828$
Comparing orders of growth (n is the problem size)

\( \Theta(b^n) \) Exponential growth. Recursive \texttt{fib} takes \( \Theta(\phi^n) \) steps, where \( \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \)

Incrementing the problem scales \( R(n) \) by a factor
Comparing orders of growth (n is the problem size)

$\Theta(b^n)$ Exponential growth. Recursive \texttt{fib} takes $\Theta(\phi^n)$ steps, where $\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828$

Incrementing the problem scales $R(n)$ by a factor

$\Theta(n^2)$
Comparing orders of growth (n is the problem size)

- $\Theta(b^n)$: Exponential growth. Recursive $\text{fib}$ takes $\Theta(\phi^n)$ steps, where $\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828$
  Incrementing the problem scales $R(n)$ by a factor

- $\Theta(n^2)$: Quadratic growth. E.g., overlap
Comparing orders of growth (n is the problem size)

$\Theta(b^n)$ Exponential growth. Recursive fib takes

$\Theta(\phi^n)$ steps, where $\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828$

Incrementing the problem scales $R(n)$ by a factor

$\Theta(n^2)$ Quadratic growth. E.g., overlap

Incrementing $n$ increases $R(n)$ by the problem size $n$
Comparing orders of growth (n is the problem size)

\[ \Theta(b^n) \] Exponential growth. Recursive \texttt{fib} takes \[ \Theta(\phi^n) \] steps, where \[ \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \]
Incrementing the problem scales R(n) by a factor

\[ \Theta(n^2) \] Quadratic growth. E.g., \texttt{overlap}
Incrementing n increases R(n) by the problem size n

\[ \Theta(n) \]
Comparing orders of growth (n is the problem size)

\( \Theta(b^n) \) Exponential growth. Recursive \texttt{fib} takes \( \Theta(\phi^n) \) steps, where \( \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \)

Incrementing the problem scales \( R(n) \) by a factor

\( \Theta(n^2) \) Quadratic growth. E.g., \texttt{overlap}

Incrementing \( n \) increases \( R(n) \) by the problem size \( n \)

\( \Theta(n) \) Linear growth. E.g., slow \texttt{factors} or \texttt{exp}
Comparing orders of growth (n is the problem size)

\( \Theta(b^n) \) Exponential growth. Recursive \texttt{fib} takes

\[ \Theta(\phi^n) \] steps, where \( \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \)

Incrementing the problem scales \( R(n) \) by a factor

\( \Theta(n^2) \) Quadratic growth. E.g., \texttt{overlap}

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\( \Theta(n) \) Linear growth. E.g., slow \texttt{factors} or \texttt{exp}

\( \Theta(\sqrt{n}) \)
Comparing orders of growth (n is the problem size)

\[ \Theta(b^n) \] Exponential growth. Recursive \( \text{fib} \) takes
\[ \Theta(\phi^n) \] steps, where \( \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \)
Incrementing the problem scales \( R(n) \) by a factor

\[ \Theta(n^2) \] Quadratic growth. E.g., overlap
Incrementing \( n \) increases \( R(n) \) by the problem size \( n \)

\[ \Theta(n) \] Linear growth. E.g., slow factors or exp

\[ \Theta(\sqrt{n}) \] Square root growth. E.g., factors\_fast
Comparing orders of growth (n is the problem size)

\[ \Theta(b^n) \]  Exponential growth. Recursive \texttt{fib} takes \[ \Theta(\phi^n) \] steps, where \[ \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \]
Incrementing the problem scales \( R(n) \) by a factor

\[ \Theta(n^2) \]  Quadratic growth. E.g., \texttt{overlap} 
Incrementing \( n \) increases \( R(n) \) by the problem size \( n \)

\[ \Theta(n) \]  Linear growth. E.g., slow \texttt{factors} or \texttt{exp} 

\[ \Theta(\sqrt{n}) \]  Square root growth. E.g., \texttt{factors_fast} 

\[ \Theta(\log n) \]
Comparing orders of growth (n is the problem size)

\( \Theta(b^n) \) Exponential growth. Recursive \texttt{fib} takes \( \Theta(\phi^n) \) steps, where \( \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \)

Incrementing the problem scales \( R(n) \) by a factor

\( \Theta(n^2) \) Quadratic growth. E.g., \texttt{overlap}

Incrementing \( n \) increases \( R(n) \) by the problem size \( n \)

\( \Theta(n) \) Linear growth. E.g., slow \texttt{factors} or \texttt{exp}

\( \Theta(\sqrt{n}) \) Square root growth. E.g., \texttt{factors_fast}

\( \Theta(\log n) \) Logarithmic growth. E.g., \texttt{exp_fast}
Comparing orders of growth (n is the problem size)

\[ \Theta(b^n) \]

Exponential growth. Recursive \texttt{fib} takes \[ \Theta(\phi^n) \] steps, where \( \phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \)

Incrementing the problem scales \( R(n) \) by a factor

\[ \Theta(n^2) \]

Quadratic growth. E.g., \texttt{overlap}

Incrementing \( n \) increases \( R(n) \) by the problem size \( n \)

\[ \Theta(n) \]

Linear growth. E.g., slow \texttt{factors} or \texttt{exp}

\[ \Theta(\sqrt{n}) \]

Square root growth. E.g., \texttt{factors_fast}

\[ \Theta(\log n) \]

Logarithmic growth. E.g., \texttt{exp_fast}

Doubling the problem only increments \( R(n) \).
Comparing orders of growth (n is the problem size)

$\Theta(b^n)$ Exponential growth. Recursive fib takes $\Theta(\phi^n)$ steps, where $\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828$

Incrementing the problem scales $R(n)$ by a factor

$\Theta(n^2)$ Quadratic growth. E.g., overlap

Incrementing $n$ increases $R(n)$ by the problem size $n$

$\Theta(n)$ Linear growth. E.g., slow factors or exp

$\Theta(\sqrt{n})$ Square root growth. E.g., factors_fast

$\Theta(\log n)$ Logarithmic growth. E.g., exp_fast

Doubling the problem only increments $R(n)$.

$\Theta(1)$
Comparing orders of growth (n is the problem size)

**Θ(b^n)**  Exponential growth. Recursive fib takes

Θ(φ^n) steps, where  \( φ = \frac{1 + \sqrt{5}}{2} \approx 1.61828 \)

Incrementing the problem scales R(n) by a factor

**Θ(n^2)**  Quadratic growth. E.g., overlap

Incrementing n increases R(n) by the problem size n

**Θ(n)**  Linear growth. E.g., slow factors or exp

**Θ(\sqrt{n})**  Square root growth. E.g., factors_fast

**Θ(log n)**  Logarithmic growth. E.g., exp_fast

Doubling the problem only increments R(n).

**Θ(1)**  Constant. The problem size doesn't matter
Comparing orders of growth (n is the problem size)

- $\Theta(b^n)$: Exponential growth. Recursive `fib` takes $\Theta(\phi^n)$ steps, where $\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828$.
  - Incrementing the problem scales $R(n)$ by a factor of $b^n$.

- $\Theta(n^2)$: Quadratic growth. E.g., `overlap`.
  - Incrementing $n$ increases $R(n)$ by the problem size $n^2$.

- $\Theta(n)$: Linear growth. E.g., slow `factors` or `exp`.

- $\Theta(\sqrt{n})$: Square root growth. E.g., `factors_fast`.

- $\Theta(\log n)$: Logarithmic growth. E.g., `exp_fast`.
  - Doubling the problem only increments $R(n)$.

- $\Theta(1)$: Constant. The problem size doesn't matter.
Comparing orders of growth (n is the problem size)

- **$\Theta(b^n)$**: Exponential growth. Recursive `fib` takes $\Theta(\phi^n)$ steps, where $\phi = \frac{1 + \sqrt{5}}{2} \approx 1.61828$
  
  Incrementing the problem scales $R(n)$ by a factor

- **$\Theta(n^2)$**: Quadratic growth. E.g., `overlap`
  
  Incrementing $n$ increases $R(n)$ by the problem size $n$

- **$\Theta(n)$**: Linear growth. E.g., slow `factors` or `exp`

- **$\Theta(\sqrt{n})$**: Square root growth. E.g., `factors_fast`

- **$\Theta(\log n)$**: Logarithmic growth. E.g., `exp_fast`
  
  Doubling the problem only increments $R(n)$.

- **$\Theta(1)$**: Constant. The problem size doesn't matter