61A Lecture 28

Friday, November 7

Announcements

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- Homework 7 due Wednesday 4/8 @ 11:59pm

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Scheme Recursive Art Contest: Start Early!

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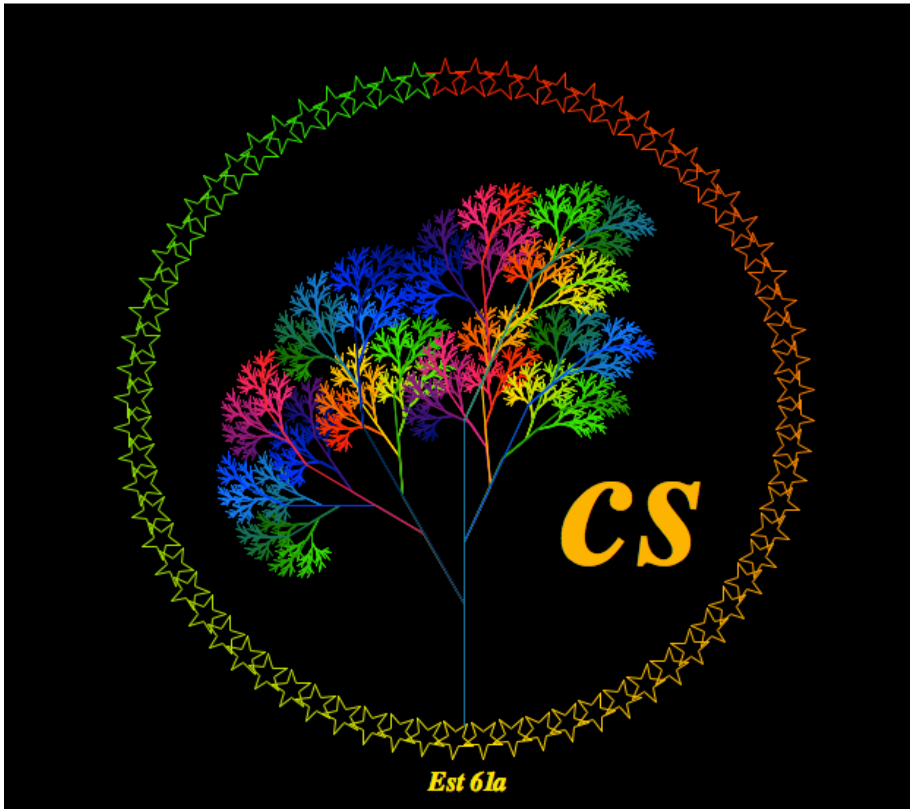

Fall 2012 Featherweight Winner
176 Scheme Tokens

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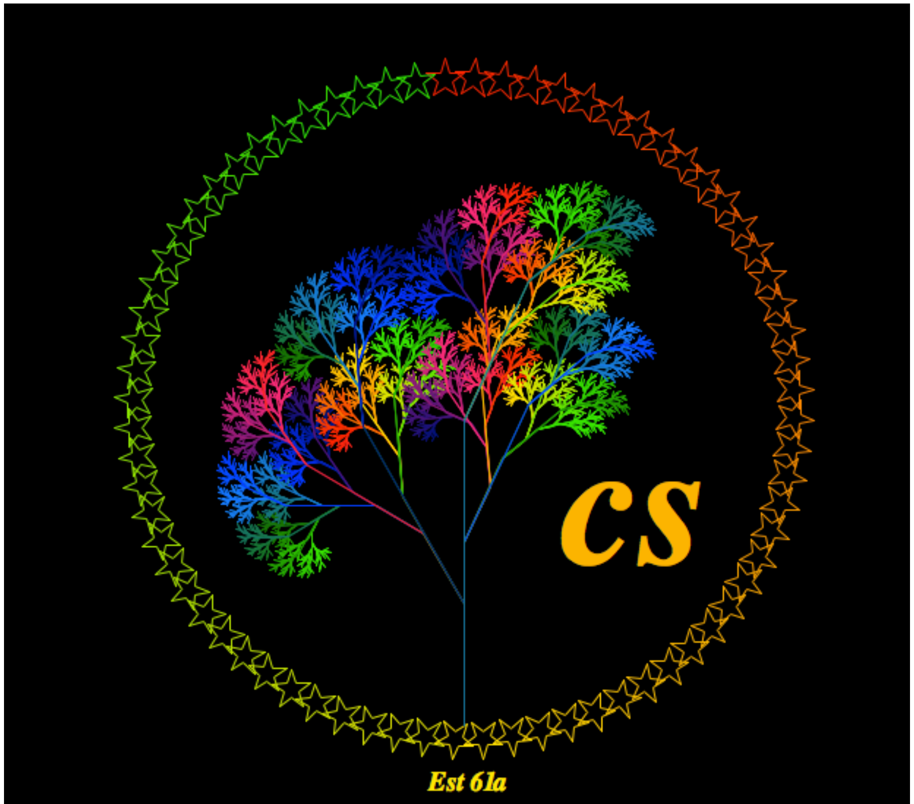

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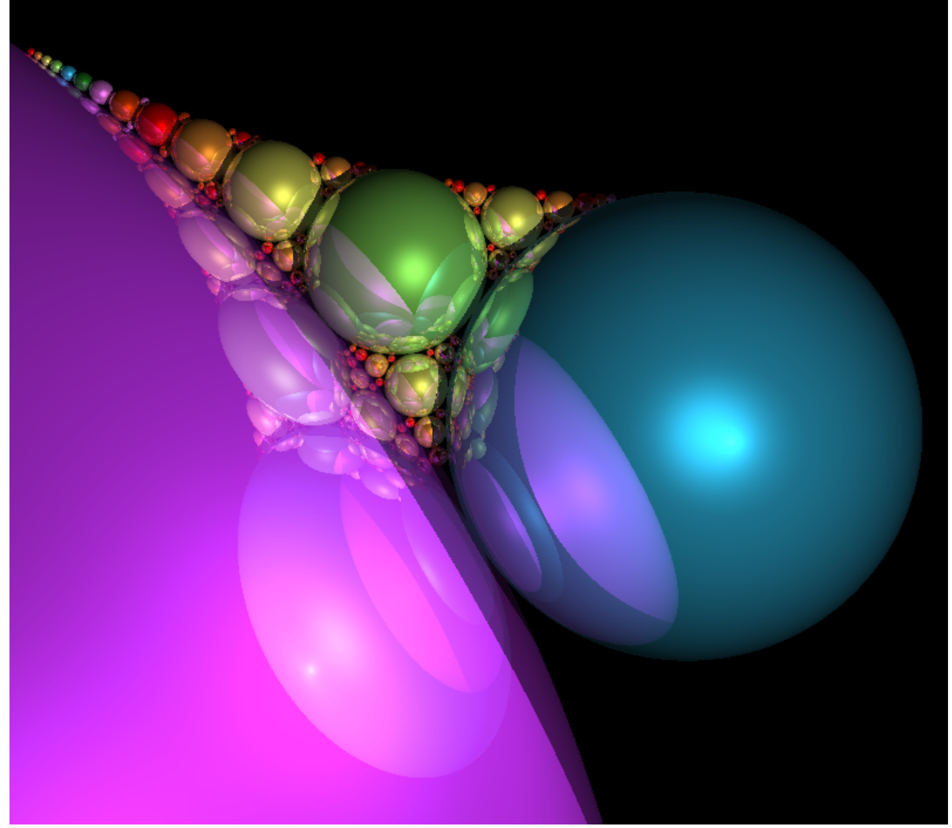

Fall 2013 Heavyweight Winner
1857 Scheme Tokens

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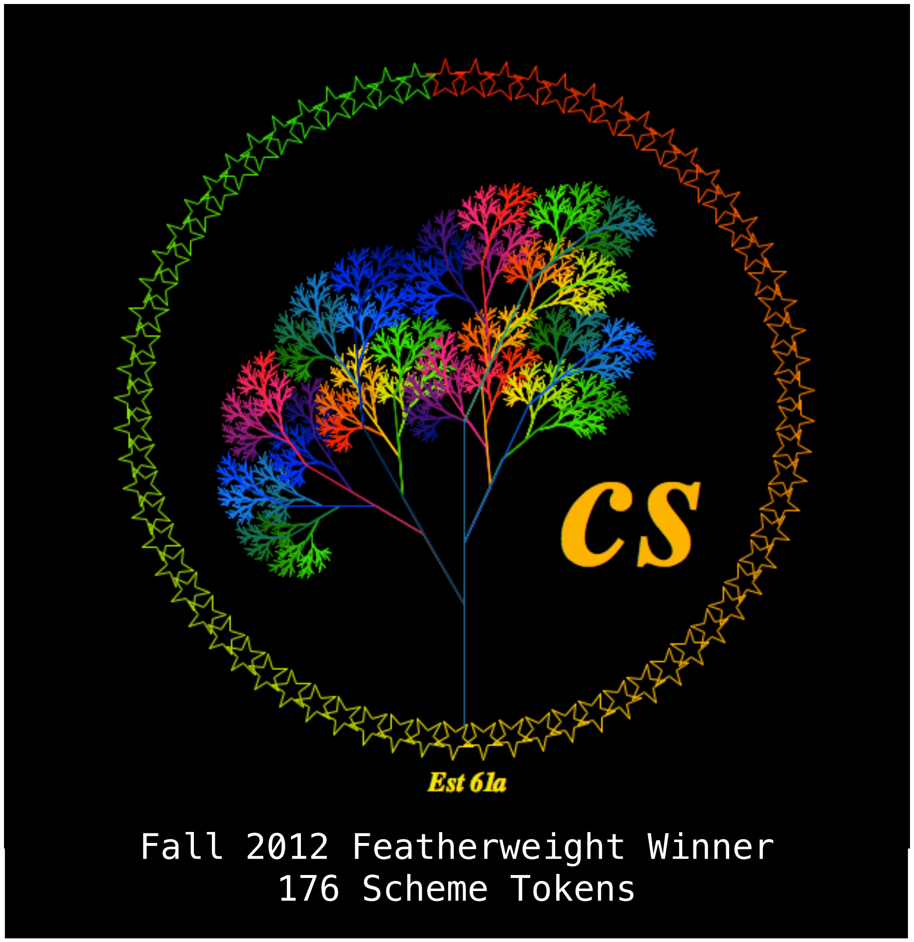

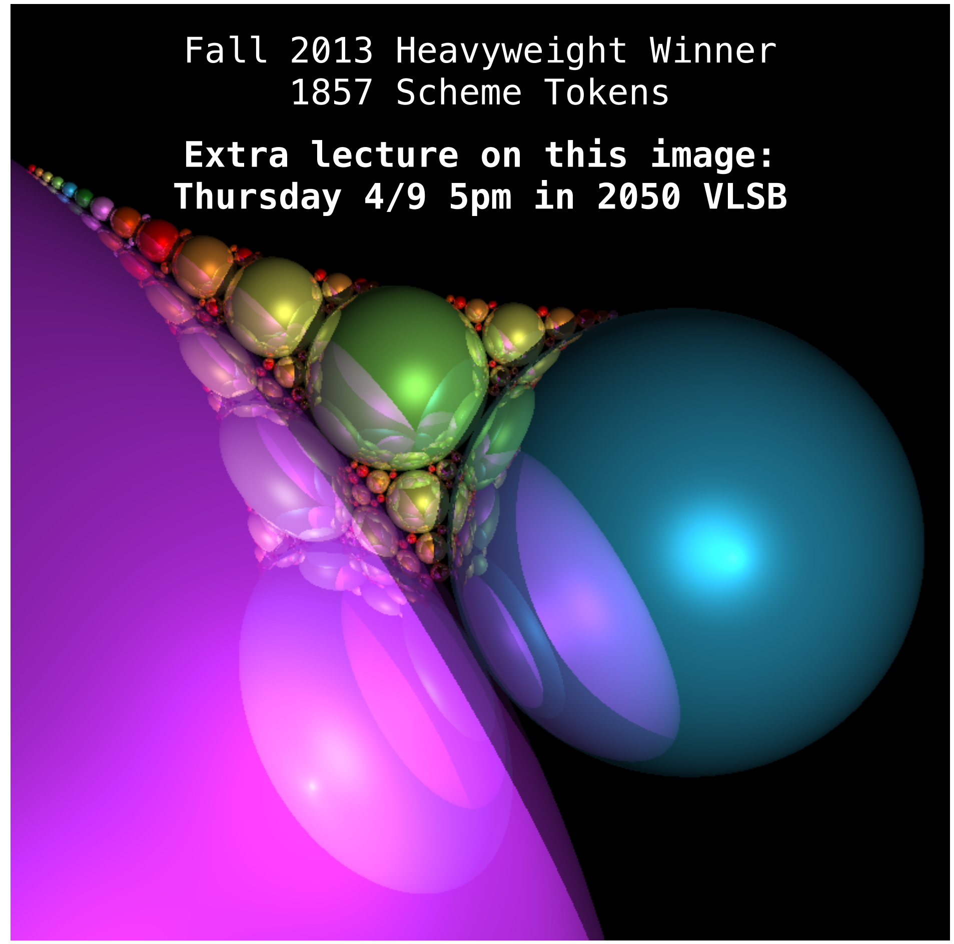

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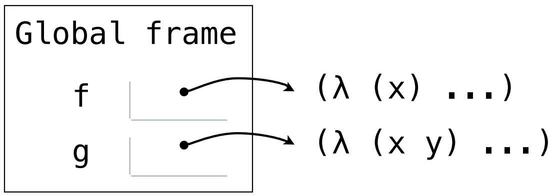


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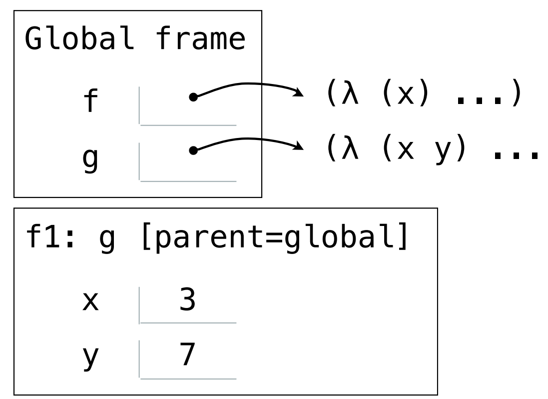


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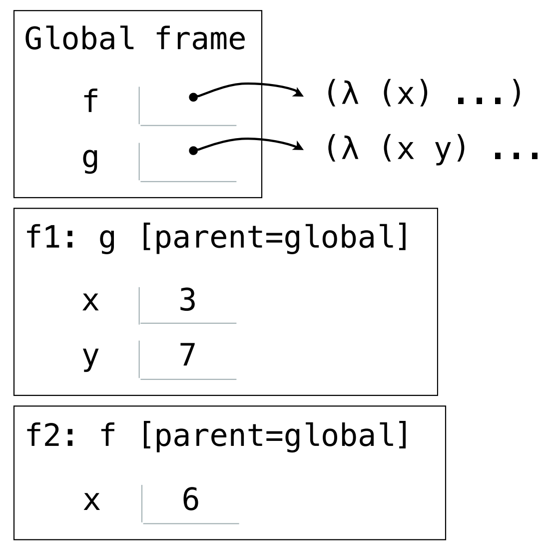

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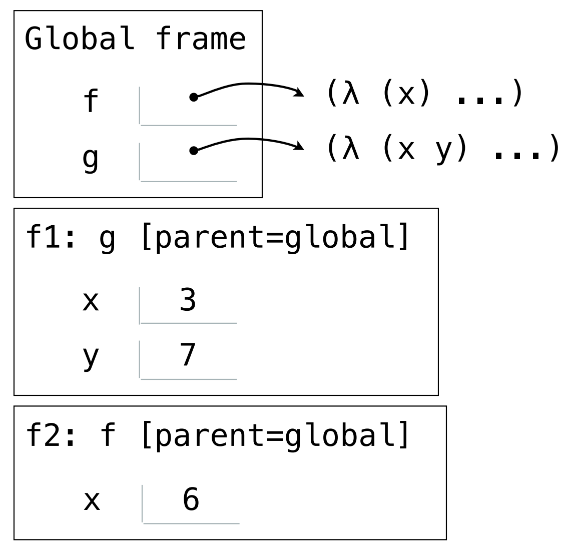

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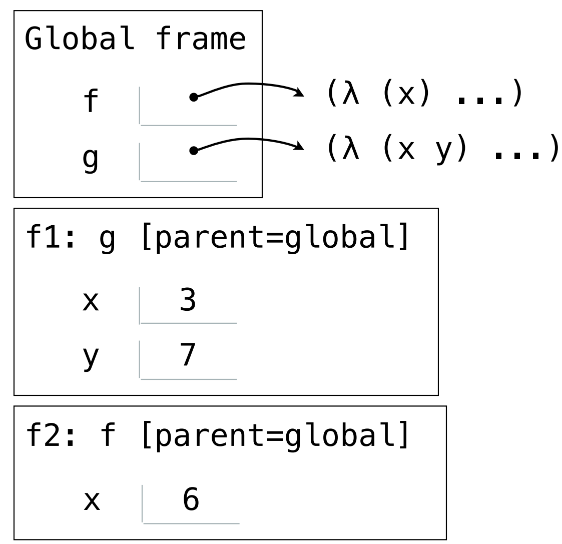

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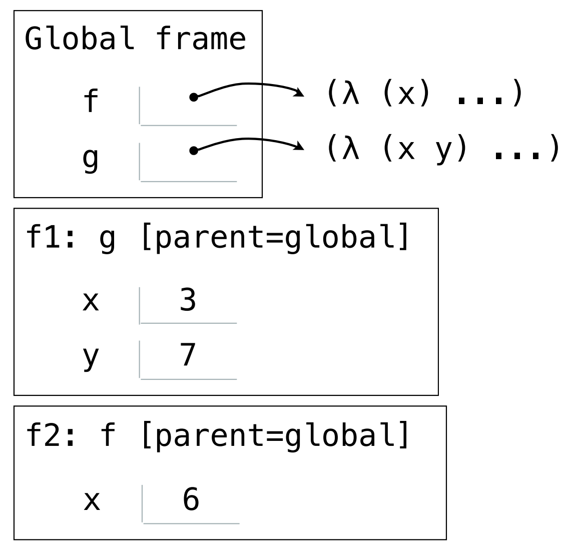

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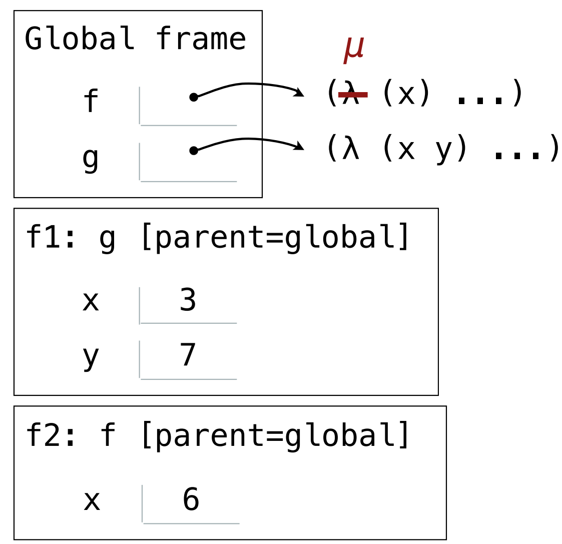

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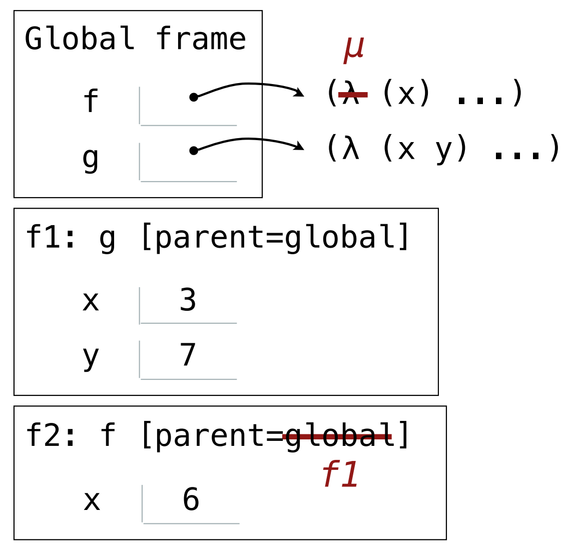

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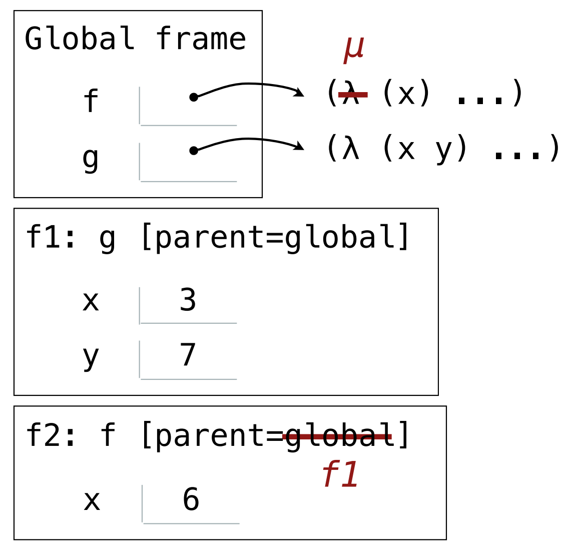

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13

Tail Recursion

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But... no for/while statements! Can we make basic iteration efficient? Yes!

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(Demo)

Interactive Diagram

Tail Calls

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```

Tail Calls

A procedure call that has not yet returned is active. Some procedure calls are tail calls. A Scheme interpreter should support an unbounded number of active tail calls using only a constant amount of space.

A tail call is a call expression in a tail context:

- The last body sub-expression in a lambda expression
- Sub-expressions 2 & 3 in a tail context if expression
- All non-predicate sub-expressions in a tail context cond
- The last sub-expression in a tail context and or or
- The last sub-expression in a tail context begin

```
(define (factorial n k)
  (if (= n 0) k
    (factorial (- n 1)
               (* k n)) ) )
```

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Recursive call is a tail call

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Tail Recursion Examples

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Which of the following procedures run in constant space? $\Theta(1)$

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Map and Reduce

Example: Reduce

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Example: Reduce

```
(define (reduce procedure s start)
  (if (null? s) start
   (reduce procedure
           (cdr s)
           (procedure start (car s)) ) ) )
```

Recursive call is a tail call

Space depends on what procedure requires

```
(reduce * '(3 4 5) 2)                                    120

(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))         (5 4 3 2)
```

Example: Map with Only a Constant Number of Frames

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```
(define (map procedure s)
```

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```
(define (map procedure s)
  (if (null? s)
```

Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
```

Example: Map with Only a Constant Number of Frames

```
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
```

Example: Map with Only a Constant Number of Frames

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(map (lambda (x) (- 5 x)) (list 1 2))
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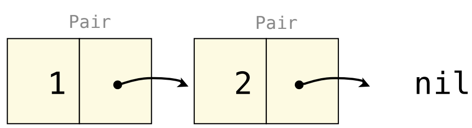

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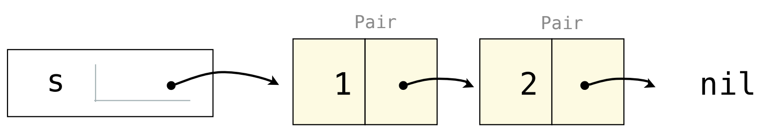

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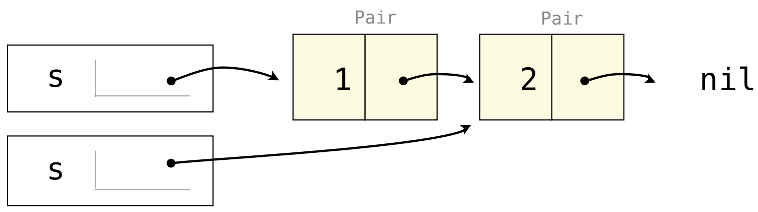

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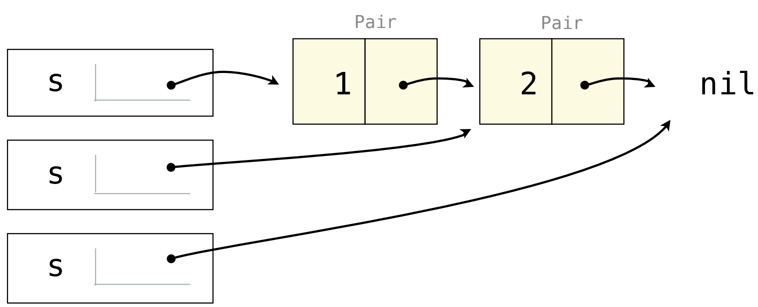

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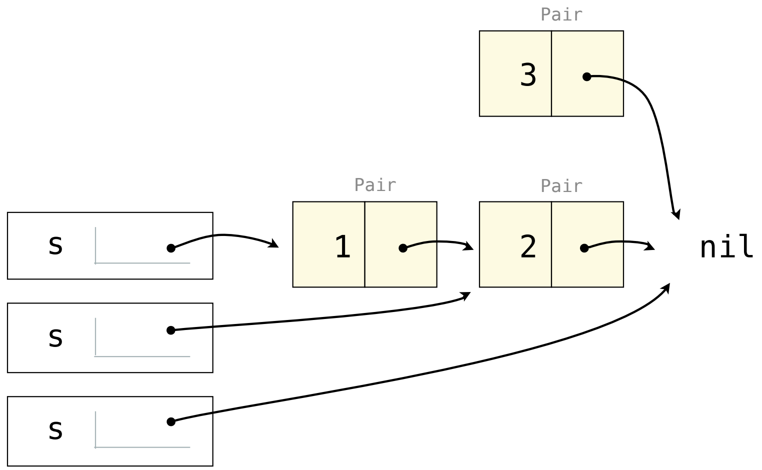

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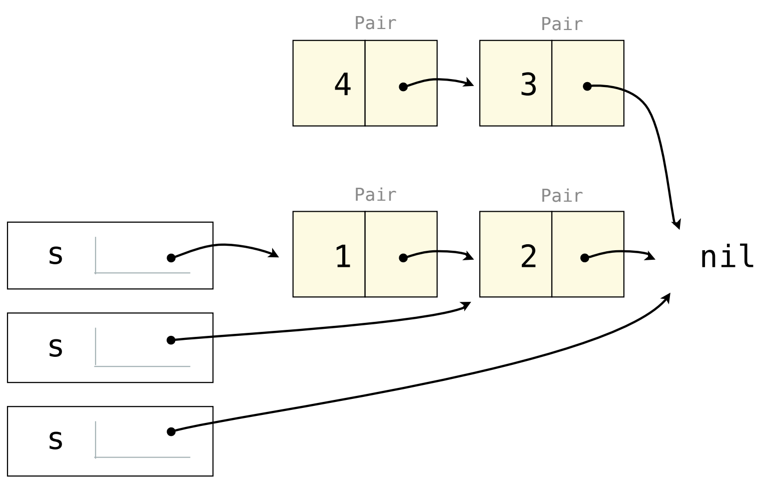

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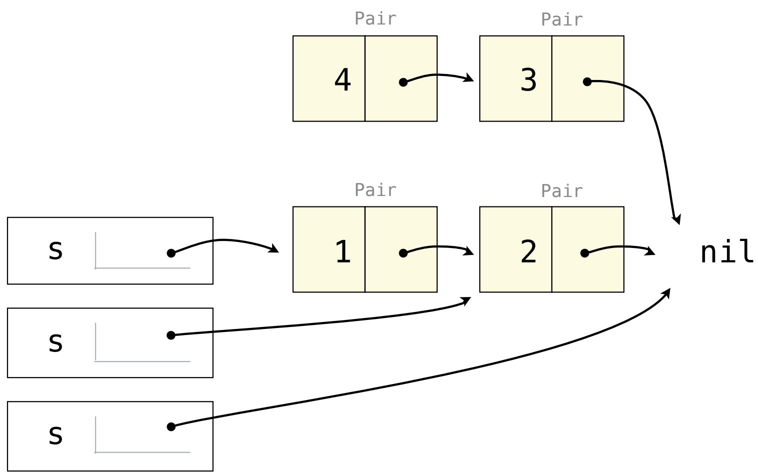

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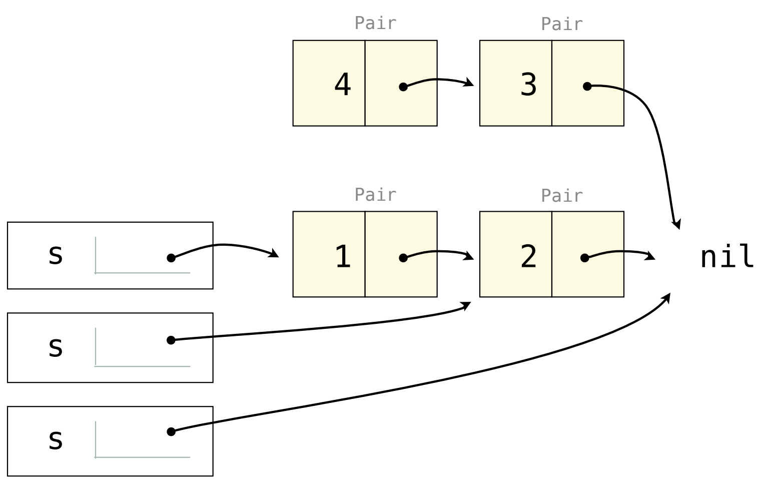

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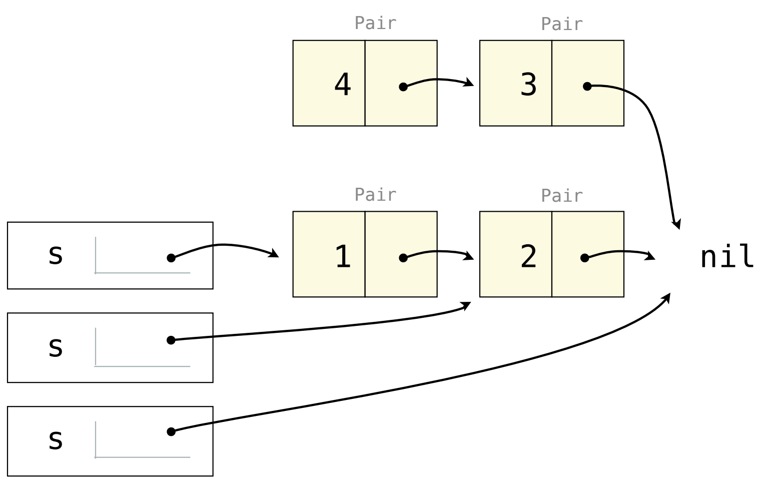


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```

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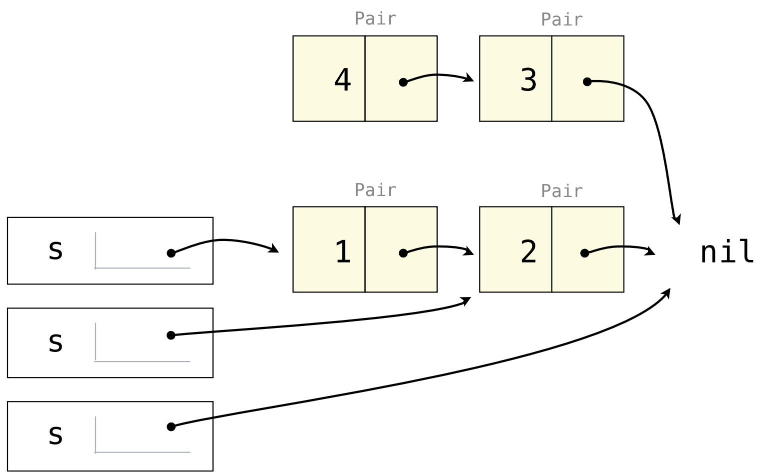

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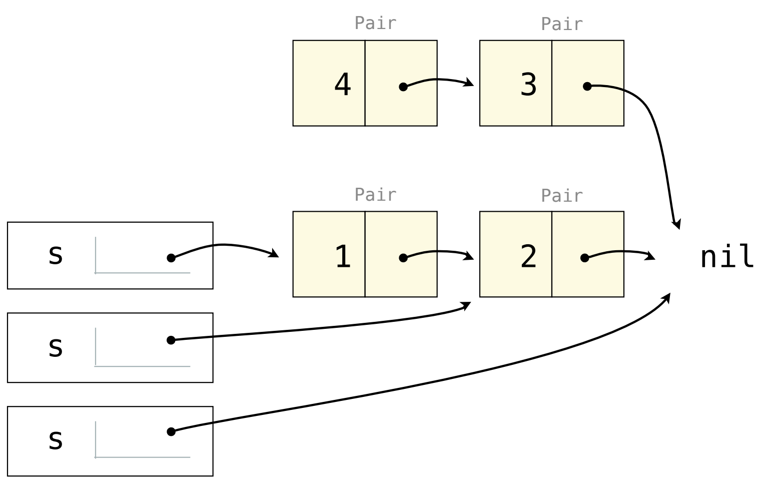

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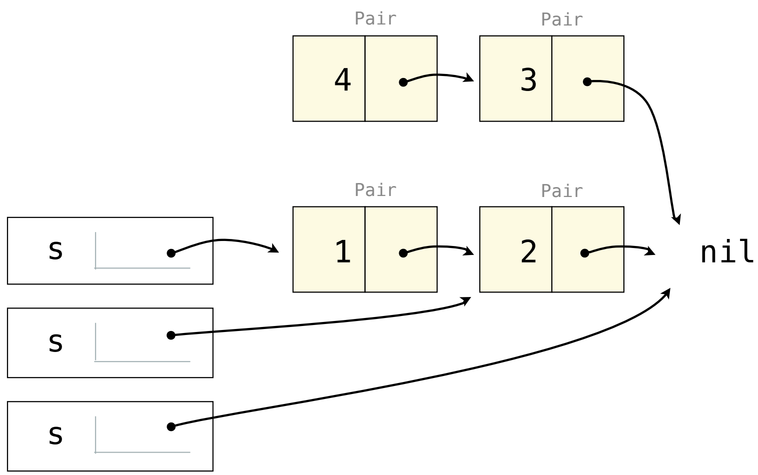

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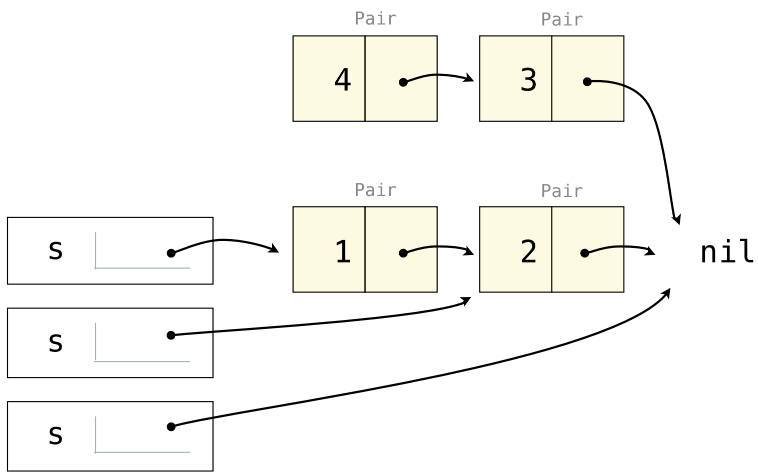

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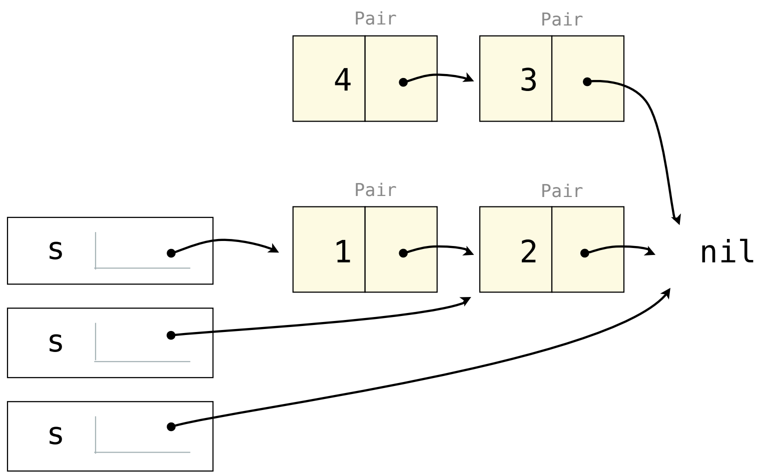


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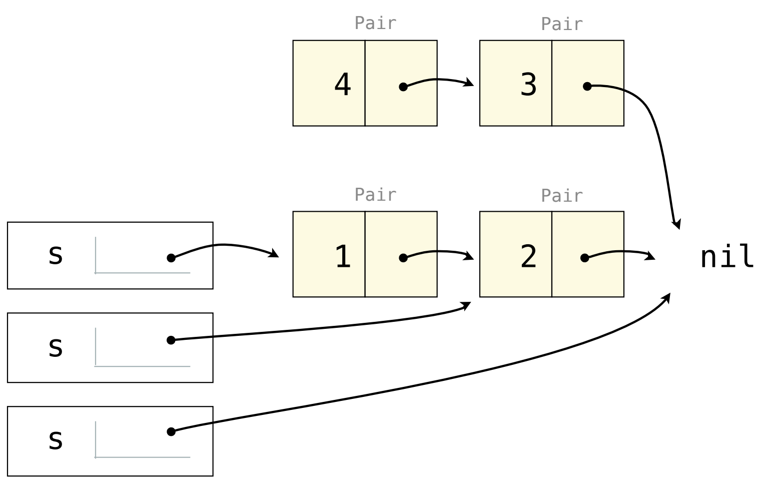

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```

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```

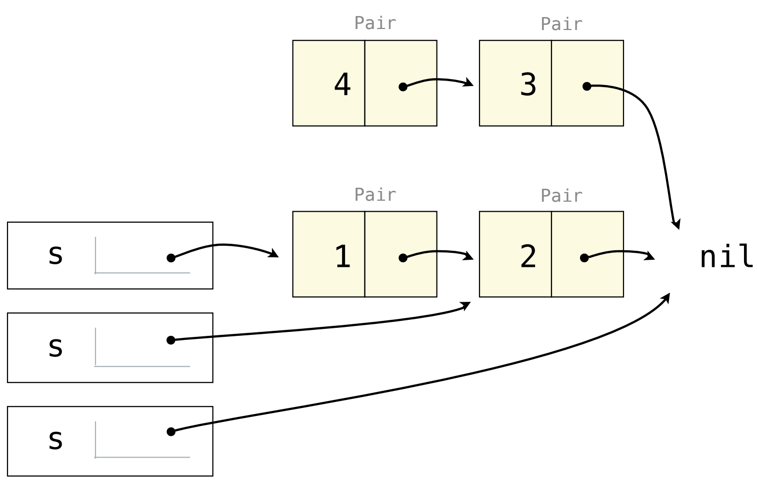


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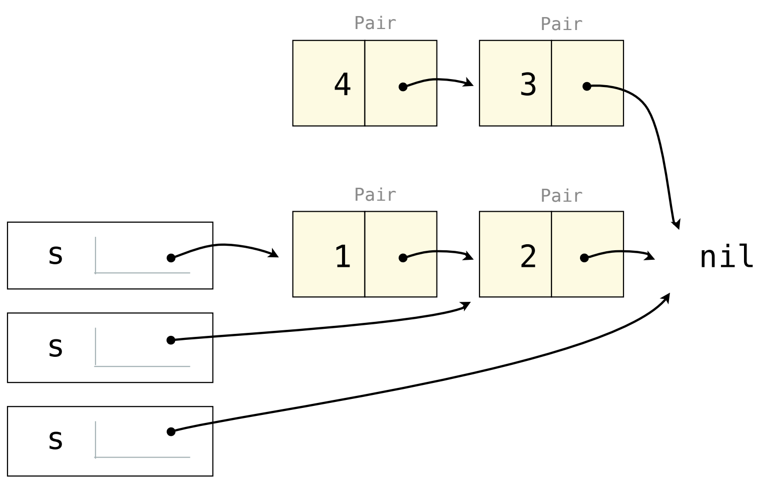


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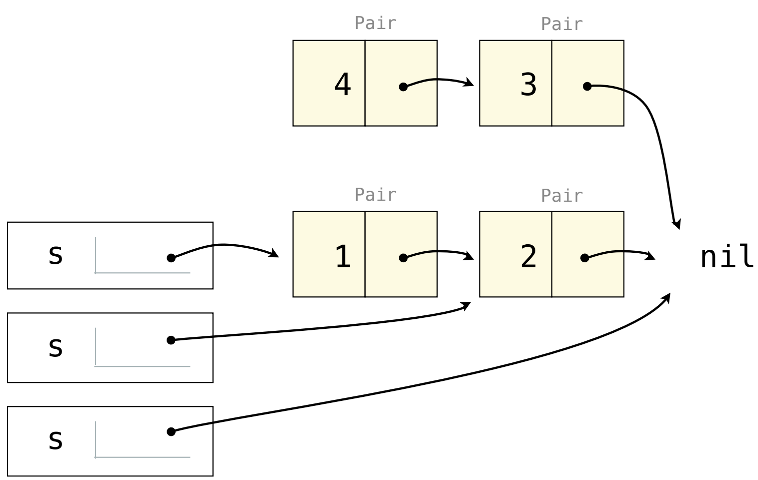


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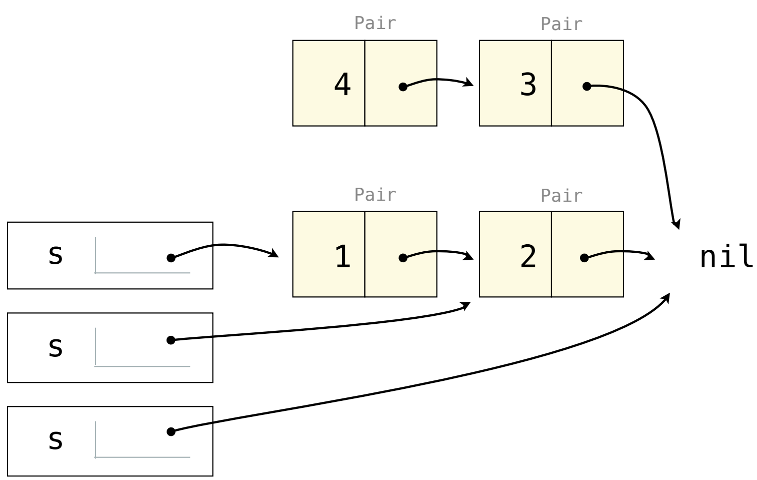


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  (define (reverse-iter s r)
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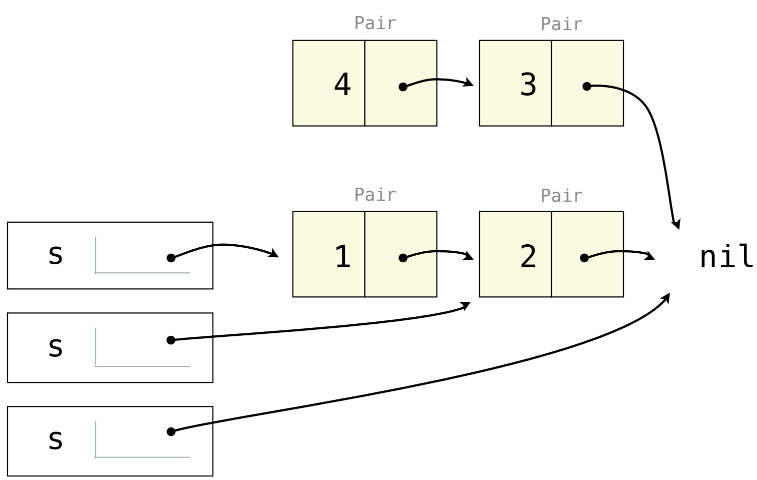


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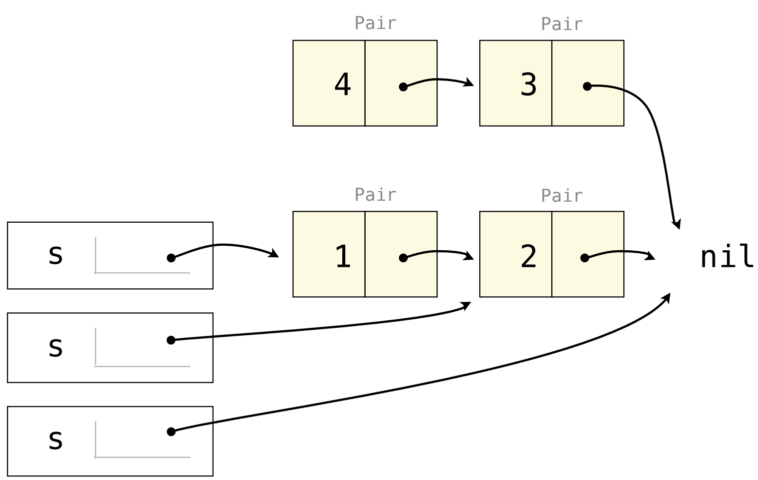


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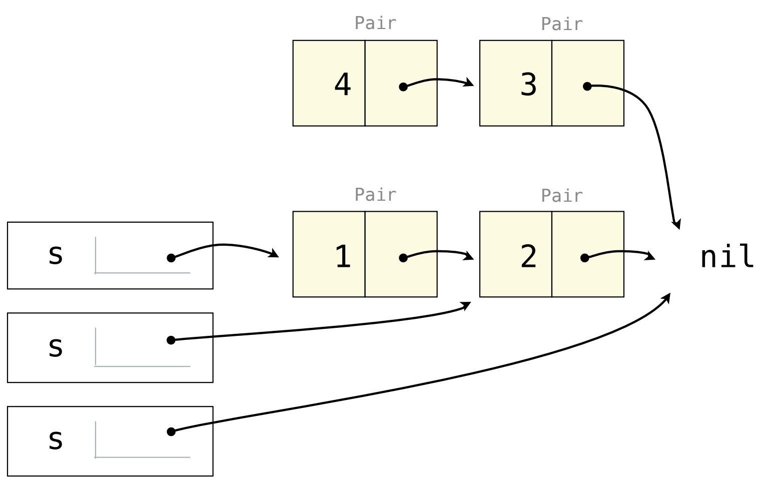


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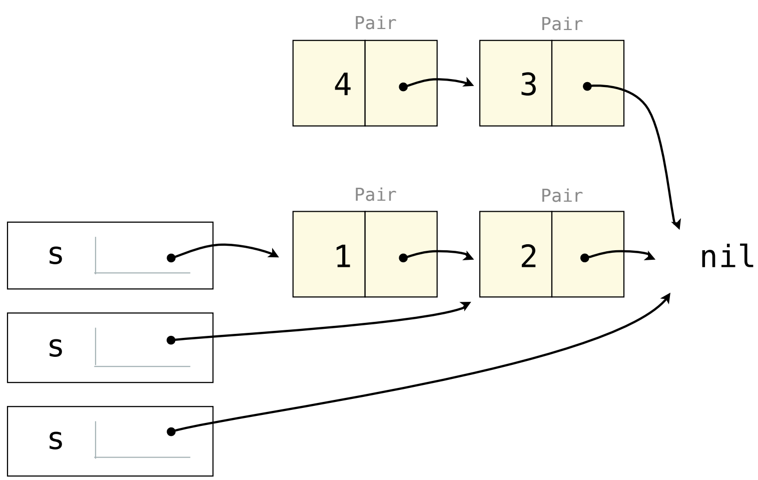


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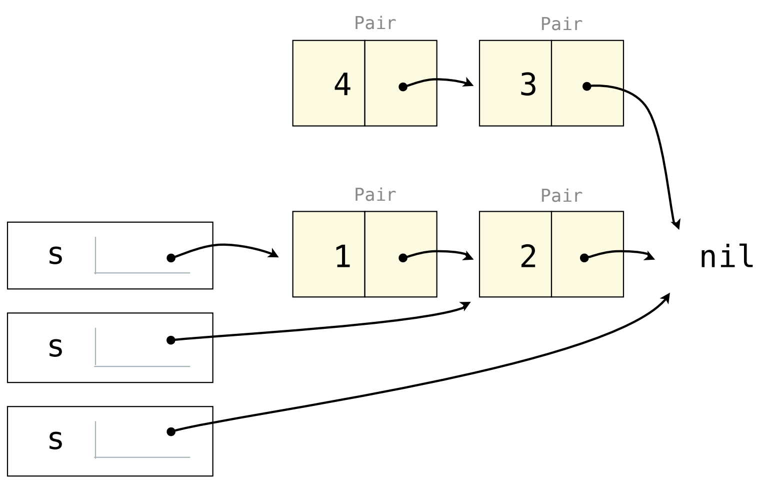


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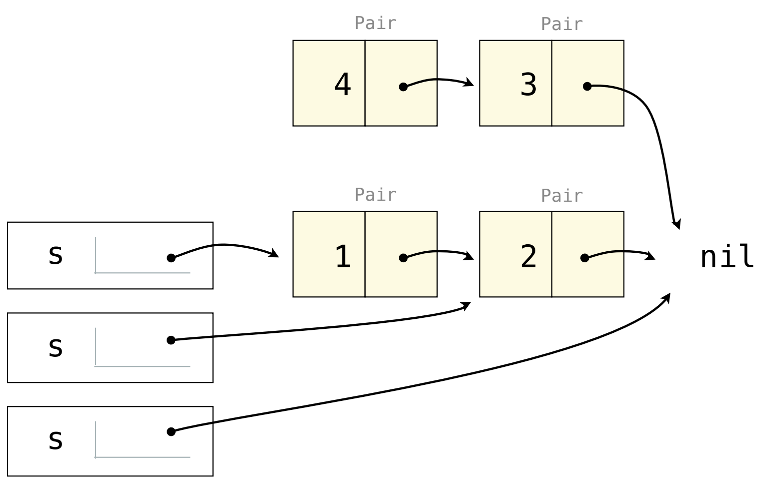


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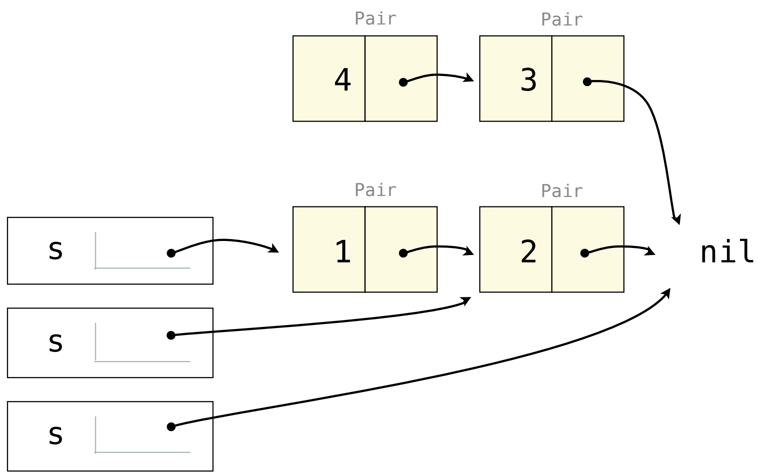


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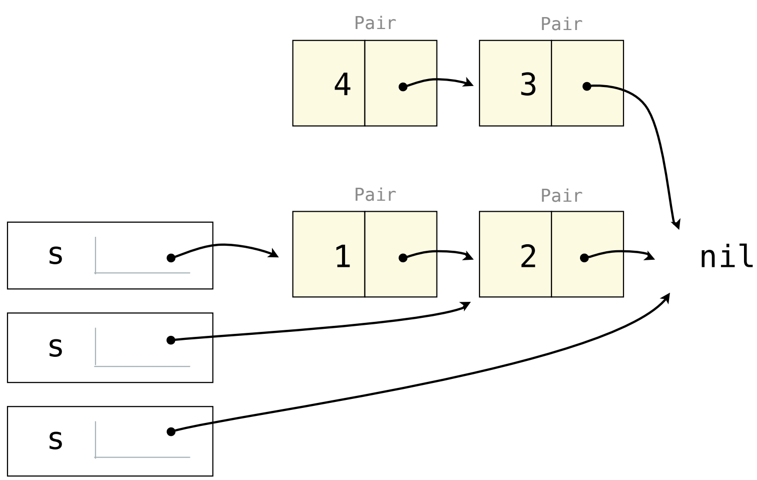


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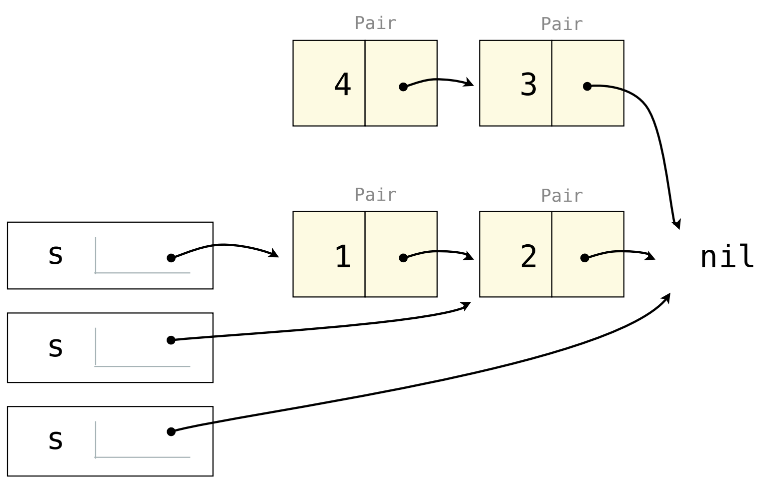


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General Computing Machines

An Analogy: Programs Define Machines

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Programs specify the logic of a computational device

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Programs specify the logic of a computational device

factorial

An Analogy: Programs Define Machines

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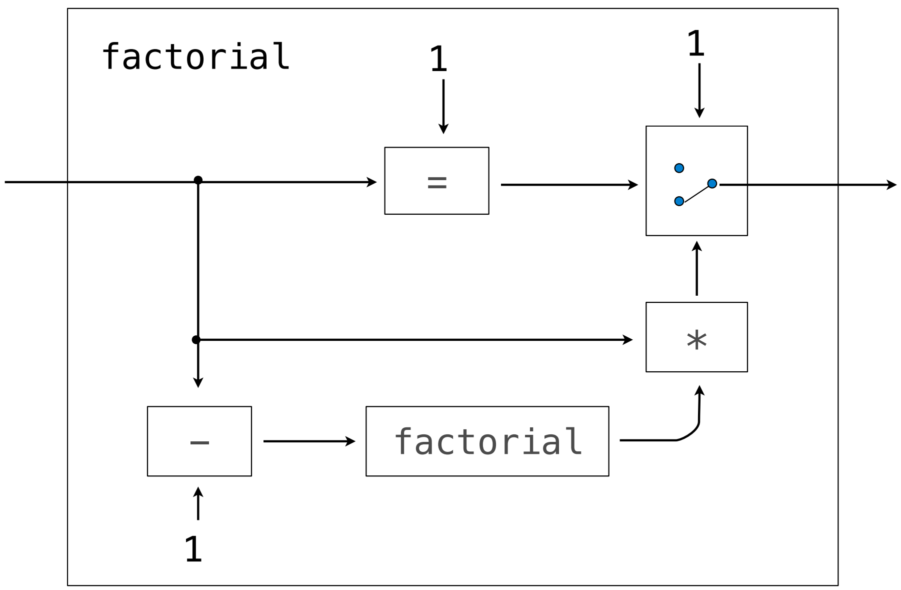

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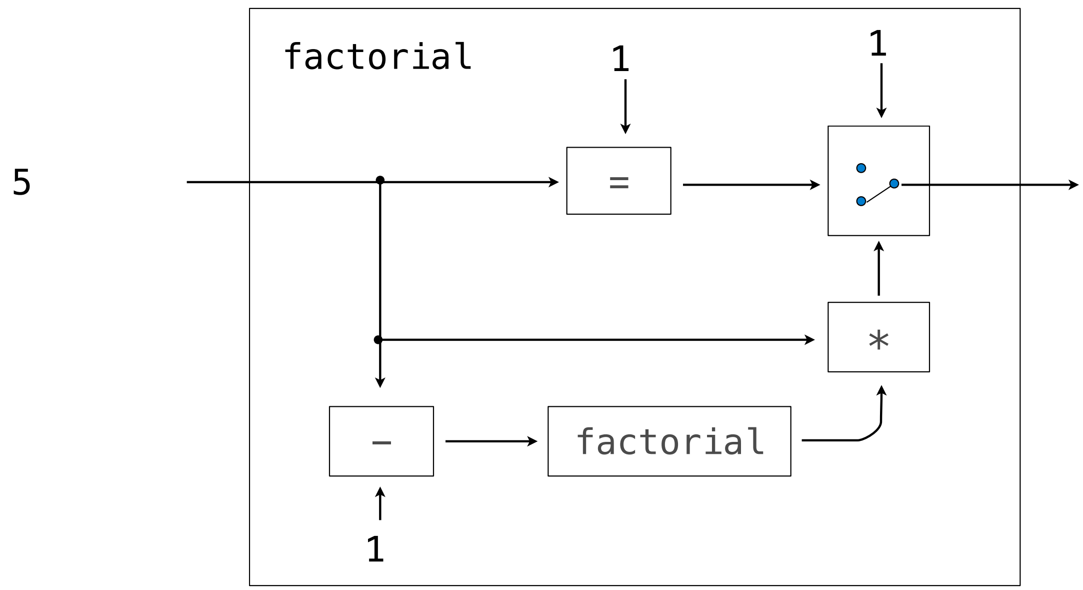
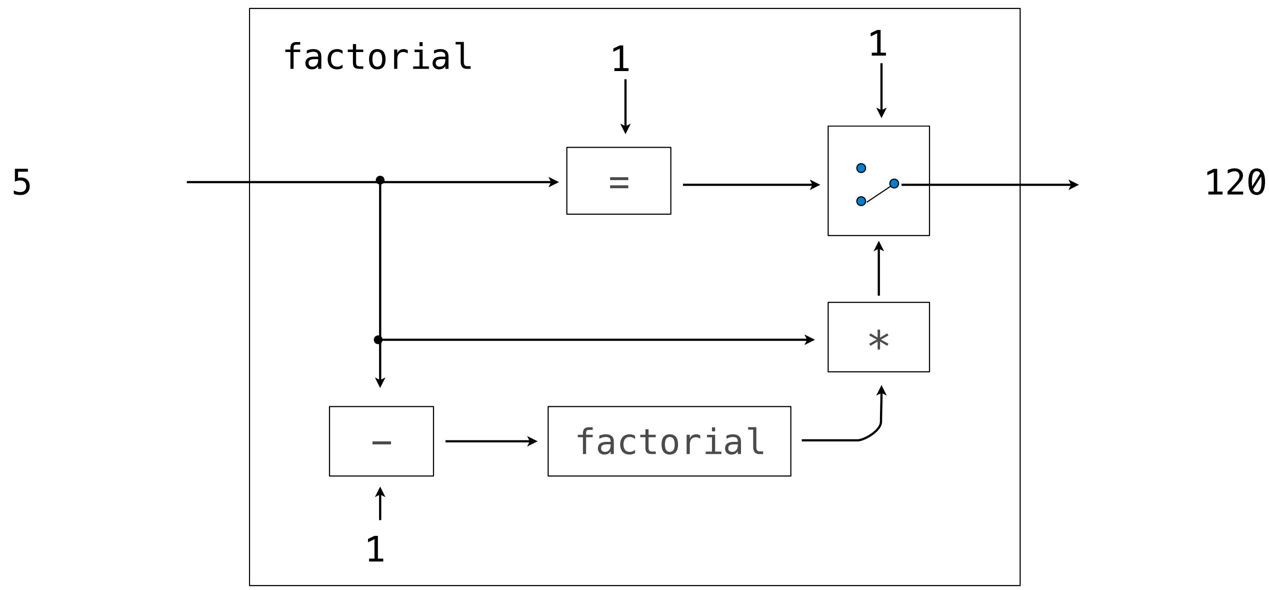


An Analogy: Programs Define Machines

Programs specify the logic of a computational device



Interpreters are General Computing Machine

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An interpreter can be parameterized to simulate any machine

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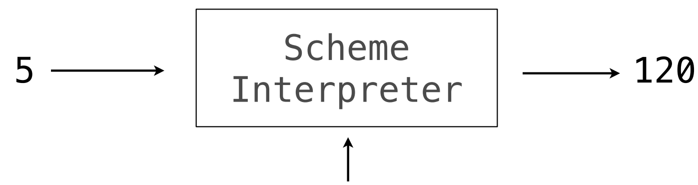


```
(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
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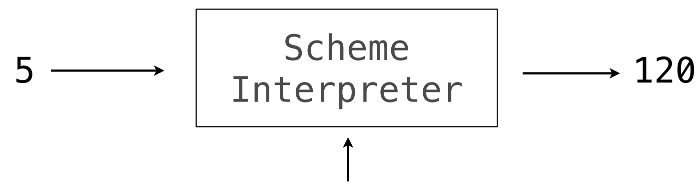


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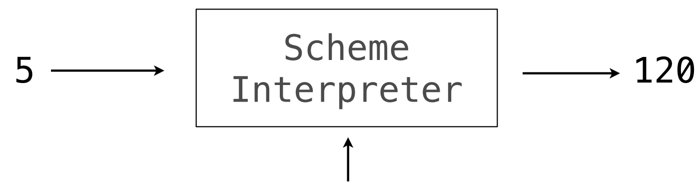


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```

Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

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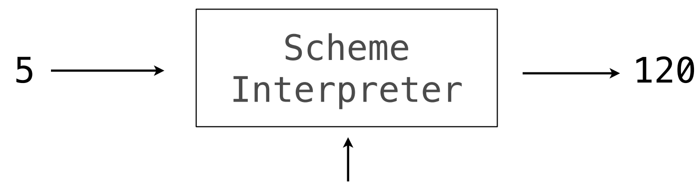


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Internally, it is just a set of evaluation rules