INSTRUCTIONS

• You have 3 hours to complete the exam.
• The exam is open book, open notes, closed computer, closed calculator.
• Mark your answers on the exam itself. We will not grade answers written on scratch paper.

<table>
<thead>
<tr>
<th>Last name</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First name</td>
<td></td>
</tr>
<tr>
<td>Student ID number</td>
<td></td>
</tr>
<tr>
<td>CalCentral email (@berkeley.edu)</td>
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<tr>
<td>TA</td>
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</tr>
<tr>
<td>Name of the person to your left</td>
<td></td>
</tr>
<tr>
<td>Name of the person to your right</td>
<td></td>
</tr>
<tr>
<td>Room in which you are taking exam</td>
<td></td>
</tr>
<tr>
<td>Seat number in the exam room</td>
<td></td>
</tr>
</tbody>
</table>

I pledge my honor that during this examination I have neither given nor received assistance. (please sign)
# Linked Lists

class Link: 
    """A linked list cell."
    >>> L = Link(0, Link(1))
    >>> L.first
    0
    >>> L.rest
    Link(1)
    >>> L.first = 2
    >>> L
    Link(2, Link(1))
    >>> L.rest = Link.empty
    >>> L
    Link(2)
    """
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest is Link.empty:
            return "Link({})".format(self.first)
        else:
            return "Link({}, {})".format(self.first, self.rest)

# Trees

class Tree:
    """A tree node."""

    def __init__(self, label, branches=[]):
        for c in branches:
            assert isinstance(c, Tree)
        self.label = label
        self.branches = branches

    def is_leaf(self):
        return not self.branches

class BinTree:
    empty = ()

    def __init__(self, label, left=empty, right=empty):
        self.label = label
        self.left = left
        self.right = right
# Streams

class Stream:
    """An infinite, lazily computed linked list."""
    def __init__(self, first, rest):
        """A stream whose .first element is FIRST and whose .rest is either
        REST, if it is a Stream, or else the Stream resulting from calling
        REST(). In the latter case, the function is called only once when
        first needed, and its result is memoized."""

    def combine_streams(func, sa, sb):
        """Return the stream of values (FUNC(sa1, sb1), FUNC(sa2, sb2), ...)
        where SA is (sa1, sa2, ...) and SB is (sb1, sb2, ...)."""

    def filter_stream(pred, s):
        """Return the substream of S consisting of values, si, for
        which PRED(si)."""

    def make_integer_stream(start):
        """Return the stream containing START, START+1, START+2, ...."""

    def stream_to_list(s, n):
        """Return a list containing the first N elements of Stream S."""
1. (8 points) What Would Python Do?

For each of the expressions in the table below, write the output displayed by the interactive Python interpreter when the expression is evaluated. The output may have multiple lines. If an error (exception) occurs, write “Error”. If an expression yields (or prints) a function, write “<Function>”. The first two rows have been provided as examples.

**Important:** The statements in the table are cumulative—assume that all preceding statements in the table have been executed before each entry.

Assume that `python3` has executed the statements on the left initially:

```python
def g(L):
    def k():
        yield iter(L)
        yield iter(L)
    return k

def inc(x):
    x += 1
    return x

def f1():
    def h(x):
        y = x
        return h
    return h

def f2():
    def h(x):
        S[2] = x
        return h
    return h

S = [1, 2, 3]
p = g(S)()
```

<table>
<thead>
<tr>
<th>Expression</th>
<th>Interactive Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>pow(2, 3)</code></td>
<td>8</td>
</tr>
<tr>
<td><code>print(4, 5) + 1</code></td>
<td>4 5</td>
</tr>
<tr>
<td><code>p[0]</code></td>
<td>Error</td>
</tr>
<tr>
<td><code>t1 = next(p)</code></td>
<td></td>
</tr>
<tr>
<td><code>t2 = next(p)</code></td>
<td></td>
</tr>
<tr>
<td><code>next(t1)</code></td>
<td>list(t1)</td>
</tr>
<tr>
<td><code>list(t1)</code></td>
<td>next(t1)</td>
</tr>
<tr>
<td><code>next(t2)</code></td>
<td>next(t2)</td>
</tr>
<tr>
<td><code>y = 4</code></td>
<td></td>
</tr>
<tr>
<td><code>z = inc(y)</code></td>
<td></td>
</tr>
<tr>
<td><code>print(z, y)</code></td>
<td></td>
</tr>
<tr>
<td><code>y = 4</code></td>
<td></td>
</tr>
<tr>
<td><code>z = f1()(6)</code></td>
<td></td>
</tr>
<tr>
<td><code>print(z, y)</code></td>
<td></td>
</tr>
<tr>
<td><code>f2()(6)</code></td>
<td></td>
</tr>
<tr>
<td><code>a = next(t2)</code></td>
<td></td>
</tr>
<tr>
<td><code>b = next(t2)</code></td>
<td></td>
</tr>
<tr>
<td><code>print(a, b)</code></td>
<td></td>
</tr>
</tbody>
</table>
2. (8 points) What’s the Point(er)?

(a) (4 pt) Fill in code below that, when executed, yields the situation shown in the diagram. The composite boxes in the diagram are Python list objects. Single boxes with labels to their left denote variables, not list objects. Boxes with diagonal lines through them contain None. You might not need the second blank line.

```
P = ________________________________ ______________________
    ________________________________ ______________________
P____________________________ = _____________________ ______________________
P____________________________ = _____________________ ______________________
P____________________________ = _____________________ ______________________
```

```
P = _________________________________________________
    _________________________________________________
```

```
P____________________________ = _____________________
```

```
P____________________________ = _____________________
```

```
P____________________________ = _____________________
```

```
P____________________________ = _____________________
```
(b) (4 pt) In the top diagram below, the two-slot objects are \texttt{Links} (see page 2). There are $2N > 0$ of these \texttt{Link} objects altogether. Slots with diagonal lines through them contain \texttt{Link.empty}. Fill in the blanks to modify the boxes on the top into the state shown in the bottom diagram. That is, modify the list $M$ destructively so that node $k$ points to node $k + 2$ if $k$ is odd, and to node $k - 2$ if $k$ is even. The only exceptions are that node $2N - 1$ continues to point to node $2N$, and the resulting list ends at node 2.

Do not create any new \texttt{Links} and do not modify any \texttt{.first} fields.

\[
\begin{array}{c}
M: \\
\text{1} \quad \text{2} \quad \text{3} \quad \text{4} \quad \cdots \\
\end{array}
\]

\[
\begin{array}{c}
M: \\
\text{1} \quad \text{2} \quad \text{3} \quad \text{4} \quad \cdots \\
\end{array}
\]

\[
\begin{array}{c}
to 5 \\
from 2N-3 \\
\end{array}
\]

\[
\begin{array}{c}
to 2N-2 \\
from 6 \\
\end{array}
\]

\[
\begin{array}{c}
\text{from } 2N-3 \\
\text{to } 5 \\
\end{array}
\]

\[
\begin{array}{c}
\text{def relink}(L, \text{prev}): \\
\text{t} = L.\text{rest} \quad \# \text{L and t are never empty lists}. \\
\text{if } \ldots:\n\text{-------------} = \text{relink}(\ldots) \\
\text{-------------} = \text{prev} \\
\text{return } L \\
\end{array}
\]

\[
\begin{array}{c}
M = \text{relink}(M, \text{Link.empty})
\end{array}
\]
3. (8 points) Environmental Issues

(a) (5 pt) Fill in the environment diagram that results from executing the code below until the entire program is finished. Fill in the function values (func ... [parent=...]) with all function values created during execution.

```python
def g():
    f = None
    def h(x):
        nonlocal f
        def inc():
            nonlocal x
            x += 1
            return x
        if x > 0:
            f = inc
        h(x-2)
    h(2)
    return f
r = g()()
```

Global frame

```
g
```

`func g() [parent=Global]`

```
f1: [parent=__________]
```

Return value

```
f2: [parent=__________]
```

Return value

```
f3: [parent=__________]
```

Return value

```
f4: [parent=__________]
```

Return value
(b) (3 pt)

The diagram on the right below shows an environment diagram resulting from execution of a particular program. Fill in the blanks in the program on the left so as to create a situation consistent with the diagram on the right (do not change any of the environment diagram.)

```python
1 def f(n, g):
2     if n > 0:
3         return ______________________
4     else:
5         return ______________________
6     x = f(2, lambda: 1)()
```

```
Global frame
   func f(n,g) [parent= Global]
      func λ () [parent= Global]
         func λ () [parent= f1]
            func λ () [parent= f2]
               func λ () [parent= f3] [parent= Global]
                  func λ () [parent= f4] [parent= Global]
                     func λ () [parent= Global]
```

```python
1 def f(n, g):
2     if n > 0:
3         return ______________________
4     else:
5         return ______________________
6     x = f(2, lambda: 1)()
```
4. (8 points) Spring Pruning

Fill in the function `prune` below so that given a Tree (see page 2) with integer labels, it (destructively) deletes all nodes whose label is strictly less than that of their parent if their parent is at even depth, or whose label is strictly greater than that of their parent if their parent is at odd depth. Deleting a node deletes the entire subtree below it. The root of the entire tree is at depth 0. For example, given the tree on the left, your function should produce the tree on the right without creating any new Tree nodes. The function `prune` does not return a value.

```python
def prune(t):
    def prune_level(t, d):

        if ________________________________ :

            t.branches = ________________________________________

        else :

            t.branches = ________________________________________

        for ________________________________________________ ___________:

            _____________________________________________________ _________

        prune_level(t, 0)
```

5. (1 points) Sum of Human Knowledge

In what work do we find the sentence, “His series of judgements in F sharp minor, given \textit{andante} in six-eight time, are among the most remarkable effects ever achieved in a Court of Chancery.”?

Answer: ________________________________

6. (6 points) Stream

What is printed by the following statements? The \texttt{Stream} class is defined on page 3.

```python
from operator import add, mul
	nums = make_integer_stream(1)
	astream = \
	Stream(1, 
		Stream(2, 
			Stream(3, 
				Stream(4, 
					lambda: 
					combine_streams(mul, bstream, 
						combine_streams(add, bstream.rest, nums)))))

bstream = filter_stream(lambda x: x % 2 == 0, astream)

print(stream_to_list(bstream, 5))
```

Answer: ________________________________
7. (8 points) Raises

Every employee in a company is given a raise every year by a percentage specific to that employee. Assume that there is a table ‘current’ of employee names and salaries, and another table ‘raises’ listing employee names and the fraction by which their salaries increase each year:

<table>
<thead>
<tr>
<th>current name</th>
<th>salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ben Bitdiddle</td>
<td>60000</td>
</tr>
<tr>
<td>Alyssa P Hacker</td>
<td>40000</td>
</tr>
<tr>
<td>Cy D Fect</td>
<td>35000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>raises name</th>
<th>raise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ben Bitdiddle</td>
<td>0.03</td>
</tr>
<tr>
<td>Alyssa P Hacker</td>
<td>0.02</td>
</tr>
<tr>
<td>Cy D Fect</td>
<td>0.01</td>
</tr>
</tbody>
</table>

For example, Ben Bitdiddle has a current salary of $60000, which is raised by 0.03 (3%) per year. From these tables, create two new tables ‘next’ and ‘future’ giving employee names and salaries next year and in $K$ years, respectively. For example, given the tables above and taking $K = 2$, we should have

<table>
<thead>
<tr>
<th>next name</th>
<th>salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ben Bitdiddle</td>
<td>61800</td>
</tr>
<tr>
<td>Alyssa P Hacker</td>
<td>40800</td>
</tr>
<tr>
<td>Cy D Fect</td>
<td>35350</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>future name</th>
<th>salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ben Bitdiddle</td>
<td>63654</td>
</tr>
<tr>
<td>Alyssa P Hacker</td>
<td>41616</td>
</tr>
<tr>
<td>Cy D Fect</td>
<td>35703.5</td>
</tr>
</tbody>
</table>

For the ‘future’ table, write ‘$K$’ wherever you need to put the number of years in the future (that is, your solution should work for any value of $K$.)

CREATE TABLE next AS

```
SELECT ------------------------------- FROM -----------------------------
WHERE -------------------------------;
```

CREATE TABLE future AS

```
WITH temp (-----------------------------) AS ( )
SELECT ------------------------------- FROM ----------------------------- UNION
SELECT ------------------------------- FROM -----------------------------
WHERE -------------------------------

SELECT ------------------------------- FROM temp WHERE -------------------------------;
```
8. (6 points) Pair Up

Fill in the Scheme `pairs` function so that `(pairs L)`, where `L` is a list, produces a list of lists, where each of these lists contains a pair of elements from `L`. The function must be tail-recursive. You need not define (or use) the `reverse` function.

```
scm> (pairs '(1 2 3 4))
  ((1 2) (3 4))
scm> (pairs '(1 2 3 4 5)) ; Odd element at end put in singleton list.
  ((1 2) (3 4) (5))
scm> (pairs '())
  ()
```

```
(define (reverse P)  
  """Returns the reverse of list P. This function is tail-recursive""
  ;; Implementation not shown
)

(define (pairs L)

  (define (accum-pairs lst result)

    (cond (_________________________________________ result)

      (___________________________________________

      (cons _________________________________________))

      (else (accum-pairs _____________________________

        _____________________________)

        _____________________________))))

    _________________________________)

)
```
9. (10 points) Don’t Go Down!

For the purposes of this problem, define the greedy nondescending subsequence of a sequence \( S = (s_0, s_1, \ldots, s_{n-1}) \) to be the sequence \((s_{i_1}, s_{i_2}, \ldots, s_{i_m})\) where \(0 = i_1 < i_2 < \cdots < i_m < n\), \(i_k+1\) is the smallest index such that \(s_{i_k} \leq s_{i_k+1}\), and \(i_m\) is as large as possible (given the first two constraints).

(a) (5 pt) Fill in the function `upsubseq` such that `upsubseq(L)` returns an iterator that yields a greedy nondescending subsequence of \(L\), where \(L\) is a Python list or tuple. Do not use a generator. Your iterator must use constant space; do not create a new list containing the subsequence and return a standard iterator for it. You need not use all the spaces provided.

```python
class upseq_iter:
    def __init__(self, L):
        _________________________________
        _________________________________
        _________________________________

    def __iter__(self):
        return self

    def __next__(self):
        if _________________________________:
            _________________________________
            result = _________________________________
            _________________________________
        while _________________________________:
            _________________________________
            _________________________________
        return result

def upsubseq(L):
    """"""
    >>> list(upsubseq([4, 2, 3, 6, 6, 5, 7, 1, 3]))
    [4, 6, 6, 7]
    >>> list(upsubseq([]))
    []
    """""""

    return _________________________________
```
(b) (5 pt) Fill in the function genupseq so that genupseq(L) returns a generator that yields a greedy nondescending subsequence of L, where L is a Python list or tuple. Your generator must use constant space; do not create a new list containing the subsequence and return a standard iterator for it. You need not use all the spaces provided.

def genupseq(L):
    """
    >>> list(genupseq([4, 2, 3, 6, 6, 5, 7, 1, 3]))
    [4, 6, 6, 7]
    >>> list(genupseq([]))
    []
    """

    while _________________________________:
        _________________________________
        _________________________________
        _________________________________
        _________________________________
        _________________________________
        _________________________________

    while _________________________________:
        _________________________________
        _________________________________
10. (8 points) Sum Range

Fill in the function `sumrange` so that, given a binary search tree T with integer labels and two integer bounds low and high, it returns the sum of all values in T between low and high, inclusive. If the label of the root of T is within this range, your program must take time $\Theta(M)$, where $M$ is the number of labels in T that are within the bounds low and high, regardless of the number of values in T. Assume that T is of type `BinTree`, as defined on page 2. You do not need to use all lines.

```python
def sumrange(T, low, high):
    """The sum of all values in BST T that are between LOW and HIGH, inclusive.""
    >>> S = BinTree(50, BinTree(25, BinTree(10, BinTree(5), BinTree(20)),
                    BinTree(30, BinTree(28), BinTree(40))),
                    BinTree(75, BinTree(60, BinTree(55), BinTree(65)),
                    BinTree(90, BinTree(80), BinTree(100))))
    >>> sumrange(S, 20, 56)
    248
    """
    if _________________________________________________ ___________:
        return 0
    elif _______________________________________________ ____________:
        return _____________________________________________ ________
        _____________________________________________________ __
    elif _______________________________________________ _____________:
        return _____________________________________________ __________ 
        _____________________________________________________ ____
    else:
        return _____________________________________________ ___________ 
        _____________________________________________________ ___
```

```python
if _________________________________________________ ___________:
    return 0
elif _______________________________________________ ____________:
    return _____________________________________________ ________\
        _____________________________________________________ __
elif _______________________________________________ _____________:
    return _____________________________________________ \ 
        _____________________________________________________ 
else:
    return _____________________________________________ \ 
        _____________________________________________________ 
```
11. (10 points) Big Theta

(a) (2 pt) Circle all of the following that are true.

i. $\Theta(3N) = \Theta(N/3)$

ii. $\Theta(2^N) = \Theta(2^N + N^2)$

iii. $\Theta(\log(2^N)) = \Theta(\log(2^N \cdot N^2))$

iv. $\Theta(2^N) = \Theta(2^N \cdot N^2)$

v. $\Theta(f(N) + 1/N) = \Theta(f(N))$ if $f(N) > K$ for some fixed $K > 0$.

(b) (2 pt) Using $N$ as the length of the Python list $L$, give a worst-case bound for the execution time of the following code, assuming that execution time is proportional to the number of times the commented statement is executed.

```python
s = 0
for k in range(len(L)):
    j = 1
    while j < k:
        s += L[j] # Count this
        j *= 3
```

Answer: _______________________

(c) (2 pt) If $T$ is a Tree containing $N$ nodes in all, what is a worst-case bound on the time required to determine whether a value $x$ is a label of the tree, measuring the number of comparisons to $x$ that are required? Do not assume that $T$ is a search tree.

Answer: _______________________

(d) (2 pt) If $T$ is a binary search tree containing $N$ nodes in all, what is a worst-case bound on the time required to determine whether a value $x$ is a label of the tree, measuring the number of comparisons to $x$ that are required?

Answer: _______________________

(e) (2 pt) We’ve seen how to add a `__getitem__` method to the `Link` class so that one can index a linked list just as one can a Python list. That is, $L[k]$, for $L$ a `Link` (see page 2), will traverse $L$ to item $k$ and return it. Given such an implementation, what is the worst-case running time of the following program, as a function of $N$ the length of linked list $L$, where we are counting the number of times a `.rest` field is accessed? Assume $N$ is set to the length of the list.

```python
c = 0
for k in range(N):
    for j in range(k-1):
        if L[k] == L[j]:
            c += 1
```

Answer: _______________________

Extra space if needed.
Extra space if needed.