CS61A Notes – Week 8: Environments

The Attack of the Environmentalists

The biggest blow side effects deal us is that we can no longer use the substitution model of evaluation that we’ve fallen so hopelessly in love with. Consider this:

> (define (foo x) (set! x 3) x)  
> (foo 10)

If we use the substitution model, then when we do (foo 10), we will replace all occurrences of x inside foo with 10. Note that this means (foo 10) will then return 10, not 3! Obviously, we can no longer afford to be so obtuse, as values of the variables can now change at any point in time. Thus, we need to use what is called the “environment model” of evaluation. Briefly, we keep an “environment” of bindings from variable to value, and every time we encounter a variable, we look up its value in the current environment. Note that we only lookup the value when we see the variable, and so we’ll lookup the value of x only after we’ve already set x to be 3. Drawing these environment diagrams is one of the most hated aspects of CS61A, but remember this: this is what the Scheme interpreter does, and since you’re much smarter than the interpreter, if the interpreter can do it, you can do it.

An environment frame is a box that contains bindings from variables to values. An environment frame can “extend” another frame; that is, this frame can see all bindings of the frame it extends. We represent this by drawing an arrow from an environment frame to the frame it is extending. The global environment is the only environment that extends nothing. Here’s how to go about it:

1. Start with a box labeled global environment. This box starts out with bindings for all the primitives like +, cons, map, etc.
2. Set the current frame to be the global environment. The current frame is just the environment that you’re in at the moment, and of course, you start in the global environment.
3. Evaluate your expressions, one by one. I’d recommend converting all sugar-coated procedure definitions to their raw forms:
   (define (square x) (* x x)) => (define square (lambda (x) (* x x))

Evaluation rules:

- **constant** (numbers, strings, etc), they are self-evaluating so don’t do any work.
- **variable**, try to find a binding for it in the current frame. Failing that, follow what environment the current frame points to, and try to find the binding there, and so on, until you reach the global environment. If it’s still not in the global environment, then you can’t find it; it’s an error! (Recall that the global environment contains all the bindings for primitives like cons, +, etc.)
- **define expression**, first evaluate the value to bind to according to these evaluation rules, then add an entry into the current frame for the variable pointing to that value.
- **lambda expression**, draw two bubbles next to each other (usually just in the space around the boxes). The first bubble should point to the text of the lambda – the argument list and the body – and the second bubble should point to the current frame. The frame this circle points to is called the “procedure environment”.
- **let expression**, convert to a lambda and invoke this lambda on its arguments (the values to bind the new variables to).
- **set! expression**, evaluate the value to set! to. Then, for the given variable, find its closest binding from the current frame, and overwrite the old value with the new value.
- **any other special forms**, just evaluate what you’re supposed to in the current frame.
- **procedure call**, check if the procedure is a:
  - **primitive procedure**, these work by magic, so just apply the procedure intuitively in your head.
  - **compound procedure**, evaluate all the arguments first. Then, create a new box, and make this box point to the “procedure environment” box of the procedure. Note that you often do not point to the current frame! Set this new box as the new current frame, and add all the parameters into the box, and have them point to the argument values you evaluated. Evaluate the body of the procedure in the current frame. Once done, go back to the frame from which you made the procedure call.

QUESTIONS: Draw environment diagrams for the following:

IN CLASS:
SU2010: # 3, 4
1. (define x 3)  
   (let ((x 3) (y x))  
   (set! X (+ y x)) x)

Eric, Kevin, Phill, Hamilton, and Stephanie – CS61A Summer 2011– notes courtesy of Justin Chen
2.  \( \text{define (foo x)} \)
    \( \quad \text{(let ((y x) (z 3))} \)
    \( \quad \quad (+ y z)) \)
\( \text{(foo 3)} \)

3.  \( \text{define (foo x)} \)
    \( \quad \text{(let ((y x) (z 3))} \)
    \( \quad \quad \text{(let ((t (+ y z)))} \)
    \( \quad \quad \quad (+ t y z))) \)

EXTRA PRACTICE:

1.  \( \text{(define answer 0)} \)
    \( \text{(define (square f x)} \)
    \( \quad \text{(let ((answer 0))} \)
    \( \quad \quad (f x) \)
    \( \quad \quad answer) \)
    \( \text{(square (lambda (n) (set! answer (* n n))) 3)} \)

2.  \( \text{(define (hmm n) (lambda (x) (+ x y n)))} \)
\( \text{(define (uhh y)} \)
    \( \quad \text{(define hmm-y (hmm y))} \)
    \( \quad \quad \text{(hmm-y 2))} \)
\( \quad (uhh 42) \)
3.  (define a 3)
   ((lambda (a)
       ((lambda (a) (a))
        (lambda () (set! a 'myxomatosis)))
       a)
    (* a a))

4.  (define a 'scatterbrain)
   ((lambda (a b) (b) a)
    a
    (let ((b 'cuttooth))
     (lambda () (set! a b)))
    a)

6.  (define (f + -) (+ ((lambda (-) (- 3 5)) -) 10))
    (f - +)

7.  (define (slow-op-maker op)
     (let ((old-result #f))
      (lambda (x)
        (let ((return old-result))
         (let ((set! old-result (op x))
            return))))
    (define slow-sqr (slow-op-maker square))
    (slow-sqr 3)
    (slow-sqr 5)
    (define slow-cube (slow-op-maker cube))
    (slow-cube 3)
    (slow-cube 5)
8. (define (slow-op-maker op)
       (let ((old-result #f))
           (lambda (op)
               (lambda (x)
                   (let ((return old-result))
                       (set! old-result (op x))
                       return)))))

(define slow-sqr (slow-op-maker square))
(slow-sqr 3)
(slow-sqr 5)
(define slow-cube (slow-op-maker cube))
(slow-cube 3)

Imperative Perils

For those of you used to programming in C or Java or almost any other languages, the idea of “assignment” – assigning a value to a symbol – should be a dusty old hat. Certainly, there’s nothing exotic about it. But why have we been so cautious about introducing this concept to you – 9 weeks into the semester! – when, if you take an introductory CS class in any other language, you would see assignment on the first day? And why do I write such long and convoluted sentences?

Before that, note that we have now stepped out of pure “functional programming”, and now we have functional programming with side effects. Here, nice properties of functional programming disappear; more specifically, the order in which expressions are evaluated matters, and two calls to the same procedure with the same arguments may not produce the same result!

Beyond that, you should note one more thing. Up until now, whenever we make a procedure call like (square 3) or (+ 2 5), we are interested in what the procedure call returns. Such is the fundamental idea of functional programming – procedures are interesting only because they take in something and produce something else (deterministically, might I add!). This is not always true now. For example, if we say (set! x 5), what do we want it to return? Well, we really don’t care what it returns, as long as it binds the value of 5 to x after it’s done! For such expressions, we’re more interested in their side effects than their return values.

Precisely because of that, sometimes we’ll want to evaluate multiple statements at once (each having some side effects), whereas before we only evaluated one. Sometimes you can do that directly:

(define (foo x) (set! x 3) (set! y 5) (cons x y))

Note that the above defined procedure, foo, has three statements, only one of which returns something interesting. STk will execute the three statements in the order of appearance, and return the value of the last expression. You can do the same with a cond statement:

(cond ((< x 5) (set! x 10) (set! y 5) '(cs61a rocks and so do toads))
      (else (set! x 5) (set! y 10) '(flan in the face)))

However, you cannot do the same for if statements, for obvious reasons:

(if (> x 3) (set! x 3) (set! y 5) (set! z 10))
What does the above mean? There’s no way for STk to tell what you intended (set! x on #t? set! x and y on #t? set! x, y and z on #t?) And we can’t simply enclose things in parentheses, as you might be tempted to do; ((set! x 3) (set! y 5)) means a procedure call where the procedure is the value of (set! x 3) and the argument to the procedure is the value of (set! y 5), resulting in pure gibberish. It seems that we need a new piece of syntax to deal with such situations!

This calls for begin, which groups together a sequence of statements, and returns the value of the last expression. So we can instead do:

(if (> x 3) (set! x 3) (begin (set! y 5) (set! z 10)))

Now, side effects are obviously not all bad; if you’re careful, it’s immensely more efficient and can often make the code more compact. The keyword, of course, is “careful”; let’s look at what havoc side effects bring.

Assigning Things to Things and Stuff (and other things)

Our way of doing assignment in Scheme is set!. Note that set! is a special form. Consider if it’s not; then, if we do:

(define x 5)
(set! x 3)

and set! is not a special form, it will evaluate what x is – 5 – and try to set the value of 5 to be 3, making no sense at all. What we really meant to say, is to set the value bound to the symbol x (not the value of x) to 3. There’s really not much to talk about here, so let’s try a few. The key is to keep the scope of variables straight!

QUESTIONS

IN CLASS:

1. Personally – and don’t let this leave the room – I think set! is useless. I mean, why do set!, when we can always just redefine a variable using a define statement? Instead of doing (set! x 3), why don’t we just do (define x 3) again? I propose the following alternative implementation of counter, similar to the one in class:

   The Old Way
   (define count
     (let ((current 0))
       (lambda()
         (set! current (+ 1 current))
         current)))
   (count) ==> 1
   (count) ==> 2

   Hamilton’s Brilliant New Way
   (define count
     (let ((current 0))
       (lambda ()
         (define current
           (+ current 1))
         current)))
   (count) ==> 1
   (count) ==> 2

   How dumb am I? What happens when I use my brilliant new implementation?

2. Consider these definitions:
   (define x 3)
   (define (z) (set! x 5) x)
   What would (list (z) x) return?
3. Define a procedure `fib` so that, every time it is called, it returns the next Fibonacci number, starting from 1:
   \( \texttt{(fib)} \Rightarrow 1; \texttt{(fib)} \Rightarrow 2; \texttt{(fib)} \Rightarrow 3; \texttt{(fib)} \Rightarrow 5; \texttt{(fib)} \Rightarrow 8, \text{ etc.} \)

EXTRA PRACTICE:

4. (SICP ex. 3.8) Keeping number 2 in mind, define a procedure `f` so that, given the procedure call
   \( (+ (f \ 0) (f \ 1)) \)
   if `STk` evaluates from left to right, it returns 0, and if `STk` evaluates from right to left, it returns 1.