CS61A Lecture 12
Immutable Trees

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Computer Science in the News

Soap bubble screen is 'the world's thinnest display'

The team says the display is the world's thinnest transparent screen

http://www.bbc.com/news/technology-18671061

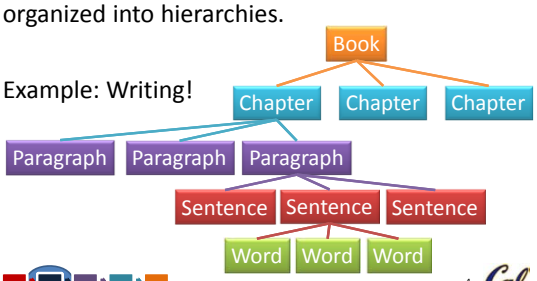
Today

- Review: Immutable Trees
- Binary Trees and Binary Search Trees
- Extra: Tuples versus IRLists

Review: Hierarchical Data

Often, we find that information is nicely organized into hierarchies.

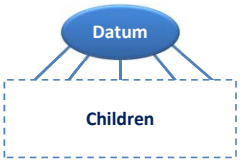
Example: Writing!



Review: Trees

A ***tree*** data structure traditionally has two parts:

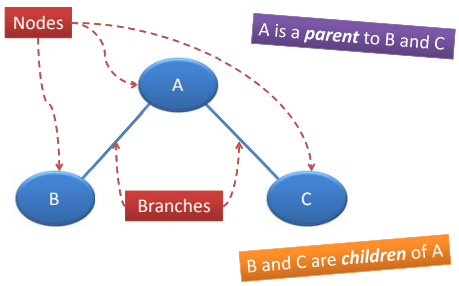
1. A ***datum***: Information stored at the top.
2. Some ***children***: Trees that appear below this tree.



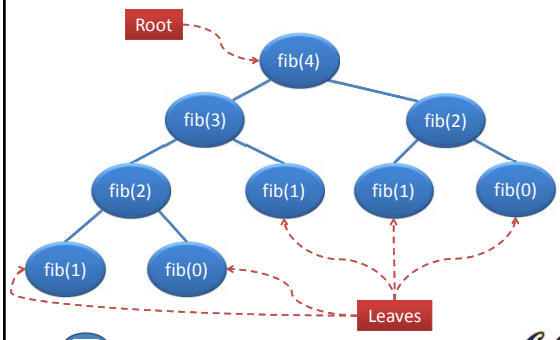
Review: Trees

Notice that trees are also *recursively defined*.
A tree is made from other trees –
these trees are its *subtrees*.

Review: Trees

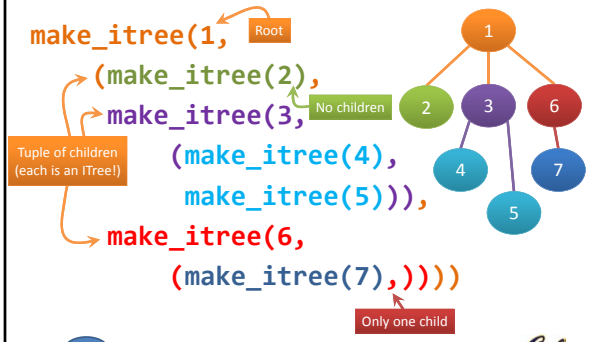


Review: Trees



Review: Immutable Trees

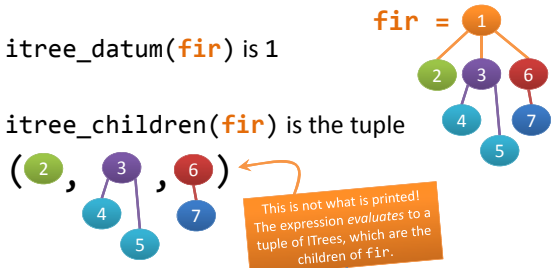
```
make_itree(1,
    (make_itree(2),
     make_itree(3,
        (make_itree(4),
         make_itree(5))),
     make_itree(6,
        (make_itree(7),))))
```



Review: Immutable Trees

`itree_datum(fir)` is 1

`itree_children(fir)` is the tuple



This is not what is printed! The expression *evaluates* to a tuple of ITrees, which are the children of `fir`.

A group of trees is called a ***forest***.

Review: ITrees

```
def make_itree(datum, children=()):
    return (datum, children)

def itree_datum(t):
    return t[0]

def itree_children(t):
    return t[1]
```

Constructor

Selectors

Review: Operating on ITrees

Write the function `itree_prod`, which takes an ITree of numbers and returns the product of all the numbers in the ITree.

```
>>> t = make_itree(1, (make_itree(2),
                       make_itree(3),
                       make_itree(4)))
>>> itree_prod(t)
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```

Example: Operating on ITrees

Idea: Split the problem into two different parts: one that handles a single tree and one that handles a forest.

```
def itree_prod(t):
    return itree_datum(t) * forest_prod(itree_children(t))

def forest_prod(f):
    if len(f) == 0:
        return 1
    return itree_prod(f[0]) * forest_prod(f[1:])
```

Returns the product of numbers in an ITree.

Datum, multiplied by…

… the product of all the numbers in all the ITrees in the forest.

These ITrees are the children.

Product of numbers in the first ITree, multiplied by…

… the product of all the numbers in all the other ITrees of the forest.

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Example: Operating on ITrees

Idea: Split the problem into two different parts: one that handles a single tree and one that handles a forest.

```
def itree_prod(t):
    return itree_datum(t) * forest_prod(itree_children(t))

def forest_prod(f):
    if len(f) == 0:
        return 1
    return itree_prod(f[0]) * forest_prod(f[1:])
```

Not a data abstraction violation. We *know* that f is a tuple of ITrees.

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Example: Operating on ITrees

Idea: Split the problem into two different parts: one that handles a single tree and one that handles a forest.

```
def itree_prod(t):
    return itree_datum(t) * forest_prod(itree_children(t))

def forest_prod(f):
    if len(f) == 0:
        return 1
    return itree_prod(f[0]) * forest_prod(f[1:])
```

This is called ***mutual recursion*** because it involves two functions recursively calling each other!

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Mutual Recursion

To find the product of the whole ITree in **itree_prod**, we need to apply **itree_prod** itself to the smaller subtrees that are the children of the provided itree.

forest_prod does this for us.

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Example: Operating on ITrees

An alternate definition for `itree_prod` does not need two separate and smaller functions:

```
from operator import mul
def itree_prod(t):
    return itree_datum(t) * \
           reduce(mul,
                  map(itree_prod,
                      itree_children(t)),
                  1)
```

Applies `itree_prod` recursively on the children.

Multiplies all the results of applying `itree_prod` on the children.

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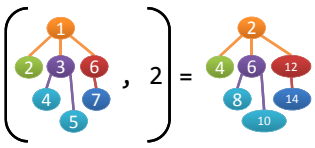
Practice: Operating on ITrees

Write a function `scale_itree` that takes an ITree of numbers and a scaling factor, and returns a new ITree that results from scaling each number in the original ITree.

`scale_itree(`



`, 2) =`

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Practice: Operating on ITrees

```
def scale_itree(itree, factor):
    return make_itree(__________________________,
                      scale_forest(____________________,
                                   ______))

def scale_forest(forest, factor):
    results = ()
    for itree in forest:
        results = results + ________________________
    return results
```


Practice: Operating on ITrees

```
def scale_itree(itree, factor):
    return make_itree(itree_datum(itree) * factor,
                      scale_forest(itree_children(itree),
                                   factor))

def scale_forest(forest, factor):
    results = ()
    for itree in forest:
        results = results + scale_itree(itree, factor)
    return results
```


Practice: Operating on ITrees

Alternative solution:

```
def scale_itree(itree, factor):
    return make_itree(itree_datum(itree) * factor,
                      scale_forest(itree_children(itree),
                                   factor))

def scale_forest(forest, factor):
    if len(forest) == 0:
        return ()
    return (scale_itree(forest[0], factor),) + \
           scale_forest(forest[1:], factor)
```

Tuple of one tree

Practice: Operating on ITrees

```
def scale_itree(itree, factor):
    return make_itree(__________________________,
                      tuple(map(_________,
                                ____________________)))
```


Practice: Operating on ITrees

```
def scale_itree(itree, factor):
    return make_itree(itree_datum(itree) * factor,
                      tuple(map(scale_itree,
                                itree_children(itree))))
```


Practice: Operating on ITrees

Write a function `count_leaves` that counts the number of leaves in the ITree provided.

`count_leaves` 

= 4

Practice: Operating on ITrees

```
def count_leaves(itree):
    if _____________:
        return 1
    return cl_forest(____________________)

def cl_forest(f):
    if len(f) == 0:
        return _
    return ___________________________________
```


Practice: Operating on ITrees

```
def count_leaves(itree):
    if is_leaf(itree):
        return 1
    return cl_forest(itree_children(itree))

def cl_forest(f):
    if len(f) == 0:
        return 0
    return count_leaves(f[0]) + cl_forest(f[1:])
```

```
def is_leaf(itree):
    return len(itree_children(itree)) == 0
```


Practice: Operating on ITrees

```
def count_leaves(itree):
    if _____________:
        return 1
    return reduce(___,
                  map(____________,
                      ____________________),
                  _)
```


Practice: Operating on ITrees

```
def count_leaves(itree):
    if is_leaf(itree):
        return 1
    return reduce(add,
                  map(count_leaves,
                      itree_children(itree)),
                  0)
```


Announcements

- Homework 6 is due tomorrow, **July 10**.
  - Starting with the next homework, we will mark questions as *core* and *reinforcement*.
  - The *core* questions are ones that we suggest you work on to understand the idea better.
  - The *reinforcement* questions are extra problems that you can practice with, but are not as critical.
- Project 2 is due on Friday, **July 13**.
- Homework 0 for the staff is now available.

Announcements

- Midterm 1 is **tonight**.
  - *Where*? 2050 VLSB.
  - *When*? 7PM to 9PM.
- Closed book and closed electronic devices.
- One 8.5" x 11" 'cheat sheet' allowed.
- Group portion is 15 minutes long.
- Post-midterm potluck on Wednesday, **July 11** in the **Wozniak Lounge** from **6:30pm to 10pm**.
  - Bring food, drinks and board games!
  - Drop by if and when you can.

Announcements

http://berkeley.edu/map/maps/ABCD345.html

CAMPANILE

Break

Problem Solving: Searching

Searching is a problem that shows up frequently in many different (and daily!) settings.

Problem: We have a collection of data and we need to find a certain datum.

Examples:

- Searching for a person in a phone book.
- Searching for a webpage on the Internet.

... and so on.

http://www.spacepub.com/wp-content/uploads/2011/06/The-Truth-Is-Out-There.jpg

Problem Solving: Searching

Write a predicate function `has_num` that determines if a number is in a tuple of numbers.

```
def has_num(tup, num_to_find):
    for item in tup:
        if _____:
            return ____
    return _____
```


Problem Solving: Searching

Write a predicate function `has_num` that determines if a number is in a tuple of numbers.

```
def has_num(tup, num_to_find):
    for item in tup:
        if item == num:
            return True
    return False
```


Problem Solving: Searching

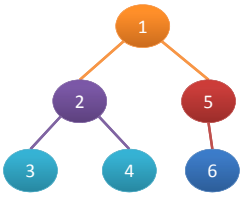
How does the running time of `has_item` scale as n, the length of the tuple, scales?

Can we do better?

http://i2.kym-cdn.com/photos/images/original/000/221/792/cute-puppy-pictures-anything-you-can-do-i-can-do-better.jpg

Trees in Real Life: Binary Trees

Trees where each node has *at most* two children are known as ***binary trees***.



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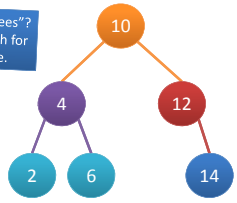
Trees in Real Life: Binary Search Trees

Binary search trees are binary trees where all of the items to the ***left*** of a node are ***smaller***, and the items to the ***right*** are ***larger***.

Why the term "*search* trees"? They speed up the search for an item in a sequence.

How?



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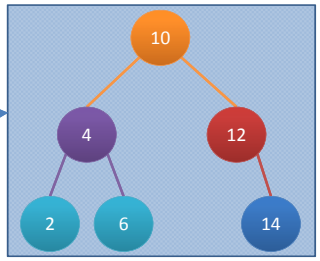
Trees in Real Life: Binary Search Trees

Let us try to find 4 in this binary search tree:

It could be anywhere here.



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Trees in Real Life: Binary Search Trees

Let us try to find 4 in this binary search tree:

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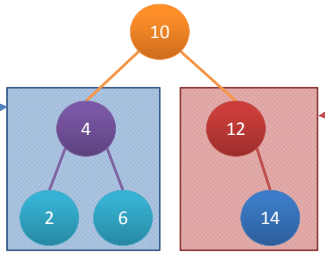

Discard this half of the data. It can't be here! They're too big.

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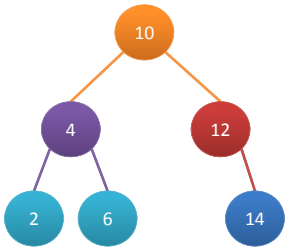
Trees in Real Life: Binary Search Trees

Let us try to find 4 in this binary search tree:

It is present in the BST. ☺



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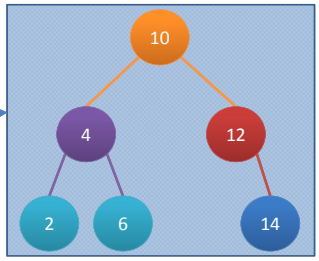
Trees in Real Life: Binary Search Trees

Let us try to find 14 in this binary search tree:

It could be anywhere here.



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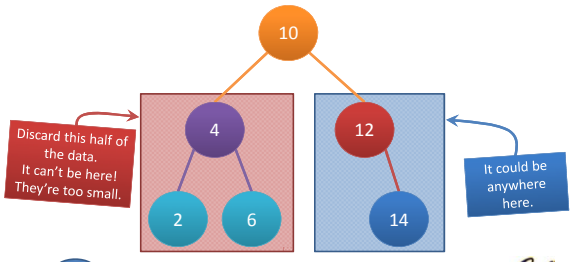
Trees in Real Life: Binary Search Trees

Let us try to find 14 in this binary search tree:



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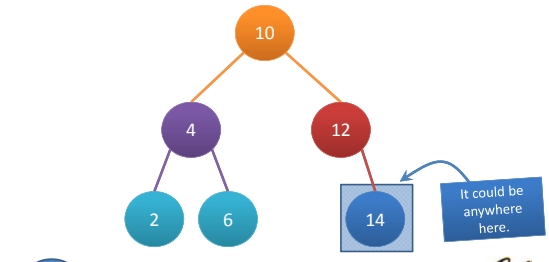
Trees in Real Life: Binary Search Trees

Let us try to find 14 in this binary search tree:



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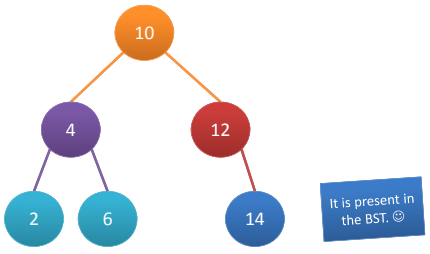
Trees in Real Life: Binary Search Trees

Let us try to find 14 in this binary search tree:



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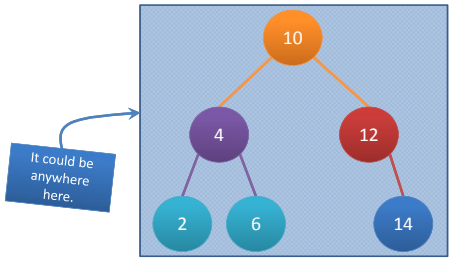
Trees in Real Life: Binary Search Trees

Let us try to find 11 in this binary search tree:



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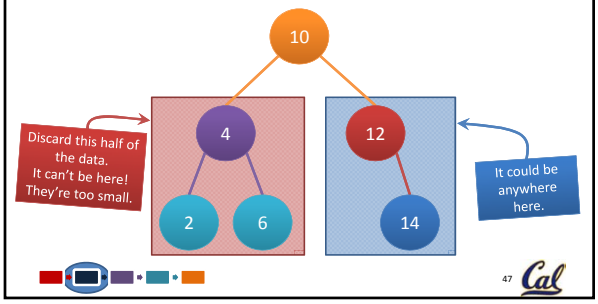
Trees in Real Life: Binary Search Trees

Let us try to find 11 in this binary search tree:



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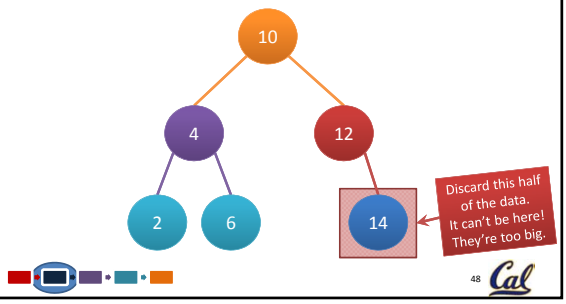
Trees in Real Life: Binary Search Trees

Let us try to find 11 in this binary search tree:



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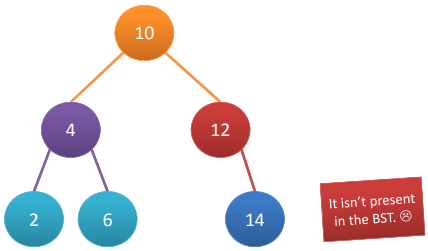
Trees in Real Life: Binary Search Trees

Let us try to find 11 in this binary search tree:



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Trees in Real Life: Binary Search Trees

Two important things when searching for an item in a binary search tree (BST):

1. As we go down the tree, from one level to another, from parent to child, we discard *approximately half* the data each time.
2. In the *worst case*, we go down all the way to the leaves.

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Trees in Real Life: Binary Search Trees

The ***height*** of a tree is the length of the *longest path* from the root to any leaf.

If there are n nodes in a binary search tree, and if the binary search tree is *balanced*, then the height of the tree is approximately $\log_2 n$.

Approximately as many nodes on the left of any node as there are on the right of that node.

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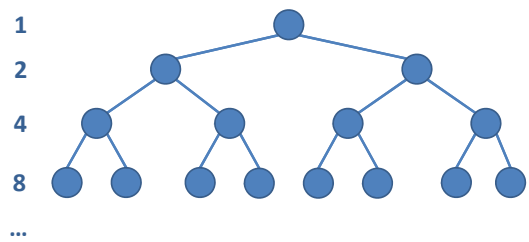
Logarithmic Algorithm

Why approximately $\log_2 n$?



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Logarithmic Algorithm

Why approximately $\log_2 n$?

For a tree of height h (or $h + 1$ levels), the total number of nodes is

$1 + 2 + 4 + 8 + \ldots + 2^h = n$

or, $2^{h+1} - 1 = n$

or, $h \approx \log_2(n)$

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Logarithmic Algorithm

Why approximately $\log_2 n$?

Alternatively, notice that when we go *down one level*, we are discarding ***approximately half*** the data.

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Trees in Real Life: Binary Search Trees

In the *worst case*, we go down to the leaves, visiting each level of nodes ***once***.

There are approximately $\log_2 n$ levels.

The runtime to find an item in a binary search tree (BST) with n nodes is $O(\log n)$.

Exponentially faster than $O(n)$!

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BST ADT

```
empty_bst = None
def make_bst(datum, left=empty_bst,
                    right=empty_bst):
    return (left, datum, right)

def bst_datum(b):
    return b[1]
def bst_left(b):
    return b[0]
def bst_right(b):
    return b[2]
```

Constructor

Selectors

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BST ADT

`bst_is_leaf(b)` returns True if the BST is a leaf:

```
def bst_is_leaf(b):
    return bst_left(b) == empty_bst and
           bst_right(b) == empty_bst
```

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BST ADT

```
def has_num(b, num):
    if b == empty_bst:
        return False
    elif bst_datum(b) == item:
        return ____
    elif bst_datum(b) > item:
        return has_num(___________,
                       num)
    return has_num(____________,
                   num)
```

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BST ADT

```
def has_num(b, num):
    if b == empty_bst:
        return False
    elif bst_datum(b) == item:
        return True
    elif bst_datum(b) > item:
        return has_num(bst_left(b),
                       num)
    return has_num(bst_right(b),
                   num)
```

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Conclusion

- ITrees are useful for representing general tree-structures.
- Binary search trees allow us to search through data faster.
- ***Preview:*** Object-oriented programming: a brand new paradigm.

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Good Luck Tonight!

http://www.whosawesome.com/images/awesome.jpg

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Extra: Tuples versus IRLists

Tuples and IRLists are two *different* ways of representing *lists of items*: they are two different data structures, each with their own advantages and disadvantages.

You will compare them further in CS61B, but here is a preview.

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Extra: Tuples versus IRLists

IRLists are recursively defined, tuples are not.

This gives IRLists an advantage when you want to perform a naturally recursive task, such as mapping a function on a sequence.

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Extra: Tuples versus IRLists

Which of the following methods is better?

```
def map_irlist(fn, irl):
    return make_irlist(fn(irlist_first(irl)),
                       map_irlist(fn,
                                  irlist_rest(irl)))

def map_tuple(fn, tup):
    results = ()
    for item in tup:
        results = results + (fn(item),)
    return results
```

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Extra: Tuples versus IRLists

`map_irlist` is better than `map_tuple`.

`map_tuple` creates and discards temporary tuples, since *tuples are immutable*.

`map_irlist` "tacks on" new items to the front of an existing (and growing) IRList.

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Extra: Tuples versus IRLists

Tuples are unchanging sequences of data, which gives them an advantage when all you want to do is grab information from a sequence.

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EXTRA: TUPLES VERSUS IRLISTS

Which of the following methods is better?

```
def getitem_tuple(tup, pos):
    return tup[pos]

def getitem_irlist(irl, pos):
    if pos == 0:
        return irlist_first(irl)
    return getitem_irlist(irlist_rest(irl), pos-1)
```


EXTRA: TUPLES VERSUS IRLISTS

`getitem_tuple` is better than `getitem_irlist`.

Indexing into a tuple takes (*on average*) constant time, but indexing into an IRList takes linear time.