Our Sequence Abstraction
Recall our previous sequence interface:

- A sequence has a finite, known length
- A sequence allows element selection for any element

In the cases we’ve seen so far, satisfying the sequence interface requires storing the entire sequence in a computer’s memory

Problems?

- Infinite sequences - primes, positive integers
- Really large sequences - all Twitter tweets, votes in a presidential election

Implicit Sequences

- We compute each of the elements on demand
- Don’t explicitly store each element
- Called an implicit sequence

A Python Example

Example: The `range` class represents a regular sequence of integers

- The range is represented by three values: `start`, `end`, and `step`
- The length and elements are computed on demand
- Constant space for arbitrarily long sequences

```
length = max(\left\lceil \frac{\text{end} - \text{start}}{\text{step}} \right\rceil, 0)

elem(k) = \text{start} + k \cdot \text{step} \quad (\text{for } k \in [0, \text{length}])
```

A Range Class

```python
class Range(object):
    def __init__(self, start, end=None, step=1):
        if end is None:
            end, start = 0, start
        self.start = start
        self.end = end
        self.step = step

    def __len__(self):
        return max(0, ceil((self.end - self.start) / self.step))

    def __getitem__(self, k):
        if k < 0:
            k = len(self) + k
        if k < 0 or k >= len(self):
            raise IndexError('index out of range')
        return self.start + k * self.step
```

The Iterator Interface

An iterator is an object that can provide the next element of a (possibly implicit) sequence

The iterator interface has two methods:

- `__iter__(self)` returns an equivalent iterator
- `__next__(self)` returns the next element in the sequence

* If no next, raises `StopIteration` exception

There are also built-in functions `next` and `iter` that call the corresponding method on their argument.
RangerIter

class RangerIter(object):
    def __init__(self, start, end, step):
        self.current = start
        self.end = end
        self.step = step
        self.sign = 1 if step > 0 else -1

    def __next__(self):
        if self.current * self.sign >= self.end * self.sign:
            raise StopIteration
        result = self.current
        self.current += self.step
        return result

    def __iter__(self):
        return self

For now, always returns self! (Why do we have this then?)

Fibonacci

class FibIter(object):
    def __init__(self):
        self.prev = -1
        self.current = 1

    def __next__(self):
        self.prev, self.current = (self.current, self.prev + self.current)
        return self.current

    def __iter__(self):
        return self

Generators and Generator Functions

Generators:
- An iterator backed by a function, called a generator function

Generator Functions:
- A function that returns a generator
- Can tell by looking for the yield keyword
- Another example of a continuation

A simple generator

def ones_generator():
    while True:
        yield 1

    The yield keyword is what marks this as a generator function
    Calling this function won’t do anything besides return a generator object (an iterator)
    Each time we ask for a value from the iterator, it runs the function until it reaches a yield statement and gives whatever value was yielded
    The next time we ask for a value, it picks up where it left off
    This iterator will keep giving you ones forever!

Iterating over an Rlist

We can iterate over a sequence even if it has no __iter__ method

Python uses __getitem__ instead, iterating until IndexError is raised

class Rlist(object):
    def __init__(self, first, rest=empty):
        self.first, self.rest = first, rest

    def __getitem__(self, k):
        if k == 0:
            return self.first
        if self.rest is Rlist.empty:
            raise IndexError('index out of range')
        return self.rest[k - 1]

How long does it take to iterate over an Rlist of n items? $\Theta(n^2)$
Iterating over an Rlist

We can define an iterator for Rlists using a generator function

class Rlist(object):
    def __init__(self, first, rest=empty):
        self.first, self.rest = first, rest
    def __getitem__(self, k):
        if k == 0:
            return self.first
        if self.rest is Rlist.empty:
            raise IndexError('index out of range')
        return self.rest[k – 1]
    def __iter__(self):
        current = self
        while current is not Rlist.empty:
            yield current.first
            current = current.rest

How long does it take to iterate over an Rlist of n items?

Fibonacci Generator

A generator function that lazily computes the Fibonacci sequence:

def fib_generator():
    yield 0
    prev, current = 0, 1
    while True:
        yield current
        prev, current = current, prev + current

A generator expression is like a list comprehension, but it produces a lazy generator rather than a list:

double_fibs = (fib * 2 for fib in fib_generator())

Map and Filter

def map_gen(fn, iterable):
    iterator = iter(iterable)
    while True:
        yield fn(next(iterator))

def filter_gen(fn, iterable):
    iterator = iter(iterable)
    while True:
        item = next(iterator)
        if fn(item):
            yield item

Bitstring Generator

from itertools import product

def bitstrings():
    """Generate bitstrings in order of increasing size."
    size = 0
    tuples = product(('0', '1'), repeat=size)
    while True:
        for elem in tuples:
            yield ''.join(elem)
        size += 1

Break
Infinite Sequences with Selection

We now have implicit sequences in the form of iterators

Such sequences may be infinite, and they might be lazily evaluated

What if we want to support element selection on infinite sequences?

Let's try creating a list out of an infinite sequence

```python
>>> list(fib_generator())
Oops! Infinite loop!
```

A list provides immediate access to all elements

But an Rlist only provides immediate access to its first element

The rest can be computed lazily!

### Streams

A stream is a recursive list with an explicit first element and a lazily computed rest-of-the-list

```python
class Stream(Rlist):
    '''A lazily computed recursive list.'''
    def __init__(self, first, compute_rest=lambda: Stream.empty):
        assert callable(compute_rest)
        self.first = first
        self._compute_rest = compute_rest
        self._rest = None
    @property
    def rest(self):
        '''Return the rest of the stream, computing it if necessary.'''
        if self._compute_rest is not None:
            self._rest = self._compute_rest()
            self._compute_rest = None
        return self._rest
```

### Integer Streams

An integer stream is a stream of consecutive integers

An integer stream starting at \( k \) consists of \( k \) and a function that returns the integer stream starting at \( k+1 \)

```python
def integer_stream(first=1):
    '''Return a stream of consecutive integers, starting with first.'''
    >>> s = integer_stream(3)
    >>> s.first
    3
    >>> s.rest.first
    4,
    def compute_rest():
        return integer_stream(first+1)
    return Stream(first, compute_rest)
```

### Filtering a Stream

When filtering a stream, processing continues until an element is kept in the output

```python
def filter_stream(fn, s):
    '''Filter stream s with predicate function fn.'''
    if s is Stream.empty:
        return s
    def compute_rest():
        return filter_stream(fn, s.rest)
    if fn(s.first):
        return Stream(s.first, compute_rest)
    else:
        return compute_rest()
```

### Mapping a Function over a Stream

Mapping a function over a stream applies a function only to the first element right away

The rest is computed lazily

```python
def map_stream(fn, s):
    '''Map fn over the elements of stream s.'''
    if s is Stream.empty:
        return s
    def compute_rest():
        return map_stream(fn, s.rest)
    return Stream(fn(s.first), compute_rest)
```

### A Stream of Primes

The stream of integers not divisible by any \( k < n \) is:

- The stream of integers not divisible by any \( k < n \)
- Filtered to remove any element divisible by \( n \)
- This recurrence is called the Sieve of Eratosthenes

\[
2, 3, \ldots, 5, 7, 8, 9, 10, 11, 12, 13
\]

```python
def primes(istream):
    '''Return a stream of primes, given a stream of consecutive integers.'''
    def compute_rest():
        not_divisible = lambda x: x % istream.first != 0
        return primes(filter_stream(not_divisible, istream.rest))
    return Stream(istream.first, compute_rest)
```
Try it

• Write a function add_streams that takes two streams and returns a new stream formed by summing corresponding elements in the argument streams.
• Bonus: see if you can use add_streams to define the Fibonacci stream!