INSTRUCTIONS

- You have 2 hours and 50 minutes to complete the exam.
- The exam is closed book, closed notes, closed computer, closed calculator, except one 8.5" × 11" cheat sheet of your own creation.
- Mark your answers on the exam itself. We will not grade answers written on scratch paper.

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All the work on this exam is my own. (please sign)
1. (6 points) Olympic Games

(a) (4 pt) For each of the statements below, write the output displayed by the interactive Python interpreter when the statement is executed. The output may have multiple lines. No answer requires more than four lines.
Assume that you have started python3 and executed the following statements:

```python
gold = [0, 1]

def go(dream, team):
    print('Steps to success: ')
    def medal(lion):
        return dream(team, lion)
    return medal

def win(big):
    print('Eat vegetables.')
    return big[-1] + big[-2]

def get(it, done):
    do = it
    while done < 2:
        print("Don’t cheat!")
        do, done = do + [win(do)], done + 1
    return do

>>> print(4, 5) + 1
4 5
Error

>>> gold[-2]
0

>>> win(gold)
Eat vegetables.
1

>>> riooo = go(get, gold)
Steps to success:

Don’t cheat!
Eat vegetables.

>>> usa = riooo(0)
Don’t cheat!
Eat vegetables.

>>> usa
[0, 1, 1, 2]

(b) (2 pt) What would be the return value of riooo(-2), after evaluating the expressions above?
[0, 1, 1, 2, 3, 5]
2. (6 points) The Name Game

Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. You may not need to use all of the spaces or frames.

A complete answer will:

- Add all missing names and parent annotations to all frames.
- Add all missing values created or referenced during execution.
- Show the return value for each local frame.

```python
def person(name, age):
    def brain(ask):
        if ask == 'name':
            return name
        elif ask == 'age':
            return age
        return brain
    return brain

def name(guy):
    return guy('name')

def age(gal):
    return gal('age')

my_name, my_age = 'Marv', 21
marv = person(my_name, my_age)
dan = person('Dan', age(marv) + 7)
```
3. (3 points) High Scores
Write expressions involving the list \texttt{lst} that evaluate to the following values. Your expressions \textbf{must} use \texttt{lst}. For the second and third lines, you \textbf{must} use a list comprehension.

\begin{verbatim}
>>> lst = [[1, 2], [3, 4], [5, 6], [7, 8], [9, 10]]
>>> len(lst)
5
>>> [[11 - x, 11 - y] for x, y in lst]
[[10, 9], [8, 7], [6, 5], [4, 3], [2, 1]]
>>> [2 * x for x in lst[0]]
[2, 4]
\end{verbatim}

4. (4 points) Kerbal Space Program
Define the function \texttt{is\_sorted} that takes in a non-negative integer \texttt{n} and returns \texttt{True} if the digits of \texttt{n} are in non-increasing order from left to right, and \texttt{False} otherwise. See the doctests for details.

\textbf{You may only use the lines provided. You may not use any Python built-in sorting functions.}

\begin{verbatim}
def is_sorted(n):
    """Returns whether the digits in n are in non-increasing order
    from left to right."
    if n < 10:
        return True
    elif n % 10 > (n // 10) % 10:
        return False
    else:
        return is_sorted(n // 10)

>>> is_sorted(4)
True
>>> is_sorted(55555)
True
>>> is_sorted(9876543210)
True
>>> is_sorted(9087654321)
False
    """
\end{verbatim}
5. (6 points) Katamari

(a) (4 pt) Define the function `aggregate` that takes in a two-argument function `fn`, a sequence `seq`, and a predicate function `pred` and returns the result of using `fn` to combine the elements in `seq` for which `pred` returns `True`. See the doctests for details. You may assume `fn` is commutative and never returns `None`. You may only use the lines provided. You may not need to fill all the lines. You may not use an import statement.

```python
def aggregate(fn, seq, pred):
    """Aggregates using fn the elements in seq that satisfy pred."

    result = None
    for elem in seq:
        if pred(elem):
            if result == None:
                result = elem
            else:
                result = fn(result, elem)
    return result
```

(b) (2 pt) Use the `aggregate` function above, which you can assume works correctly, to define the `fact` function that returns the factorial of non-negative integers. Use only one line of code. If you need more space, you can continue your line of code on the second blank. The `add` and `mul` functions, which may be useful, have been imported for you.

*Hint:* You may need the ternary operator `<expr1> if <cond> else <expr2>.

```python
>>> from operator import add, mul

>>> fact = lambda n: aggregate(mul, range(1, n+1), lambda x: True) if n > 0 else 1

>>> fact(0)
1
>>> fact(5)
120
```
6. (6 points) Legend of Zelda

This question uses the following linked list data abstraction. We have provided an example linked list.

```python
>>> empty = 'empty'
```

```python
>>> link(1, link(2, link(3)))
```

```python
>>> def link(first, rest = empty):
...     return [first, rest]
...
```

```python
>>> def first(lnk):
...     return lnk[0]
...
```

```python
>>> def rest(lnk):
...     return lnk[1]
```

Define the function `linked_sum` that takes in a linked list of positive integers `lnk` and a non-negative integer `total` and returns the number of combinations of elements in `lnk` that sum up to `total`. You may use each element in `lnk` zero or more times. See the doctests for details. Do not violate abstraction barriers.

You may only use the lines provided. You may not need to fill all the lines.

```python
def linked_sum(lnk, total):
    """Return the number of combinations of elements in lnk that sum up to total."

    >>> # Four combinations: 1 1 1 1 , 1 1 2 , 1 3 , 2 2
    >>> linked_sum(link(1, link(2, link(3, link(5))))), 4)
    4
    >>> linked_sum(link(2, link(3, link(5))), 1)
    0
    >>> # One combination: 2 3
    >>> linked_sum(link(2, link(4, link(3))), 5)
    1
    """
    if total == 0:
        return 1
    elif lnk == empty or total < 0:
        return 0
    else:
        with_first = linked_sum(lnk, total - first(lnk))
        without_first = linked_sum(rest(lnk), total)
        return with_first + without_first
```

7. (9 points) Game of Thrones

This question uses the following tree data abstraction. We have provided an example tree.

(a) (6 pt) Define the function `track_lineage` that takes in a tree of strings `family_tree` and a string `name`. Assume that there is a unique node with entry `name`. `track_lineage` returns a list with the entries of the parent and grandparent of that node. If the node with entry `name` does not have a parent or grandparent, return `None` for that element in the list. See the doctests for details. **Do not violate abstraction barriers.**

You may only use the lines provided. You may not need to fill all the lines.

```python
def track_lineage(family_tree, name):
    """Return the entries of the parent and grandparent of the node with entry name in family_tree."
    def tracker(t, p, gp):
        if name == entry(t):
            return [p, gp]
        for c in children(t):
            res = tracker(c, entry(t), p)
            if res:
                return res
    return tracker(family_tree, None, None)
```

---

1 The grandparent of a node is the parent of the node’s parent.
(b)  (3 pt) Assuming that \texttt{track\_lineage} works correctly, define the function \texttt{are\_cousins} that takes in a tree of strings \texttt{family\_tree} and two strings \texttt{name1} and \texttt{name2} and returns \texttt{True} if the node with entry \texttt{name1} and the node with entry \texttt{name2} are cousins in \texttt{family\_tree}. Assume that there are unique nodes with entries \texttt{name1} and \texttt{name2} in \texttt{family\_tree}. See the doctests for details.

Two nodes are cousins if they have the same grandparent but different parents.

You may only use the lines provided. You may not need to fill all the lines.

```python
def are_cousins(family_tree, name1, name2):
    """Return True if a node with entry name1 is a cousin of a node with entry name2 in family_tree."
    p1, gp1 = track_lineage(family_tree, name1)
    p2, gp2 = track_lineage(family_tree, name2)
    return p1 != p2 and gp1 is not None and gp1 == gp2
```

8.  (0 points)  Games of Berkeley

In the box below, write a positive integer. The student who writes the lowest unique integer will receive one extra credit point. In other words, write the smallest positive integer that you think no one else will write.