Lecture 18: Mutable Trees
Mitas Ray
07/21/2016

**Announcements**

---

**Trees**

**Terminology**

- **Node**: single unit containing an entry
- **Root**: top node
- **Leaf**: a node with no children
- **Children**: subtree with a parent

---

**Tree Class**

```python
class Tree:
    def __init__(self, entry, children=[]):
        for c in children:
            assert isinstance(c, Tree)
        self.entry = entry
        self.children = children

    def is_leaf(self):
        return not self.children

>>> t = Tree(3, [Tree(2, [Tree(1)]), Tree(4)])
>>> t.entry
3
>>> t.children[0].entry
1
>>> t.children[1].is_leaf()
True
```

---

**Comparison to ADT**

```python
class Tree:
    def __init__(self, entry, children=[]):
        for c in children:
            assert isinstance(c, Tree)
        self.entry = entry
        self.children = children

def tree(entry, children=[]):
    return [entry, children]

def entry(tree):
    return tree[0]

def children(tree):
    return tree[1]

>>> t_class = Tree(3, [Tree(2, ...[Tree(1)]), Tree(4)])
>>> t_adt = tree(3, ...[Tree(1)]), Tree(4)])
>>> t_class.entry == entry(t_adt)
True
>>> t_class.entry = 5
>>> entry(t_adt) = 5
SyntaxError: can't assign ...
>>> t_class.entry == entry(t_adt)
False
```
Map

- Want to apply a function \( fn \) to each element in the tree
- Main Ideas
  - Apply \( fn \) to current node (mutate tree)
  - Call map on children

```python
class Tree:
    def __init__(self, entry, children=[]):
        ...
    def map(self, fn):
        self.entry = fn(self.entry)
        for c in self.children:
            c.map(fn)
```

```
>> square = lambda x: x * x
>> t = Tree(3, [Tree(2, [Tree(1)]), Tree(4)])
>> t.map(square)
```

Existence

- Does the tree contain element \( e \)?
- Main Ideas
  - Check entry of current node
  - Otherwise, check children
  - If no children to investigate, return False

```python
class Tree:
    def __init__(self, entry, children=[]):
        ...
    def __contains__(self, e):
        if self.entry == e:
            return True
        for c in self.children:
            if e in c:
                return True
        return False
```

```
>> t = Tree(3, [Tree(2, [Tree(1)]), Tree(4)])
>> 2 in t
True
```

**Binary Search Tree**

- Each node has at most 2 children, left and right
- Left child elements are all less than or equal to entry
- Right child elements are all greater than entry
- Left child and right child are also BSTs
- Only contains numbers!
Binary Search Tree Class

```python
class BST:
    empty = ()
    def __init__(self, entry, left=empty, right=empty):
        assert left is BST.empty or isinstance(left, BST)
        assert right is BST.empty or isinstance(right, BST)
        self.entry = entry
        self.left, self.right = left, right
        if left is not BST.empty:
            assert left.max <= entry
        if right is not BST.empty:
            assert entry < right.min

@property
def max(self):  # Returns the maximum element in the BST
    @property
def min(self):  # Returns the minimum element in the BST
```

Existence

- Does the BST contain element e?
- Main Ideas
  - Check entry of current node
  - Otherwise, check left or right
  - If no children to investigate, return False

```python
class BST:
    def __init__(self, entry, left=empty, right=empty): ...
    def __contains__(self, e):
        if self.entry == e:
            return True
        elif e < self.entry and self.left is not BST.empty:
            return e in self.left
        elif e > self.entry and self.right is not BST.empty:
            return e in self.right
        return False
```

Runtime Comparison

- Is there a difference in runtime when we check existence in a tree versus a BST?
- Runtime in terms of n, the number of nodes

```python
class Tree:
    def __init__(self, entry, children=[]): ...
    def __contains__(self, e):
        if self.entry == e:
            return True
        for c in self.children:
            if e in c:
                return True
        return False
```

```python
t = Tree(7, [Tree(3, [Tree(2), Tree(5)]), Tree(13, [Tree(11), Tree(17)])])
11 in t
```

Runtime Comparison

- Trees created with a class are mutable!
- BSTs allow us to organize our data in left child and right child based on value
- BST allows for more efficient search
  - $\Theta(n)$ in regular tree
  - $\Theta(\log n)$ in BST

```python
class BST:
    def __init__(self, entry, left=empty, right=empty): ...
    def __contains__(self, e):
        if self.entry == e:
            return True
        elif e < self.entry and self.left is not BST.empty:
            return e in self.left
        elif e > self.entry and self.right is not BST.empty:
            return e in self.right
        return False
```

```python
tst = BST(7, ...
    BST(3, BST(2), BST(5)), ...
    BST(13, BST(11), BST(17)))
11 in bst
```

Summary

- Trees created with a class are mutable!
- BSTs allow us to organize our data in left child and right child based on value
- BST allows for more efficient search
  - $\Theta(n)$ in regular tree
  - $\Theta(\log n)$ in BST