Today:

- More balanced search structures \((DS(IJ), \text{Chapter 9})\)

Coming Up:

- Pseudo-random Numbers \((DS(IJ), \text{Chapter 11})\)
Really Efficient Use of Keys: the Trie

- Have been silent about cost of comparisons.
- For strings, worst case is length of string.
- Therefore should throw extra factor of key length, $L$, into costs:
  - $\Theta(M)$ comparisons really means $\Theta(ML)$ operations.
  - So to look for key $X$, keep looking at same chars of $X$ $M$ times.
- Can we do better? Can we get search cost to be $O(L)$?

**Idea:** Make a multi-way decision tree, with one decision per character of key.
The Trie: Example

- Set of keys
  \{a, abase, abash, abate, abbas, axolotl, axe, fabric, facet\}
- Ticked lines show paths followed for “abash” and “fabric”
- Each internal node corresponds to a possible prefix.
- Characters in path to node = that prefix.
Adding Item to a Trie

- Result of adding bat and faceplate.
- New edges ticked.
A Side-Trip: Scrunching

- For speed, obvious implementation for internal nodes is array indexed by character.

- Gives $O(L)$ performance, $L$ length of search key.

- [Looks as if independent of $N$, number of keys. Is there a dependence?]

- **Problem**: arrays are *sparsely populated* by non-null values—waste of space.

**Idea**: Put the arrays on top of each other!

- Use null (0, empty) entries of one array to hold non-null elements of another.

- Use extra markers to tell which entries belong to which array.
Scrunching Example

**Small example:** (unrelated to Tries on preceding slides)

- Three leaf arrays, each indexed 0..9

  A1: 0 1 2 3 4 5 6 7 8 9
     bass trout pike

  A2: 0 1 2 3 4 5 6 7 8 9
     ghee milk oil

  A3: 0 1 2 3 4 5 6 7 8 9
     salt cumin mace

- Now overlay them, but keep track of original index of each item:

  A1: 0* 1 2 3 4 5* 6 7 8 9
  A2: 0 1* 2 3 4 5* 6 7 8 9
  A3: 0 1* 2 3 4 5* 6 7 8 9

  A123: 0 -1 1 -1 2 5 5 7 6 7 9
         bass trout pike ghee milk oil mace

Probabilistic Balancing: Skip Lists

- A *skip list* can be thought of as a kind of n-ary search tree in which we choose to put the keys at “random” heights.
- More often thought of as an ordered list in which one can skip large segments.
- Typical example:

  ![Diagram of a skip list]

- To search, start at top layer on left, search until next step would overshoot, then go down one layer and repeat.
- In list above, we search for 125 and 127. Gray nodes are looked at; darker gray nodes are overshoots.
- Heights of the nodes were chosen randomly so that there are about $1/2$ as many nodes that are $\geq k$ high as there are that are $k$ high.
- Makes searches fast with high probability.
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```
∞ ↓
1 2 3
10 20 25 30 40 50 55 60 90 95 100 115 120 125 130 140 150 ∞
```

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  ![Skip List Diagram](image)

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\[
\begin{array}{cccccccccccc}
\infty & 10 & 20 & 25 & 30 & 40 & 50 & 55 & 60 & 90 & 95 & 100 & 115 & 120 & 125 & 130 & 140 & 150 & \infty
\end{array}
\]

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Example: Adding and deleting

- Starting from initial list:

- In any order, we add 126 and 127 (choosing random heights for them), and remove 20 and 40:

- Shaded nodes here have been modified.
Summary

• Balance in search trees allows us to realize $\Theta(lg N)$ performance.

• B-trees, red-black trees:
  - Give $\Theta(lg N)$ performance for searches, insertions, deletions.
  - B-trees good for external storage. Large nodes minimize # of I/O operations.

• Tries:
  - Give $\Theta(B)$ performance for searches, insertions, and deletions, where $B$ is length of key being processed.
  - But hard to manage space efficiently.

• Interesting idea: scrunched arrays share space.

• Skip lists:
  - Give probable $\Theta(lg N)$ performance for searches, insertions, deletions.
  - Easy to implement.
  - Presented for interesting ideas: probabilistic balance, randomized data structures.
Summary of Collection Abstractions

**Multiset**
- contains, iterator

**List**
- get(n)

**Set**
- Ordered Set
  - first
- Unordered Set
- Priority Queue
- Sorted Set
  - subset

**Map**
- contains, iterator
  - get

**Unordered Map**
- Ordered Map

*Blue: Java has corresponding interface*

*Green: Java has no corresponding interface*

Data Structures that Implement Abstractions

Multiset

- **List**: arrays, linked lists, circular buffers
- **Set**
  - **OrderedSet**
    - *Priority Queue*: heaps
    - *Sorted Set*: binary search trees, red-black trees, B-trees, sorted arrays or linked lists
  - **Unordered Set**: hash table

Map

- **Unordered Map**: hash table
- **Ordered Map**: red-black trees, B-trees, sorted arrays or linked lists
Corresponding Classes in Java

**Multiset** (Collection)

- **List**: ArrayList, LinkedList, Stack, ArrayBlockingQueue, ArrayDeque
- **Set**
  - OrderedSet
    - Priority Queue: PriorityQueue
    - Sorted Set (SortedSet): TreeSet
  - Unordered Set: HashSet

**Map**

- Unordered Map: HashMap
- Ordered Map (SortedMap): TreeMap