1 Basic Algorithmic Analysis

For each of the following function pairs $f$ and $g$, list out the $\Theta$, $\Omega$, $O$ relationships between $f$ and $g$, if any such relationship exists. The log function here denotes the natural logarithm.

1. $f(x) = x^2$, $g(x) = x^2 + x$
2. $f(x) = 500000x^3$, $g(x) = x^5$
3. $f(x) = \log(x)$, $g(x) = 5x$
4. $f(x) = e^x$, $g(x) = x^5$ (hint: $5 > e$)
5. $f(x) = \log(5^x)$, $g(x) = x$

2 Practice with Runtime

For each of the following functions, find the Big-Theta expression for the runtime of the function in terms of the input variable $n$.

1. For this problem, you may assume that the static method `constant` runs in $\Theta(1)$ time.

```java
public static void thisIsANestedLoop(int n) {
    for (int i = 0; i < n; i += 1) {
        for (int j = 0; j < i; j += 1) {
            System.out.println(i + j);
        }
    }
    for (int k = 0; k < n; k += 1) {
        constant(k);
    }
}
```

2. `public static void thisIsMoreConfusing(int n) {
    for (int i = 1; i <= n; i *= 2) {
        for (int j = 0; j < i; j += 1) {
            System.out.println("moo");
        }
    }
}`
3 A Bit with some Bits

Complete the following method such that it does what it is intended to do: given a list of integers, it returns an integer such that the i-th bit of the return value is 1 if and only if more than half of the integers in the list have 1 in the i-th bit. Keep in mind that Java int is 32 bits long!

Note: the solution to this question isn’t very complicated, but it’s not short! Try breaking it down into components, and ask your neighbors for help!

```java
public static int bitVote(int[] bitList) {

    for (int i = 0; i < 32; i++) {
        // For each bit index

        for (int k : bitList) {
            // For each integer

        }
    }
}
```