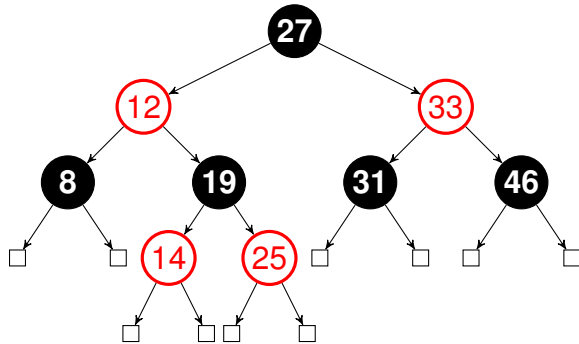


---

## 1 Balanced Search Trees

---

(a) Convert the red-black tree into a 2-4 tree. (Solid nodes are black.)



(b) Insert 13 into the resulting 2-4 tree. Assume that, if a node has 4 keys, we choose to push up the right of the 2 middle keys (so the 2<sup>nd</sup> key from the right).

(c) Convert the resulting 2-4 tree into a valid left-leaning red-black tree.

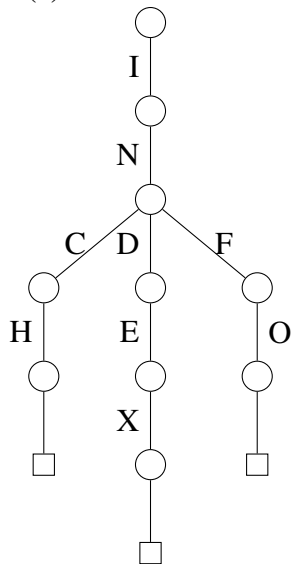
(d) Given a (2, 4) tree containing  $N$  keys, how would you obtain the keys in sorted order in worst case  $O(N)$  time? We don't need actual code—pseudo code or an unambiguous description will do.

(e) If a (2,4) tree has depth  $h$  (that is, the leaves are at distance  $h$  from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

## 2 Tries

---

(a) List the words encoded by the following trie.



(b) Draw the trie after inserting the words *indent*, *inches*, and *trie*.

### 3 Skip Lists

---

Draw the resulting skip list after adding the following numbers at the specified random height. Then highlight the links used to find 148.

Number	41	48	59	77	40	131	148	54	139	179	43	128	161	189	170
Height	1	1	1	4	2	2	1	3	1	1	3	2	3	1	2