1 Basic Algorithmic Analysis

For each of the following function pairs \( f \) and \( g \), list out the \( \Theta, \Omega, O \) relationships between \( f \) and \( g \), if any such relationship exists. For example, \( f(x) \in O(g(x)) \).

1. \( f(x) = x^2, g(x) = x^2 + x \)
2. \( f(x) = 5000000x^3, g(x) = x^5 \)
3. \( f(x) = \log(x), g(x) = 5x \)
4. \( f(x) = e^x, g(x) = x^5 \)
5. \( f(x) = \log(5^x), g(x) = x \)

2 Practice with Runtime

For each of the following functions, find the Big-Theta expression for the runtime of the function in terms of the input variable \( n \).

You may find the following relations helpful:

\[
1 + 2 + 3 + 4 + \cdots + N = \Theta(N^2)
\]
\[
1 + 2 + 4 + \cdots + N = \Theta(N)
\]

1. For this problem, assume that the static method \( \text{constant} \) runs in \( \Theta(1) \) time.

   ```java
   public static void bars(int n) {
       for (int i = 0; i < n; i += 1) {
           for (int j = 0; j < i; j += 1) {
               System.out.println(i + j);
           }
       }
       for (int k = 0; k < n; k += 1) {
           constant(k);
       }
   }
   ```

2. `public static void barsRearranged(int n) {
    for (int i = 1; i <= n; i *= 2) {
        for (int j = 0; j < i; j += 1) {
            System.out.println("mooove");
        }
    }
} `