# Asymptotic analysis

In the following questions, give the runtime of the following functions in Θ(.) or O(.) notation as requested. Your answer should be a function of N that is as simple as possible with no unnecessary leading constants or lower order terms. (MT2, Spring 2015)

(a) public static void f1(int n) {
    for (int i = 0; i < 2*n; i += 1) {
        System.out.println("hello");
    }
}

Runtime: ___Θ(n)____

(b) public static void f3(int n) {
    if (n == 0) { return; }
    f3(n/3);
    f1(n);
    f3(n/3);
    f1(n);
    f3(n/3);
}

Runtime: ___Θ(nlogn)____

(c) public static void f4(int n) {
    if (n == 0) { return; }
    f4(n-1);
    f1(17);
    f4(n-1);
}

Runtime: ___Θ(2^n)____

(d) public static void f5(int n, int m) {
    if (m <= 0) {
        return;
    } else {
        for (int i = 0; i < n; i += 1) {
            f5(n, m-1);
        }
    }
}

Runtime: ___Θ(n^m)____

(e) public static void f6(int n) {
    if (n == 0) { return; }
    f6(n / 2);
    f6(n / 2);
    f6(n / 2);
    f6(n / 2);
    g(n); // runs in Θ(N^2) time
}

Runtime: ___Θ(n^2logn)____
(f) public static void f7(int n, int m) {
    Runtime: \(O(n)\)
    if (n < 10) { return; }
    for (int i = 0; i <= n % 10; i++) {
        f7(n / 10, m / 10);
        System.out.println(m);
    }
}

2 More analysis

For each problem, give the best and worst-case runtimes in \(\Theta(.)\) notation as a function of N. Your answer should be simple with no unnecessary leading constants or summations.

(a) public static void removeIndex(int[] arr, int i) {
    // Assume i > 0
    int N = arr.length;
    for (int j = i; j < N; j++) {
        arr[j - 1] = arr[j];
    }
}

    Best Case: \(\Theta(1)\)  Worst Case: \(\Theta(N)\)

(b) public static int recurse(int N) {
    return helper(N, N / 2);
}

    private static int helper(int N, int M) {
        if (N <= 1) {
            return N;
        }
        for (int i = 1; i < M; i *= 2) {
            System.out.println(i);
        }
        return helper(N - 1, M) + helper(N - 1, M);
    }

    Best Case: \(\Theta(2^N \log N)\)  Worst Case: \(\Theta(2^N \log N)\)

(c) public static boolean find(int tgt, int[] arr) {
    int N = arr.length;
    return find(tgt, arr, 0, N);
}

    private static boolean find(int tgt, int[] arr, int lo, int hi) {
        if (lo == hi || lo + 1 == hi) {
            return arr[lo] == tgt;
        }
        int mid = (lo + hi) / 2;
        for (int i = 0; i < mid; i++) {
            System.out.println(arr[i]);
        }
        return arr[mid] == tgt || find(tgt, arr, lo, mid) || find(tgt, arr, mid, hi);
    }

    Best Case: \(\Theta(N)\)  Worst Case: \(\Theta(N^2)\)
### 3. Bit Operations

In the following questions, use bit manipulation operations to achieve the intended functionality and fill out the function details -

(a) Implement a function `isPalindrome` which checks if the binary representation of a given number is palindrome. The function returns true if and only if the binary representation of `num` is a palindrome.

For example, the function should return true for `isPalindrome(9)` since binary representation of 9 is 1001 which is a palindrome.

```java
/**
 * Returns true if binary representation of num is a palindrome
 */
public static boolean isPalindrome(int num) {
    // stores reverse of binary representation of num
    int reverse = 0;

    // do till all bits of num are processed
    int k = num;
    while (k > 0) {
        // add rightmost bit to reverse
        reverse = (reverse << 1) | (k & 1);
        k = k >> 1;  // drop last bit
    }

    return num == reverse;
}
```

(b) Implement a function `swap` which for a given integer, swaps two bits at given positions. The function returns the resulting integer after bit swap operation.

For example, when the function is called with inputs `swap(31, 3, 7)`, it should reverse the 3rd and 6th bit from the right and return 91 since 31 (00011111) would become 91 (01011011).

```java
/**
 * Function to swap bits at position a and b (from right) in integer num
 */
public static int swap(int num, int a, int b) {
    int p = a-1;
    int q = b-1;

    int bit_a = (num >> p) & 1;
    int bit_b = (num >> q) & 1;

    if (bit_a != bit_b) {  // if the bits are different
        num ^= (1 << p);
        num ^= (1 << q);
    }

    return num;
}
```