CS61B Lecture #14: Integers
# Integer Types and Literals

<table>
<thead>
<tr>
<th>Type</th>
<th>Bits</th>
<th>Signed?</th>
<th>Literals</th>
</tr>
</thead>
<tbody>
<tr>
<td>byte</td>
<td>8</td>
<td>Yes</td>
<td>Cast from <code>int</code>: (byte) 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>None. Cast from <code>int</code>: (short) 4096</td>
</tr>
<tr>
<td>short</td>
<td>16</td>
<td>Yes</td>
<td>'a' // (char) 97</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>'\n' // newline ((char) 10)</td>
</tr>
<tr>
<td>char</td>
<td>16</td>
<td>No</td>
<td>'\t' // tab ((char) 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>'\' // backslash</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>'A', '\101', '\u0041' // == (char) 65</td>
</tr>
<tr>
<td>int</td>
<td>32</td>
<td>Yes</td>
<td>123</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0100 // Octal for 64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0x3f, 0xffffffff // Hexadecimal 63, -1 (!)</td>
</tr>
<tr>
<td>long</td>
<td>64</td>
<td>Yes</td>
<td>123L, 01000L, 0x3fL</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1234567891011L</td>
</tr>
</tbody>
</table>

- Negative numerals are just negated (positive) literals.
- "\(N\) bits" means that there are \(2^N\) integers in the domain of the type:
  - If signed, range of values is \(-2^{N-1} \ldots 2^{N-1} - 1\).
  - If unsigned, only non-negative numbers, and range is \(0 .. 2^N - 1\).
Overflow

- **Problem:** How do we handle overflow, such as occurs in 10000*10000*10000?
- Some languages throw an exception (Ada), some give undefined results (C, C++)
- Java defines the result of any arithmetic operation or conversion on integer types to “wrap around”—modular arithmetic.
- That is, the “next number” after the largest in an integer type is the smallest (like “clock arithmetic”).
- E.g., (byte) 128 == (byte) (127+1) == (byte) −128
- In general,
  - If the result of some arithmetic subexpression is supposed to have type $T$, an $n$-bit integer type,
  - then we compute the real (mathematical) value, $x$,
  - and yield a number, $x'$, that is in the range of $T$, and that is equivalent to $x$ modulo $2^n$.
  - (That means that $x − x'$ is a multiple of $2^n$.)
Modular Arithmetic

• Define $a \equiv b \pmod{n}$ to mean that $a - b = kn$ for some integer $k$.

• Define the binary operation $a \mod n$ as the value $b$ such that $a \equiv b \pmod{n}$ and $0 \leq b < n$ for $n > 0$. (Can be extended to $n \leq 0$ as well, but we won’t bother with that here.) This is not the same as Java’s % operation.

• Various facts: (Here, let $a'$ denote $a \mod n$).

\[
\begin{align*}
    a'' & = a' \\
    a' + b'' & = (a' + b)' = a + b' \\
    (a' - b')' & = (a' + (-b)')' = (a - b)' \\
    (a' \cdot b')' & = a' \cdot b' = a \cdot b' \\
    (a^k)' & = ((a')^k)' = (a \cdot (a^{k-1})')', \text{ for } k > 0.
\end{align*}
\]
Modular Arithmetic: Examples

• (byte) (64*8) yields 0, since $512 - 0 = 2 \times 2^8$.

• (byte) (64*2) and (byte) (127+1) yield -128, since $128 - (-128) = 1 \times 2^8$.

• (byte) (101*99) yields 15, since $9999 - 15 = 39 \times 2^8$.

• (byte) (-30*13) yields 122, since $-390 - 122 = -2 \times 2^8$.

• (char) (-1) yields $2^{16} - 1$, since $-1 - (2^{16} - 1) = -1 \times 2^{16}$.
Modular Arithmetic and Bits

• Why wrap around?

• Java’s definition is the natural one for a machine that uses binary arithmetic.

• For example, consider bytes (8 bits):

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>1100101</td>
</tr>
<tr>
<td>× 99</td>
<td>1100011</td>
</tr>
<tr>
<td>9999</td>
<td>100111</td>
</tr>
<tr>
<td>− 9984</td>
<td>100111</td>
</tr>
<tr>
<td>15</td>
<td>00001111</td>
</tr>
</tbody>
</table>

• In general, bit \( n \), counting from 0 at the right, corresponds to \( 2^n \).

• The bits to the left of the vertical bars therefore represent multiples of \( 2^8 = 256 \).

• So throwing them away is the same as arithmetic modulo 256.
Negative numbers

• Why this representation for -1?

\[
\begin{array}{c|c}
1 & 00000001_2 \\
+ & 11111111_2 \\
= & 01|00000000_2 \\
\end{array}
\]

Only 8 bits in a byte, so bit 8 falls off, leaving 0.

• The truncated bit is in the $2^8$ place, so throwing it away gives an equal number modulo $2^8$. All bits to the left of it are also divisible by $2^8$.

• On unsigned types (char), arithmetic is the same, but we choose to represent only non-negative numbers modulo $2^{16}$:

\[
\begin{array}{c|c}
1 & 00000000000000001_2 \\
+ & 2^{16} - 1 \\
= & 2^{16} + 0 \\
\end{array}
\]

\[
\begin{array}{c|c}
1 & 000000000000000001_2 \\
+ & 1111111111111111_2 \\
= & 2^{16} + 0 \\
\end{array}
\]
Conversion

• In general Java will silently convert from one type to another if this makes sense and no information is lost from value.

• Otherwise, cast explicitly, as in (byte) x.

• Hence, given

```java
byte aByte; char aChar; short aShort; int anInt; long aLong;
```

```java
// OK:
anShort = aByte; anInt = aByte; anInt = aShort;
anInt = aChar; aLong = anInt;
```

```java
// Not OK, might lose information:
anInt = aLong; aByte = anInt; aChar = anInt; aShort = anInt;
aShort = aChar; aChar = aShort; aChar = aByte;
```

```java
// OK by special dispensation:
aByte = 13; // 13 is compile-time constant
aByte = 12+100 // 112 is compile-time constant
```
Promotion

- Arithmetic operations (+, *, ...) promote operands as needed.
- Promotion is just implicit conversion.
- For integer operations,
  - if any operand is long, promote both to long.
  - otherwise promote both to int.
- So,
  - aByte + 3 == (int) aByte + 3  // Type int
  - aLong + 3 == aLong + (long) 3  // Type long
  - 'A' + 2 == (int) 'A' + 2  // Type int
  - aByte = aByte + 1  // ILLEGAL (why?)
- But fortunately,
  - aByte += 1;  // Defined as aByte = (byte) (aByte+1)
- Common example:
  - // Assume aChar is an upper-case letter
    char lowerCaseChar = (char) ('a' + aChar - 'A');  // why cast?
Bit twiddling

- Java (and C, C++) allow for handling integer types as sequences of bits. No “conversion to bits” needed: they already are.

- Operations and their uses:

<table>
<thead>
<tr>
<th>Mask</th>
<th>Set</th>
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<tr>
<td>00101100</td>
<td>00101100</td>
<td>00101100</td>
<td></td>
</tr>
<tr>
<td>&amp; 10100111</td>
<td>10100111</td>
<td>~ 10100111</td>
<td>~ 10100111</td>
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<tr>
<td>00100100</td>
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- Shifting:

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<td>01101000</td>
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- What is:

  - \((-1) \ggg 29\)?
  - \(x << n\)?
  - \(x >> n\)?
  - \((x >>> 3) \& ((1<<5)-1)\)?
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  \((-1) \ggg 3\) = 7.

- What is:

  - \(x \ll n\)?
  - \(x \gg n\)?
  - \((x \ggg 3) \& ((1\ll 5) - 1)\)?
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(-1) >>> 29? = 7.

- What is:

\[
x << n?
\]
\[
x >> n?
\]
\[
(x >>> 3) & ((1<<5)-1)?
\]
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\[
(-1) \ggg 29? \quad = 7.
\]

\[
x \ll n? \quad = x \cdot 2^n.
\]

\[
x \gg n? \quad = \lfloor x/2^n \rfloor \text{ (i.e., rounded down).}
\]

\[
(x \ggg 3) \& ((1<<5)-1)?
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(-1) >>> 29? = 7.

x << n? = x \cdot 2^n.

x >> n? = \lfloor x/2^n \rfloor (i.e., rounded down).

(x >>> 3) & ((1<<5)-1)? 5-bit integer, bits 3-7 of x.