CS61BL

Lecture 3:

Asymptotic Analysis

Trees & Tree Traversals

Stacks and Queues

Binary Search Trees (and other trees)


WARNING
BEWARE
OF MATH

Program Efficiency

- How much memory a program uses
  - Memory is cheap
- How much time a program uses

Testing Time Efficiency

- Benchmarks
  - Hardware specs?
  - Which programming language?
  - Other processes running?
- Estimate with statement count?
  - `int a = 1; // 1 “unit” of time?`
  - `int[] b = new int[100]; // 100 “units” of time?`
  - `b[15] = 2 + 2; // 1… 2… 3 (?) “units” of time?`

Big-Oh Notation

- Classify program running times into groups
  - E.g. O(n^2), O(2^n)
- Formal definition (version 1):

$$f(n) \in O(g(n))$$

$$\exists M, N > 0 : \forall n > N, f(n) < M \times g(n)$$

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$$f(n) \in O(g(n))$$

There exist positive constants M and N such that for all $n > N$, $f(n) < M \times g(n)$

(Upper bound on worst case running time)

Big-Oh Example 1

There exist positive constants M and N such that for all $n > N$, $f(n) < M \times g(n)$

$$f(n) = 3n + 5$$

$$g(n) = n^2 + 2n + 3$$

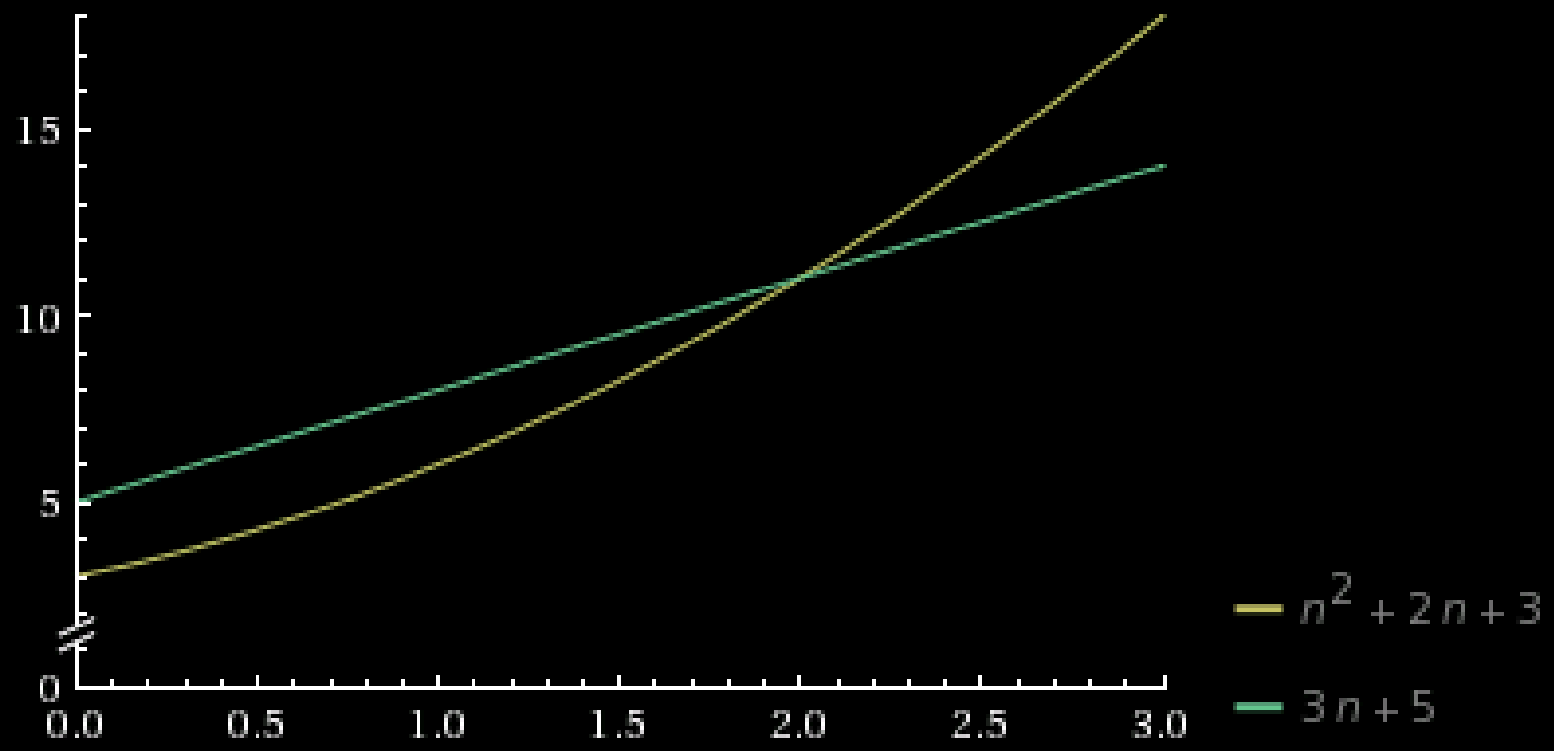
$$N = 2$$

$$M = 1$$

Input interpretation:

plot $3 + 2n + n^2$, $5 + 3n$ $n = 0$ to 3

Plot:

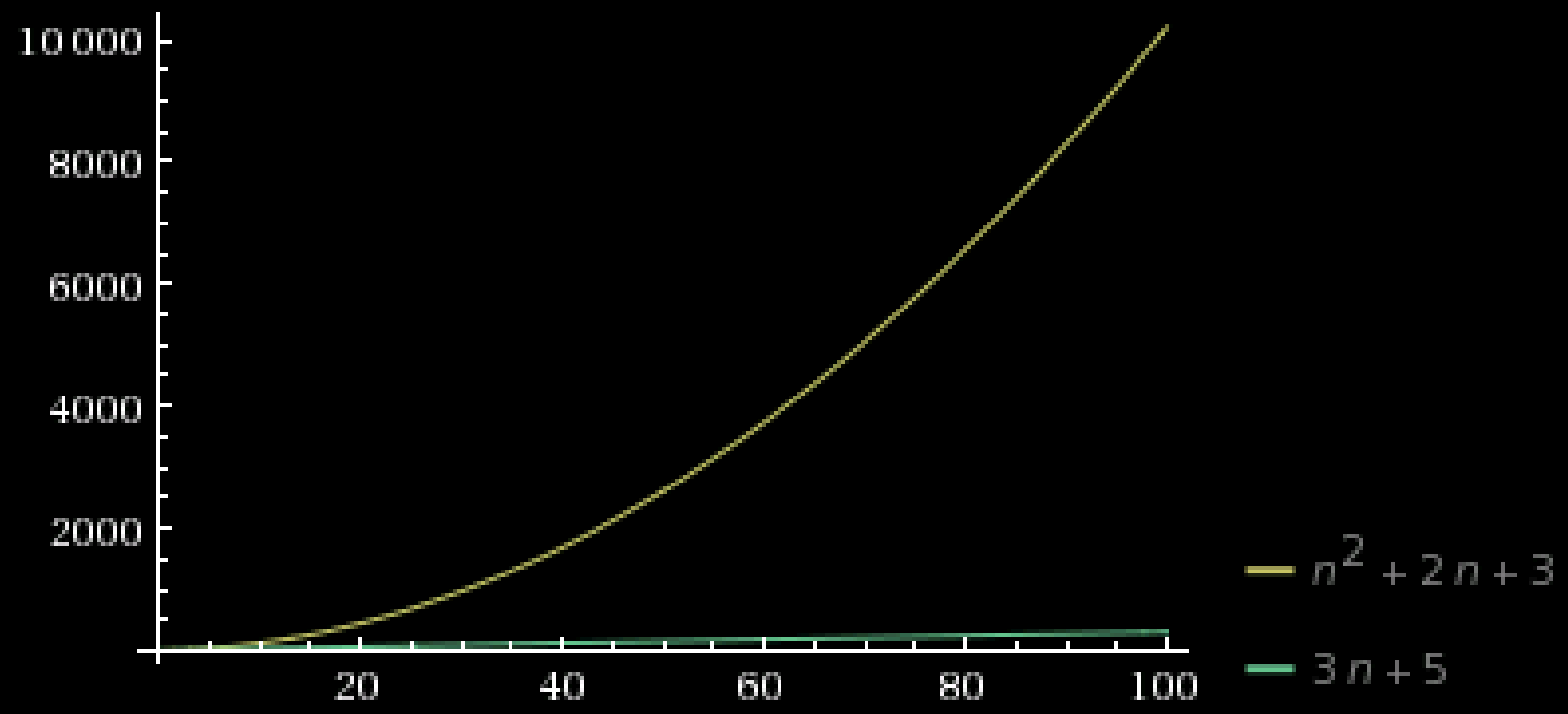


Source: wolframalpha.com

Input interpretation:

plot $3 + 2n + n^2$, $5 + 3n$ $n = 0$ to 100

Plot:



Source: wolframalpha.com

Big-Oh Notation

- Classify program running times into groups
  - E.g. O(n^2), O(2^n)
- Formal definition (version 2):

$$f(n) \in O(g(n))$$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} < \infty$$

Big-Oh Example 2

$$f(n) = 3n + 5$$

$$g(n) = n^2 + 2n + 3$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{3n+5}{n^2+2n+3} = \lim_{n\to\infty} \frac{3n}{n^2}$$

$$\lim_{n\to\infty} \frac{n}{n^2} = \lim_{n\to\infty} \frac{1}{n} = 0 < \infty$$

Shortcuts

- Get rid of additive terms with smaller growth order
  - E.g. $n^2 + 2n + 3$ becomes n^2
- Get rid of constant factors
  - E.g. $10n^2$ becomes n^2
- NOTE: Can't drop non-constant factors!
  - Common mistake: $n \log^2(n)$ becomes n

Big-Oh Example 3

$$f(n) = 3n\log_3(3n)$$

$$g(n) = 5n\log_2(n)$$

$$\lim_{n\to\infty} \frac{3n\log_3(3n)}{5n\log_2(n)} = \lim_{n\to\infty} \frac{3\log_3(3n)}{5\log_2(n)}$$

$$\lim_{n\to\infty} \frac{\log_3(3n)}{\log_2(n)} = \lim_{n\to\infty} \frac{\log_3 3 + \log_3 n}{\frac{\log_3 n}{\log_3 2}}$$

$$\lim_{n\to\infty} \frac{\log_3 n}{\log_3 n} = 1 < \infty$$

Ω

- Ω: $f(n) \in \Omega(g(n)) \iff g(n) \in O(f(n))$
  - Lower bound / best case

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0$$

Θ

- Θ: $$f(n) \in \Theta(g(n)) \iff \begin{array}{c} f(n) \in O(g(n)) \\ \text{and} \\ f(n) \in \Omega(g(n)) \end{array}$$
  - Tight bound

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = c; 0 < c < \infty$$

Big-Oh Example 4

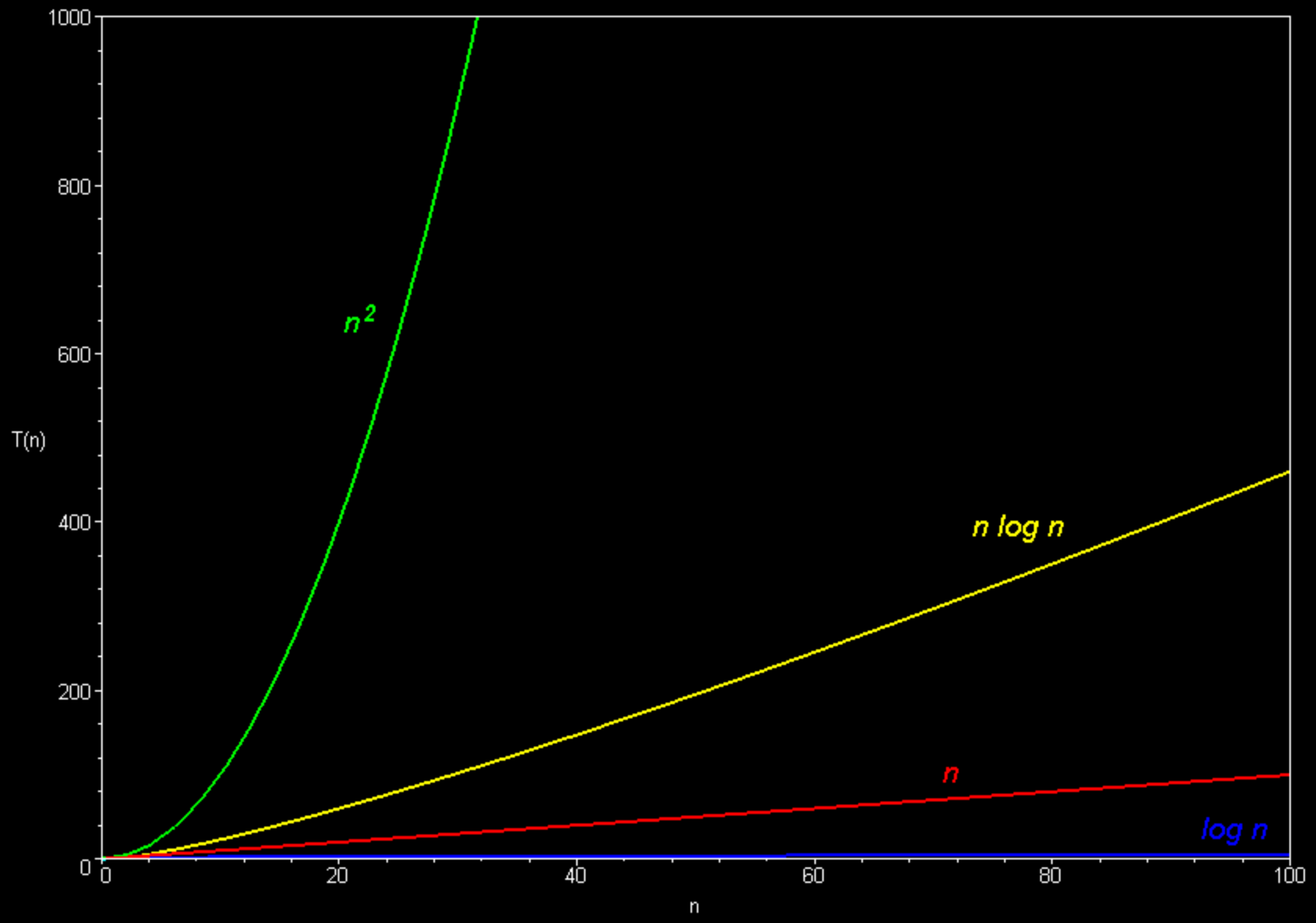
$$f(n) = 3n + 5$$

$$g(n) = n^2 + 2n + 3$$

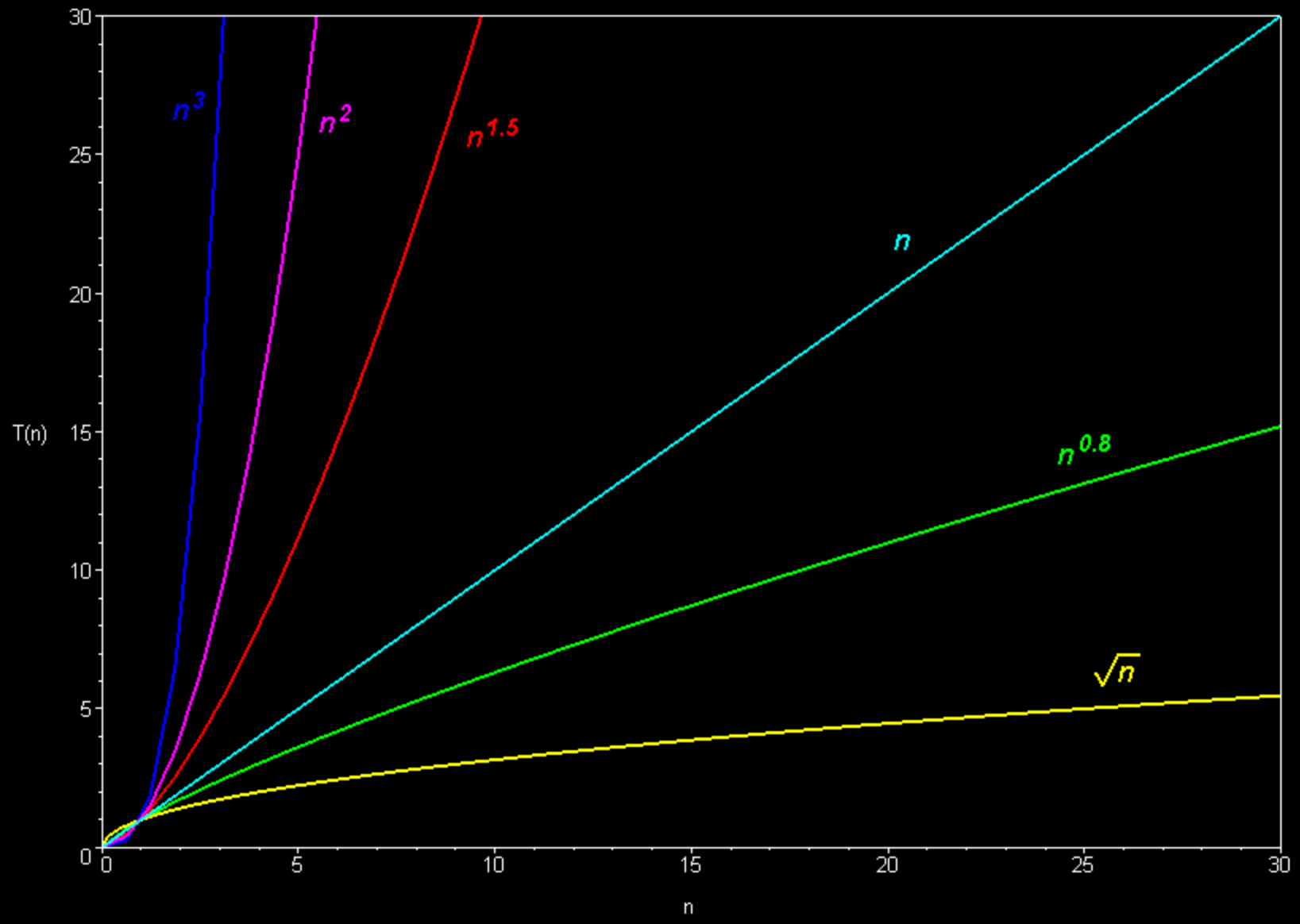
- Showed: $f(n) \in O(g(n))$
- Shortcut:
  - Show: $f(n) \in \Theta(n)$ and $g(n) \in \Theta(n^2)$
  - Use hierarchy of common time complexity classes

Time Complexity Classes

Notation	Common name
$\Theta(1)$	Constant
$\Theta(\log n)$	Logarithmic
$\Theta(n^c), 0 < c < 1$	Polynomial
$\Theta(n)$	Linear
$\Theta(n \log n)$	$n \log n$
$\Theta(n^2)$	Quadratic
$\Theta(n^3)$	Cubic
$\Theta(n^c), c > 1$	Polynomial
$\Theta(c^n), c > 1$	Exponential
$\Theta(n!)$	Factorial



Source: http://science.slc.edu/~jmarshall/courses/2002/spring/cs50/BigO/



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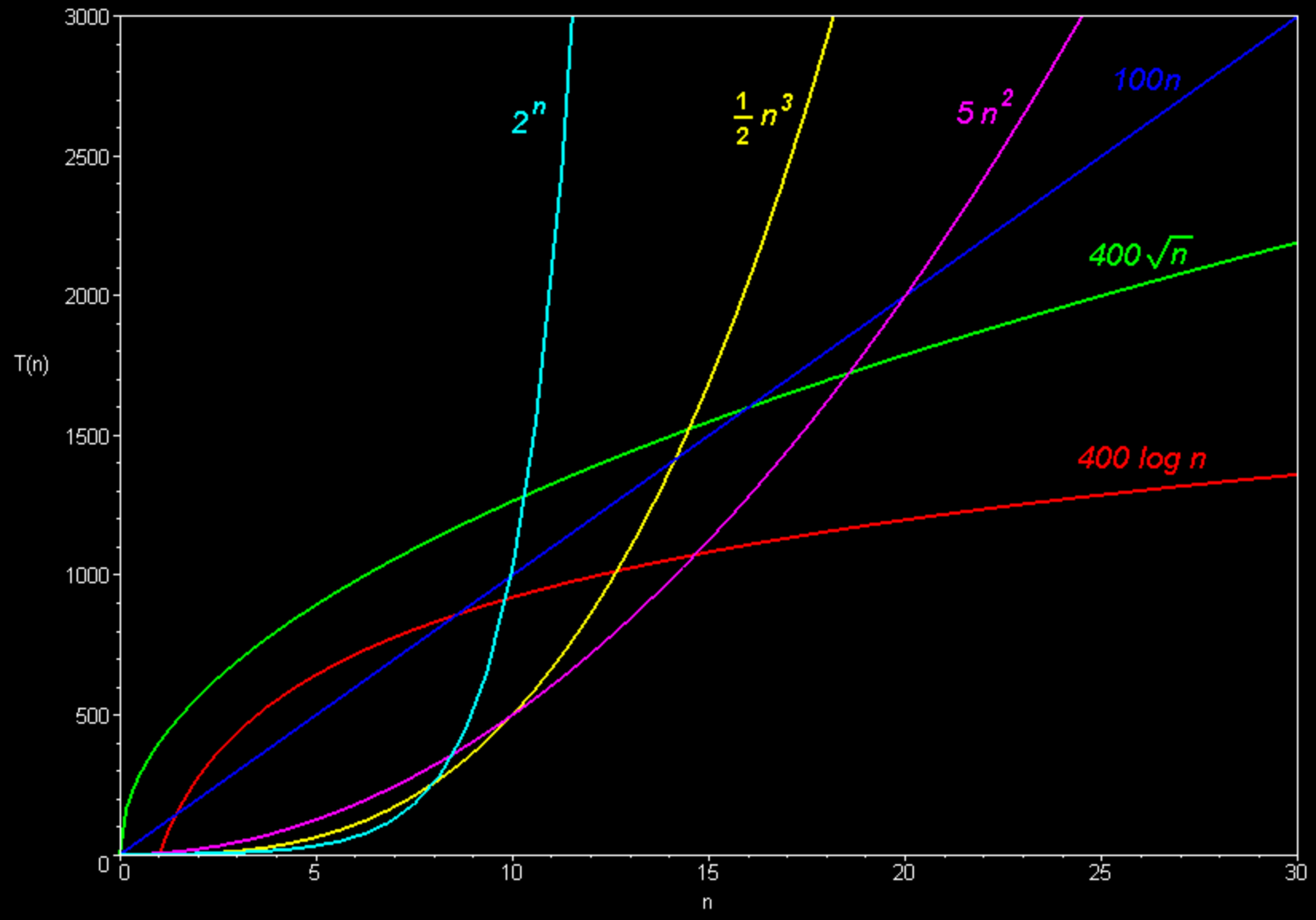

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Big O, Ω, Θ

- How well does a program scale?
  - Youtube video quality
  - Video game graphics settings
  - Google results
  - Facebook users

Story time

- Simplex algorithm
  - Developed by George Dantzig of UC Berkeley
  - Solves linear programs:

$$\min x_1 + x_2$$

$$x_1 + 2x_2 \geq 4$$

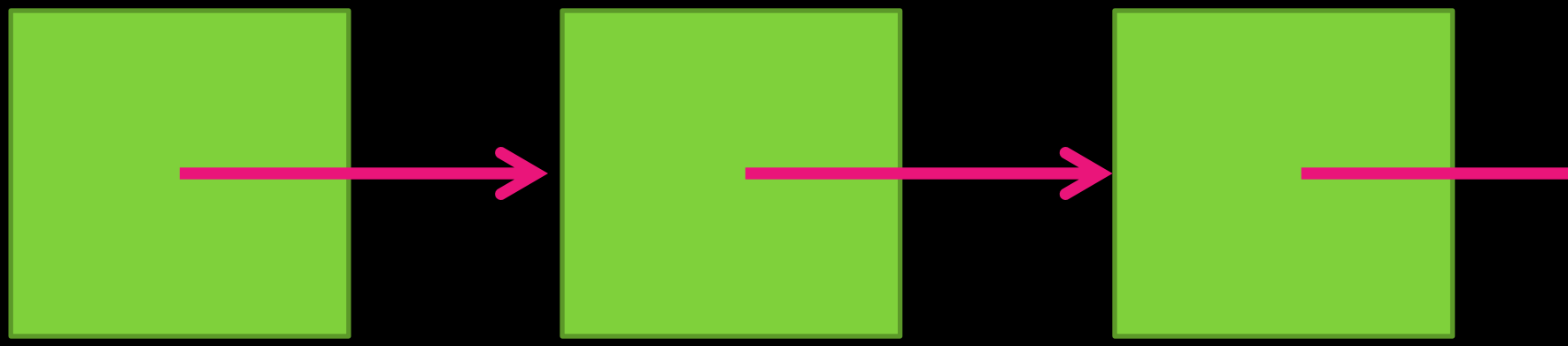
$$3x_1 + x_2 \geq 5$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

  - Worst case exponential time, but fast enough polynomial time in practice

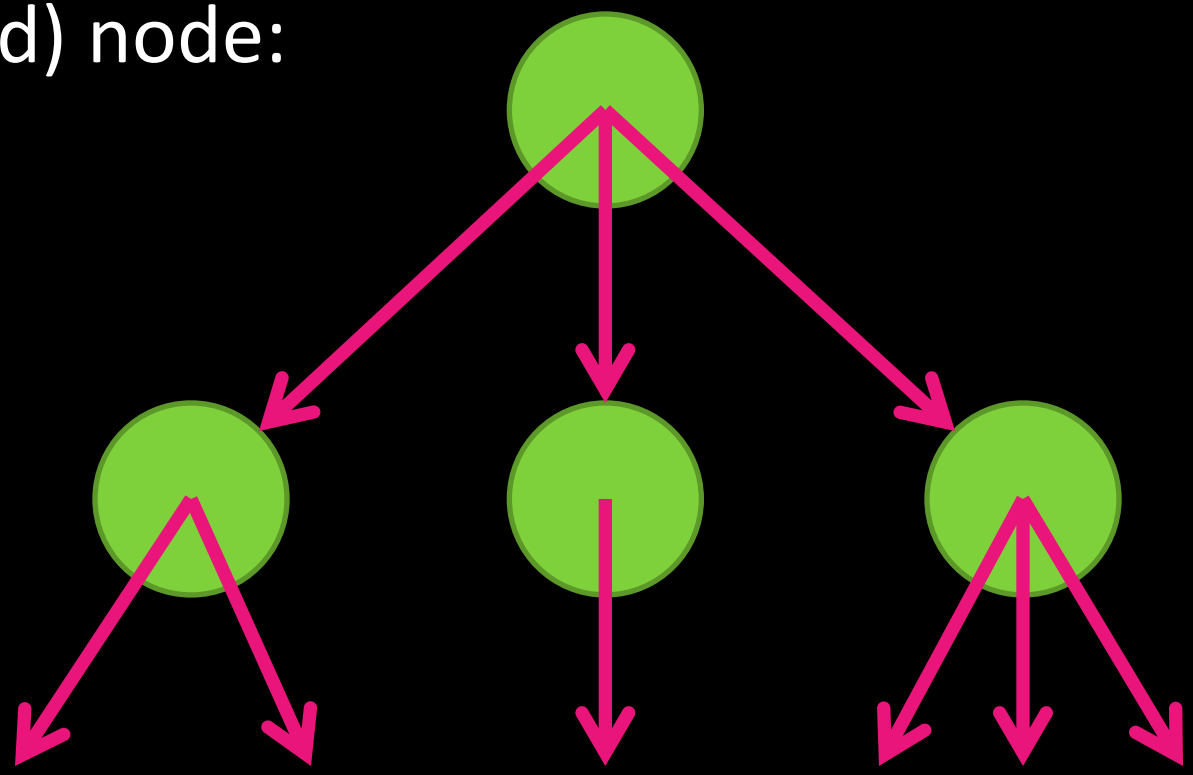
Linked List

- Each node has at most one “next” node:



Tree

- Each node can have more than one “next” (child) node:

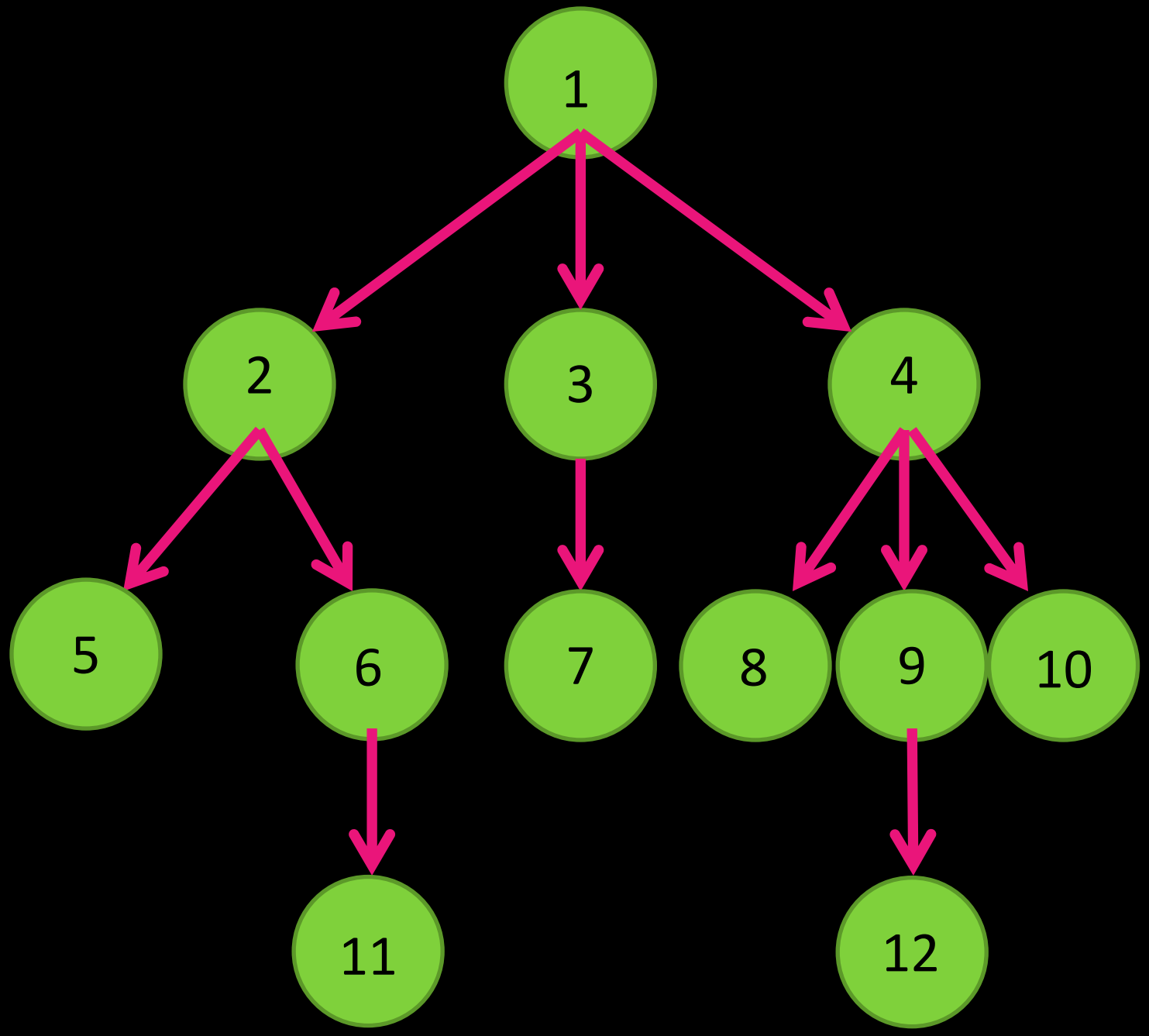


Terminology

- Node:
  - Contains an item
  - Similar to linked list node
- Root:
  - “Top” level / “first” node
  - Has no parent
  - Similar to linked list head
  - Tree can only have one root
- Leaf
  - “Bottom” / “last” node
  - Has no children
  - Similar to last node of linked list
  - Tree can have multiple leaf nodes

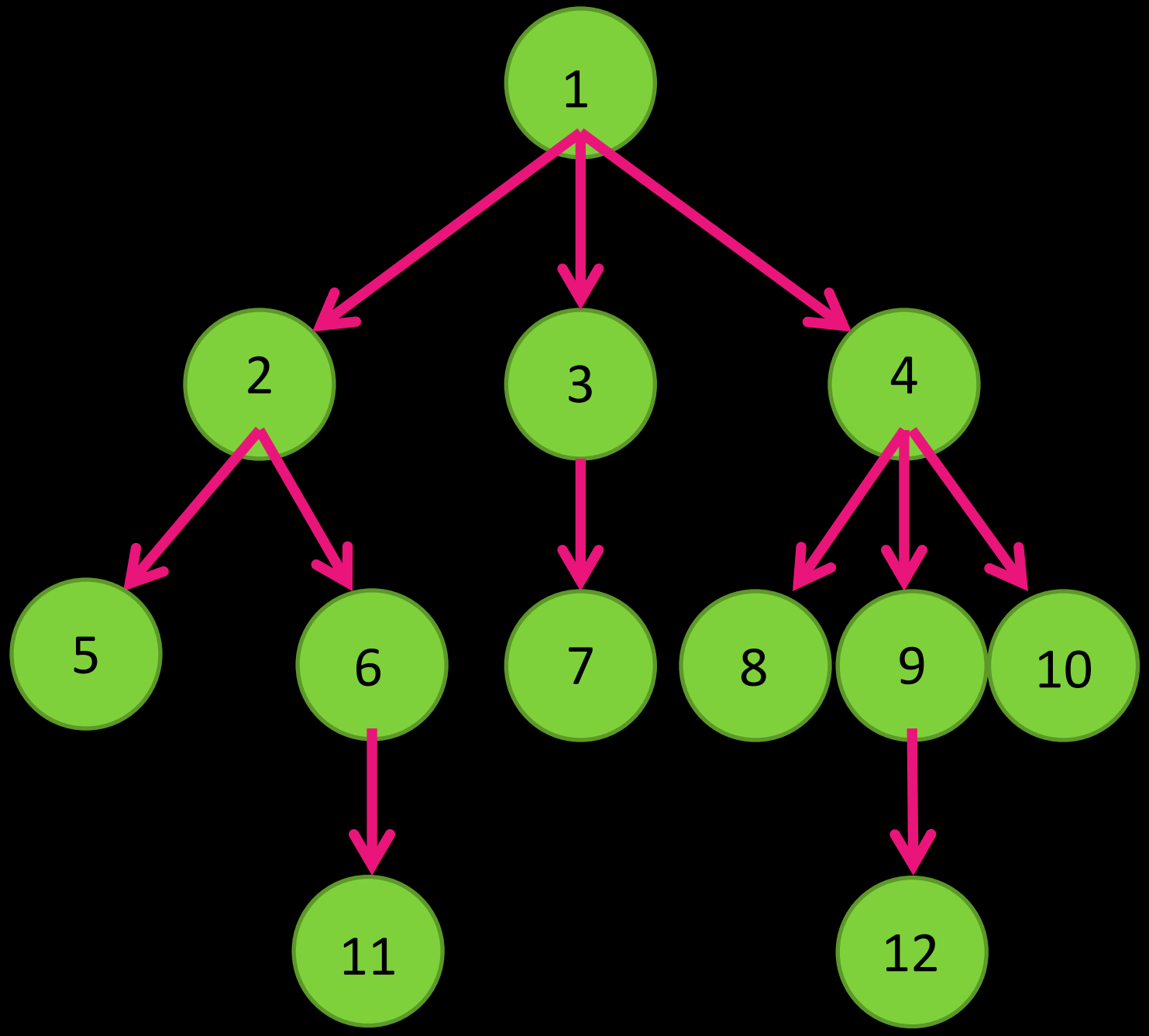
Terminology

- Children
  - Tree nodes can have multiple children
  - Similar to linked list node's "myNext"
- Parent
  - Each tree node can only have 1 parent
  - Tree nodes can share a parent (siblings)
  - Similar to linked list node's "myPrev"


1
2
3
4
5
6
7
8
9
10
11
12

Terminology

- Edge
  - Connects two nodes (pink arrows)
- Path
  - A series of edges that connect two nodes through intermediary nodes
- Depth
  - Length of path from root node
- Height
  - Length of path to deepest descendant


1
2
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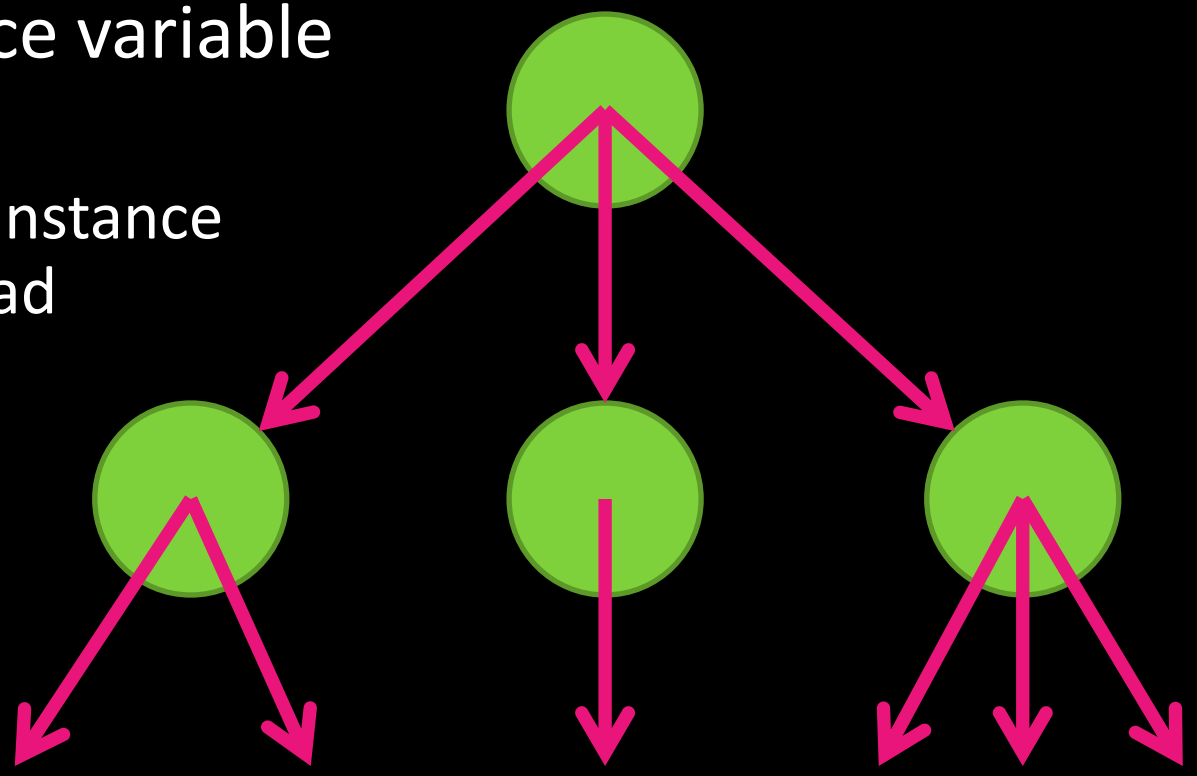
4
5
6
7
8
9
10
11
12

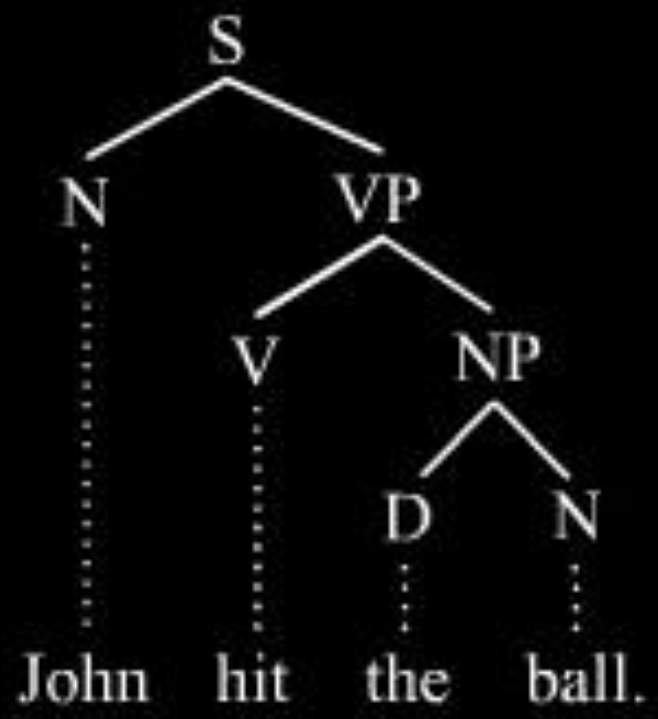
Linked List -> Tree

- TreeNode has instance variable ArrayList<TreeNode> children
  - ListNode has instance variable myNext
- Tree has instance variable myRoot
  - LinkedList has instance variable myHead
- TreeNode and ListNode both have myItem



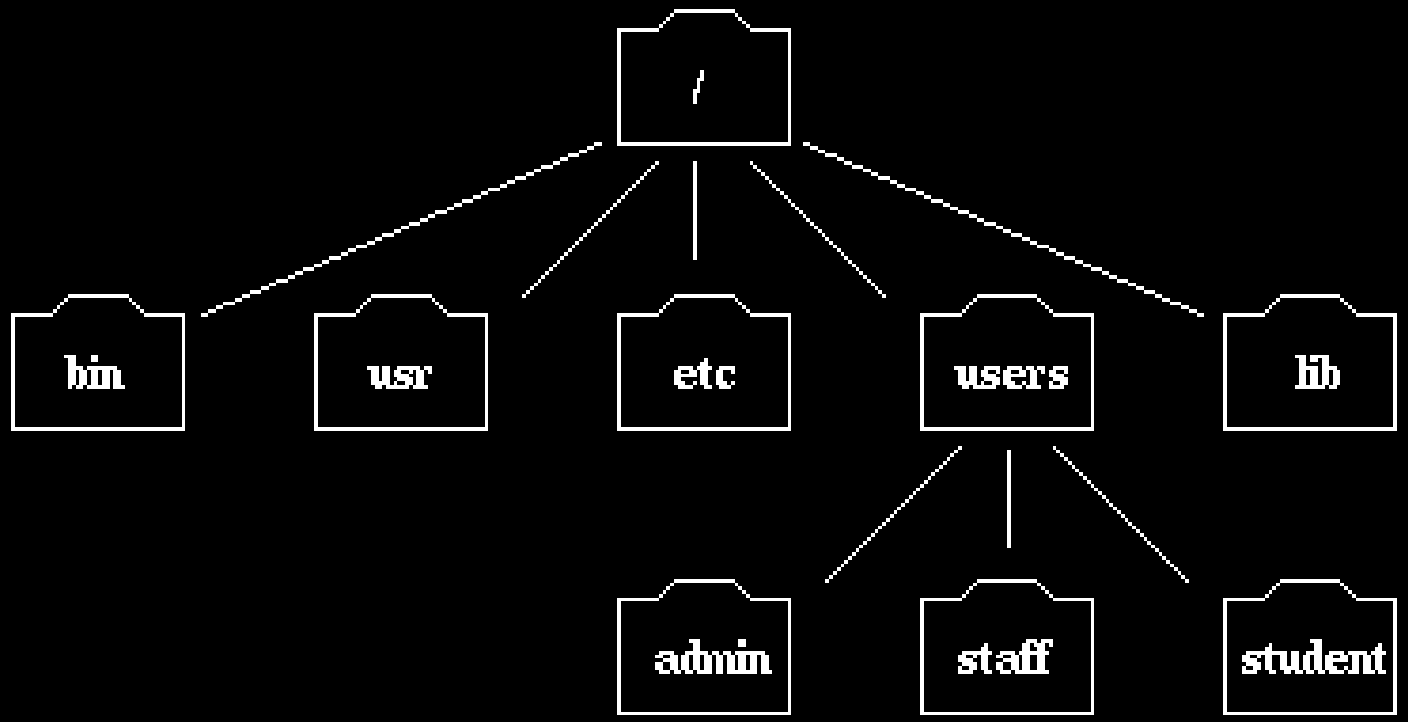
Tree Uses

- Parsing text
  - Parse tree: sentence structure (e.g. English)
  - Abstract Syntax Tree: source code structure (e.g. Java)



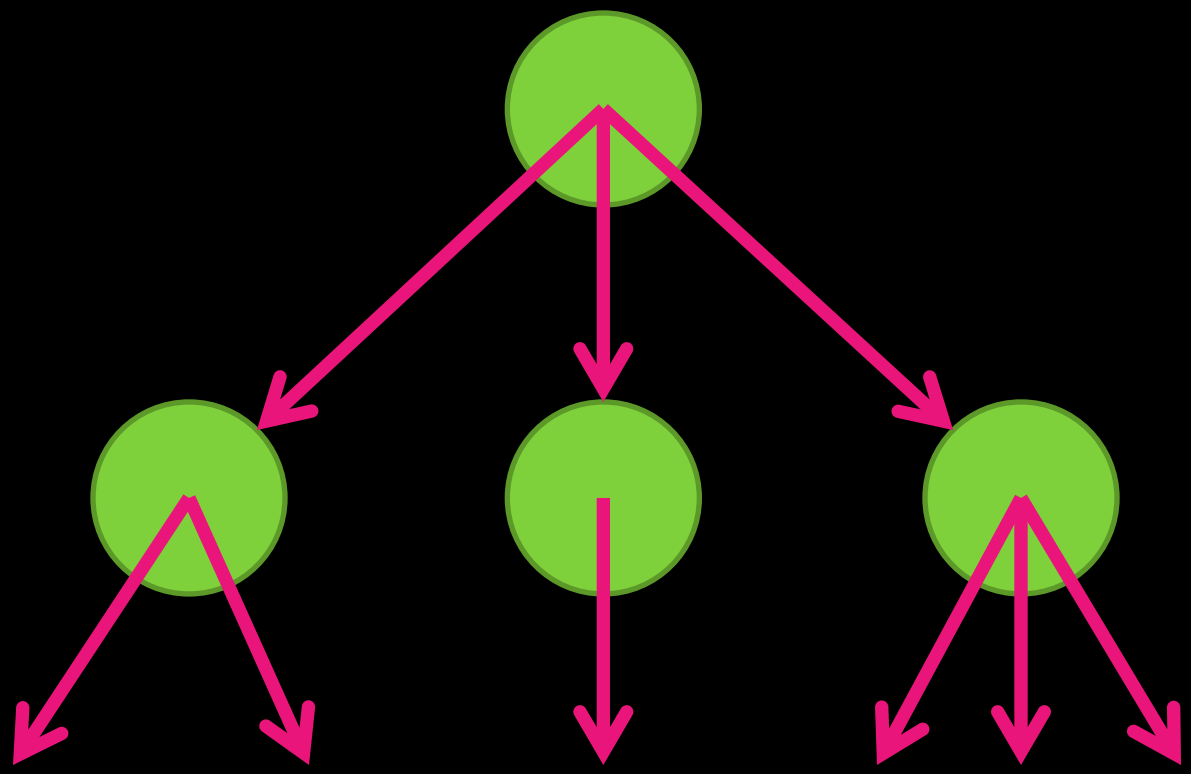
Tree Uses

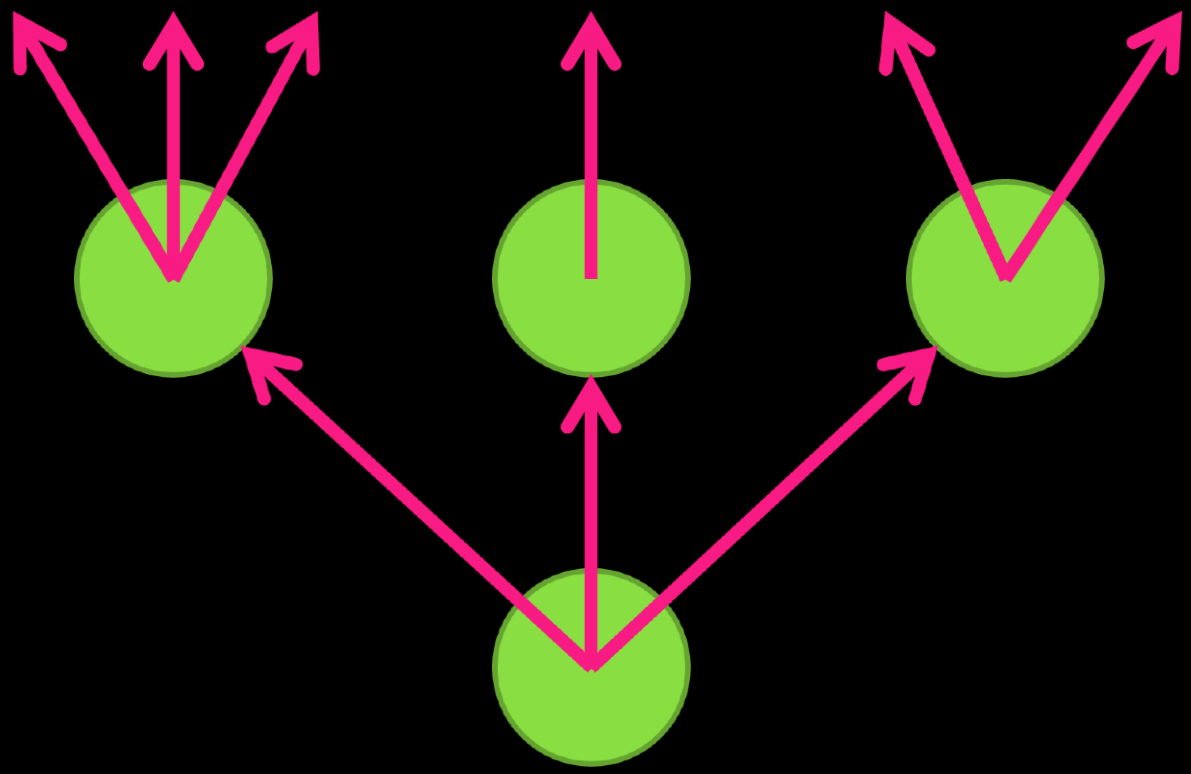
- Directory structure:
  - Children: everything inside a directory
  - Parent: directory containing stuff



Project 2

- File / folder compression
- Specs out this weekend
- Much more difficult than project 1
  - So start early!





Midterm 1 Post-mortem

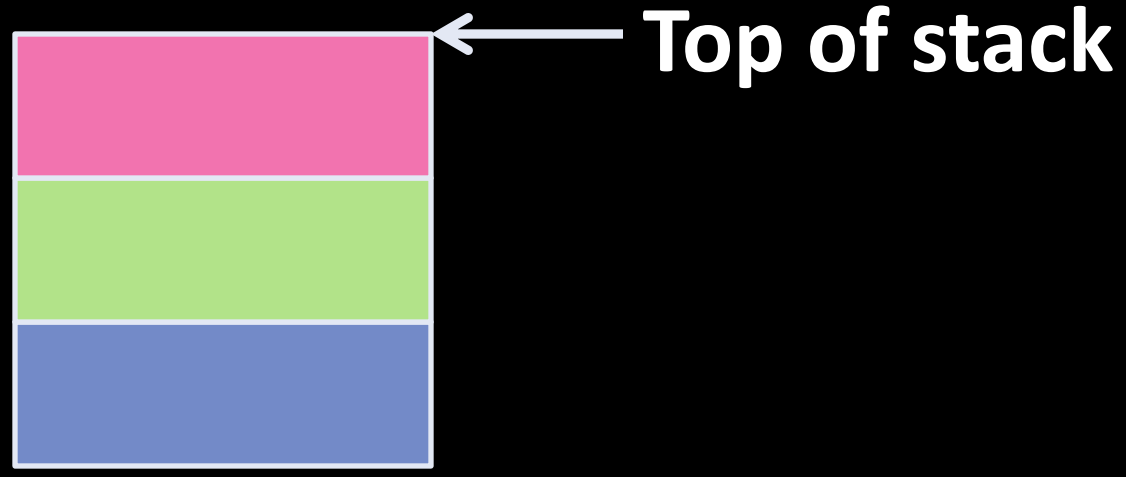
- Mean: 28/45, Standard deviation: 9

Upcoming Events

- Tonight: CS61BL dessert potluck / board game night!
  - 7pm to 11pm
- Friday: 61ABC Hackathon
  - 5pm to midnight
- Both in Wozniak Lounge (438 Soda)
- Free food!

Stack

- Last In, First Out (LIFO)
  - Most recent item added is next one removed
- Like dish tower at a buffet



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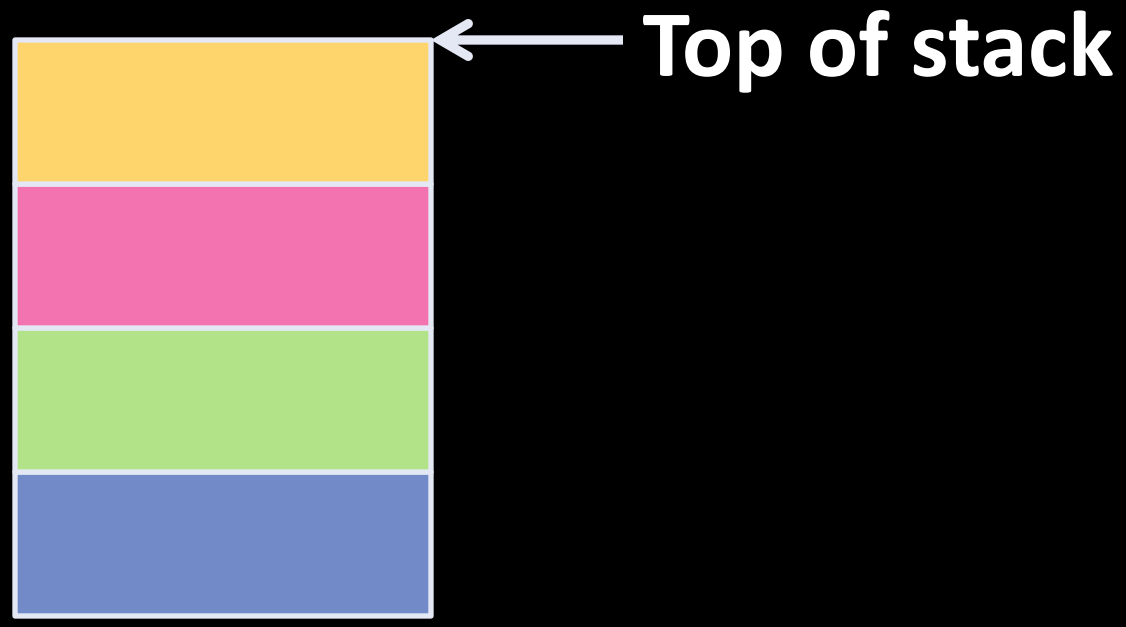

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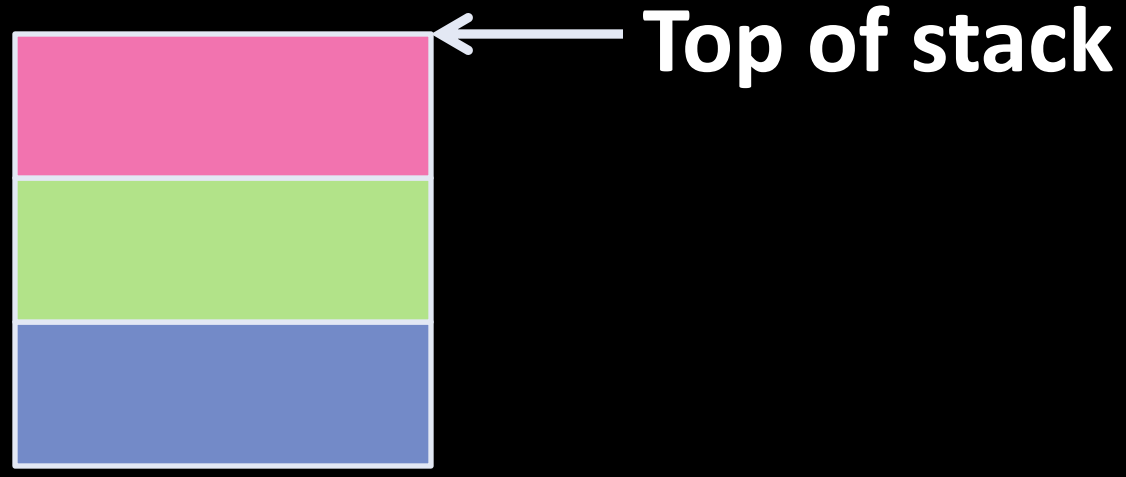

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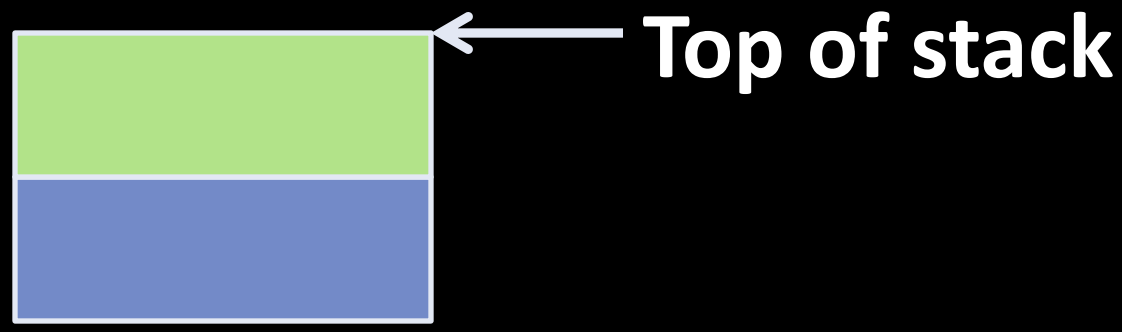


Stack Operations

- **`push`**: add item to front / top of stack
- **`pop`**: remove first item of stack
- **`peek`**: access first item of stack w/o returning it
- **`empty`**: check if stack has any items in it

Constructor Summary

Constructors

Constructor and Description
`Stack()` Creates an empty Stack.

Method Summary

All Methods | **Instance Methods** | **Concrete Methods**

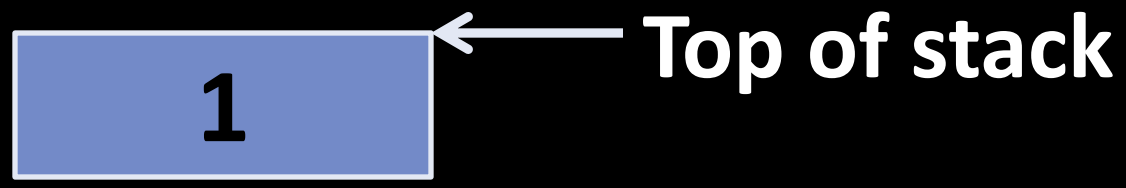
Modifier and Type	Method and Description
`boolean`	`empty()` Tests if this stack is empty.
`E`	`peek()` Looks at the object at the top of this stack without removing it from the stack.
`E`	`pop()` Removes the object at the top of this stack and returns that object as the value of this function.
`E`	`push(E item)` Pushes an item onto the top of this stack.

Stack Example 1

```
// start with empty stack
```

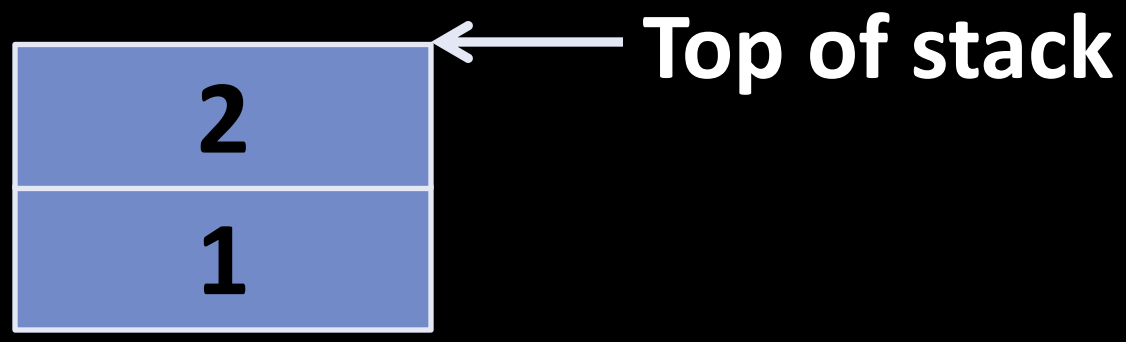
Stack Example 1

```
// start with empty stack

push(1)
```



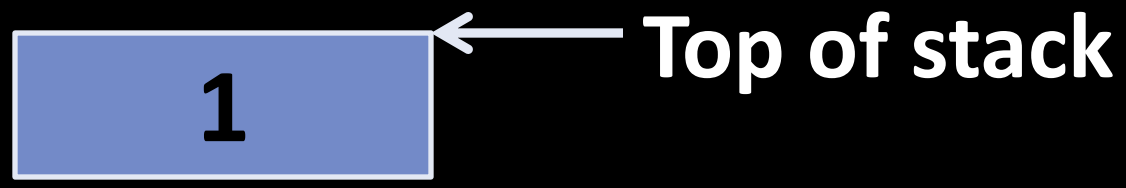
Stack Example 1

```
// start with empty stack

push(1)

push(2)
```



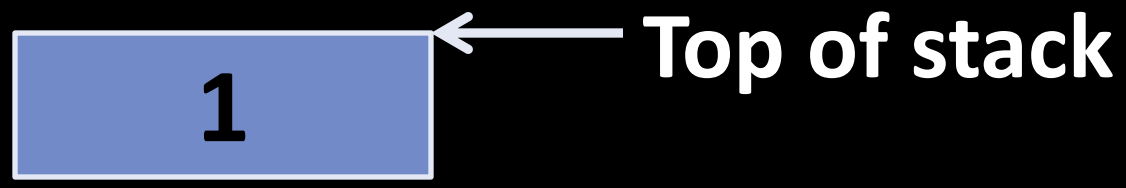
Stack Example 1

```
// start with empty stack
push(1)
push(2)
pop() // 2
```



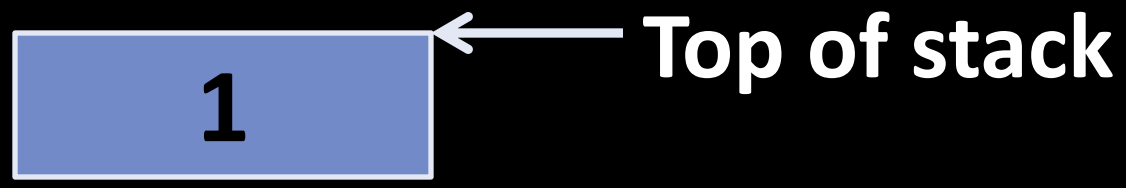
Stack Example 1

```
// start with empty stack
push(1)
push(2)
pop() // 2
peek() // 1
```



Stack Example 1

```
// start with empty stack
push(1)
push(2)
pop() // 2
peek() // 1
empty() // false
```



Stack Example 1

```
// start with empty stack
push(1)
push(2)
pop() // 2
peek() // 1
empty() // false
pop() // 1
```

← **Top of stack**

Stack Example 1

```
// start with empty stack
push(1)
push(2)
pop() // 2
peek() // 1
empty() // false
pop() // 1
empty() // true
```

← **Top of stack**

Stack Uses

- Call stack (function calls)
  - Make a function call: push stack frame onto stack
  - Exit a function: pop off stack
  - Stack overflow: too many stack frames on stack

Stack Uses

- Parenthesis matching
  - [1 + (2 * 3) / (4 – 5)]
- HTML tag matching
  - `<a> stuff </a>`
- LaTeX `\begin{} \end{}` matching

Stack Example 2

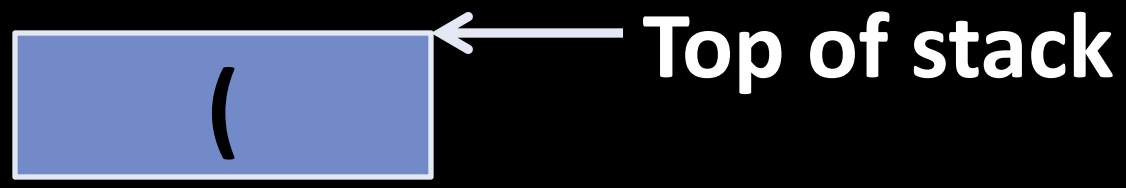
({ [ ] } ( ))

```
// start with empty stack
```

Stack Example 2

({ [ ] } ( ))

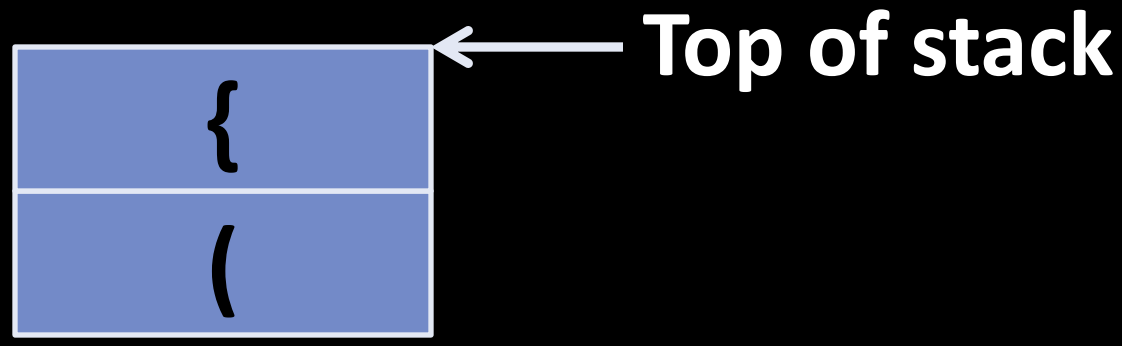
```
// start with empty stack
push (
```



Stack Example 2

({ [ ] } ( ))

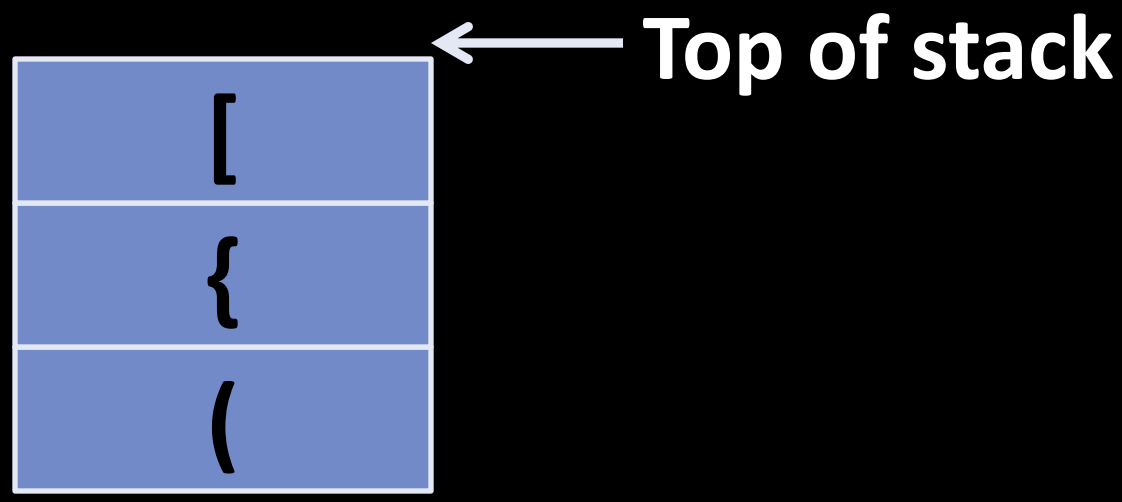
```
// start with empty stack
push (
push {
```



Stack Example 2

({ [ ] } ( ))

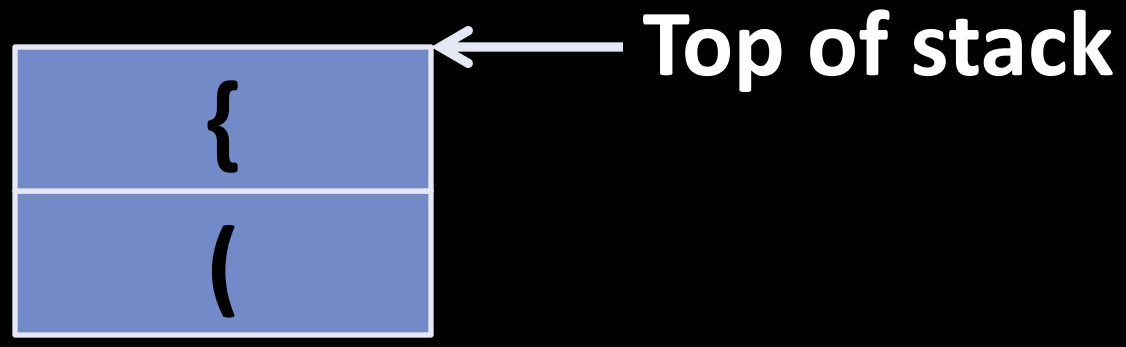
```
// start with empty stack
push (
push {
push [
```



Stack Example 2

({ [ ] } ( ))

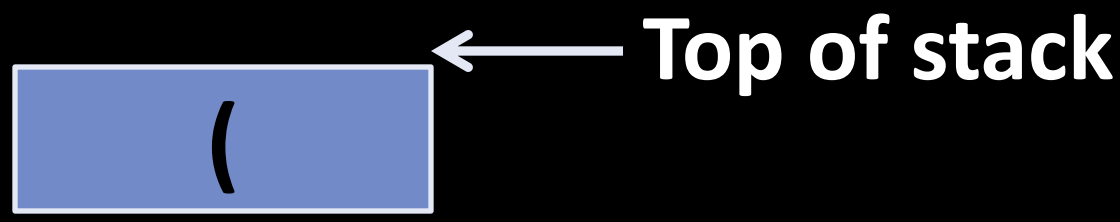
```
// start with empty stack
push (
push {
push [
pop ]
```



Stack Example 2

({ [ ] } ( ))

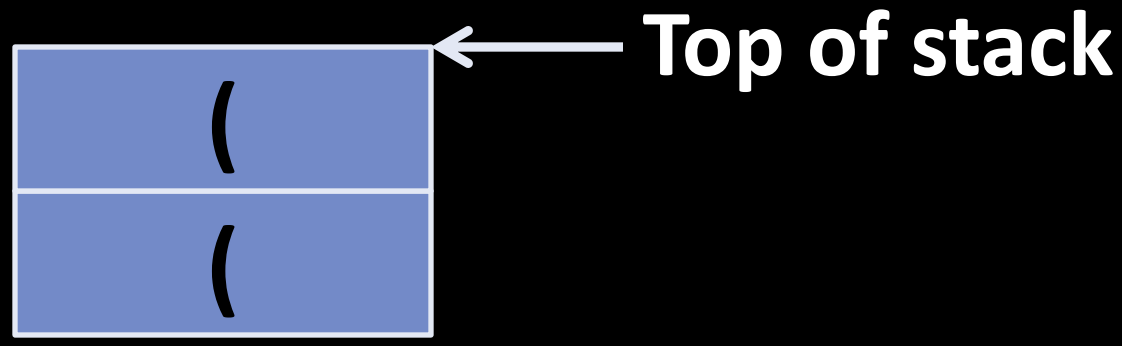
```
// start with empty stack
push (
push {
push [
pop ]
pop }
```



Stack Example 2

({ [ ] } ( ))

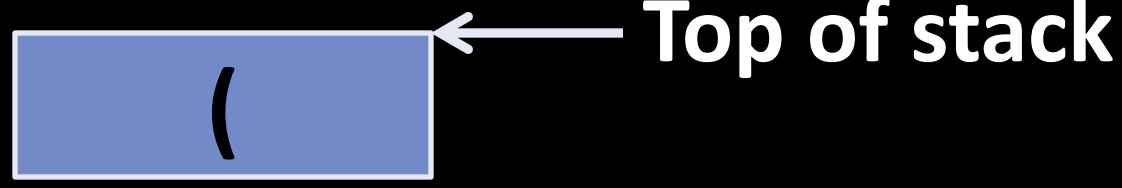
```
// start with empty stack
push (
push {
push [
pop ]
pop }
push (
```



Stack Example 2

({ [ ] } ( ))

```
// start with empty stack
push (
push {
push [
pop ]
pop }
push (
pop )
```



Stack Example 2

({ [ ] } ( ))

```
// start with empty stack
push (
push {
push [
pop ]
pop }
push (
pop )
pop )
```

← Top of stack

Stack Example 3

({] [} ( ))

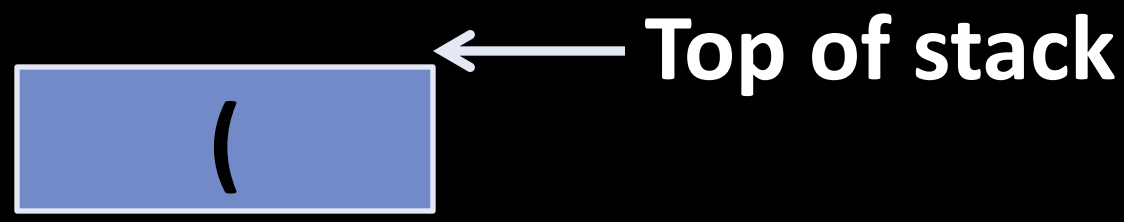
```
// start with empty stack
```

← Top of stack

Stack Example 3

({] [} ( ))

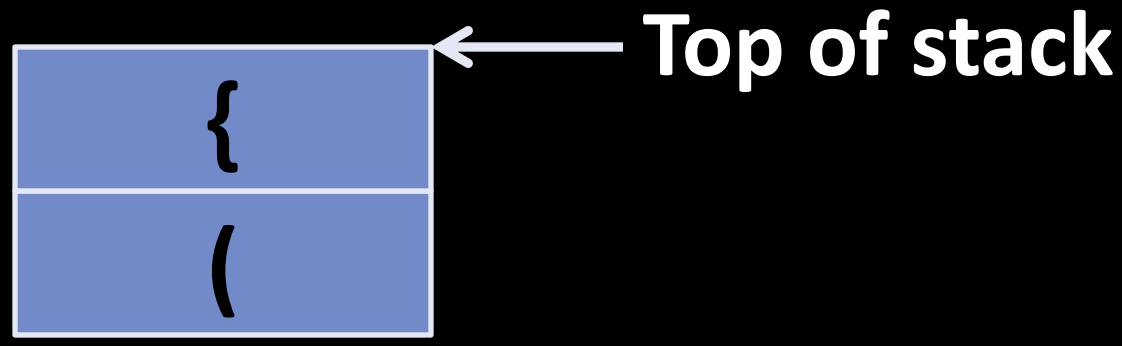
```
// start with empty stack
push (
```



Stack Example 3

({] [} ( ))

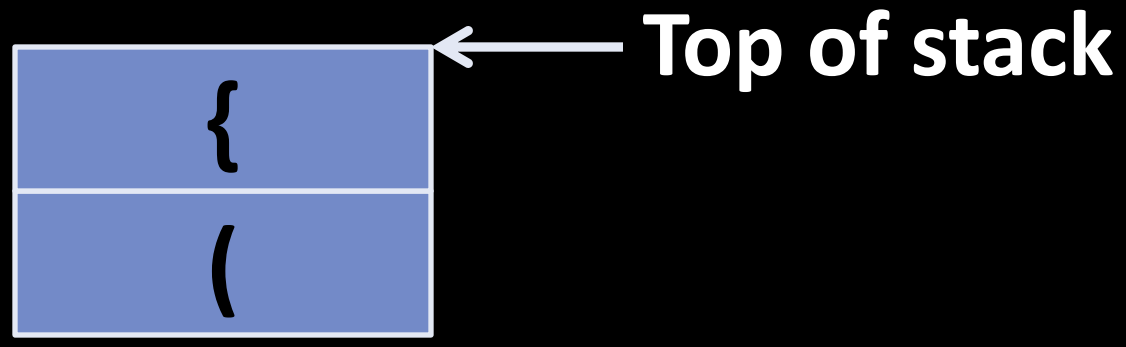
```
// start with empty stack
push (
push {
```



Stack Example 3

({] [} ( ))

```
// start with empty stack
push (
push {
// error
```



Implementing a Stack

- Linked list
  - Push: append to front
  - Pop: remove from front
- Array list
  - Keep track of last element's index
  - Push: add to end of list
  - Pop: remove from end of list

Queues

- First In, First Out (FIFO)
  - Element that has been in queue for the longest gets out first
- Like people standing in a line

Queues

- First In, First Out (FIFO)
  - Element that has been in queue for the longest gets out first
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Queues

- First In, First Out (FIFO)
  - Element that has been in queue for the longest gets out first
- Like people standing in a line

Queues

- First In, First Out (FIFO)
  - Element that has been in queue for the longest gets out first
- Like people standing in a line

Operations

- add / enqueue: add element to back of queue
- peek / element: access first element w/o removing it
- poll / remove / dequeue: remove first element and return it

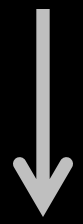
Implementing Queues

- Linked list
  - Keep a pointer to end of linked list
  - Enqueue: add to end
  - Dequeue: remove from front
  - (or vice versa)

Queue Example

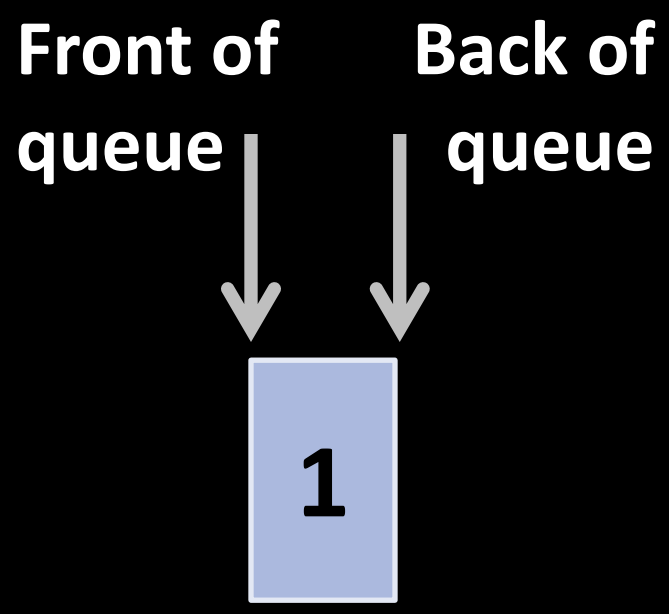
```
(start with empty queue)
```

Front / back of queue



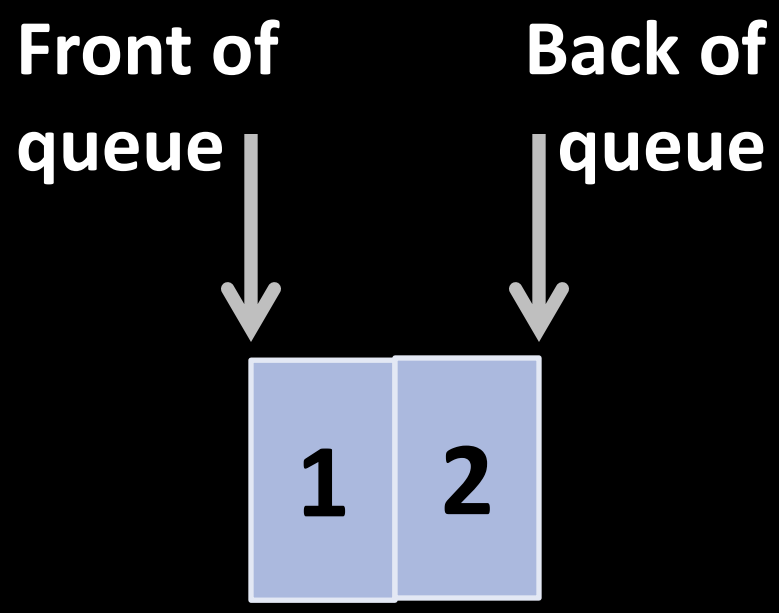
Queue Example

```
(start with empty queue)
enqueue(1)
```



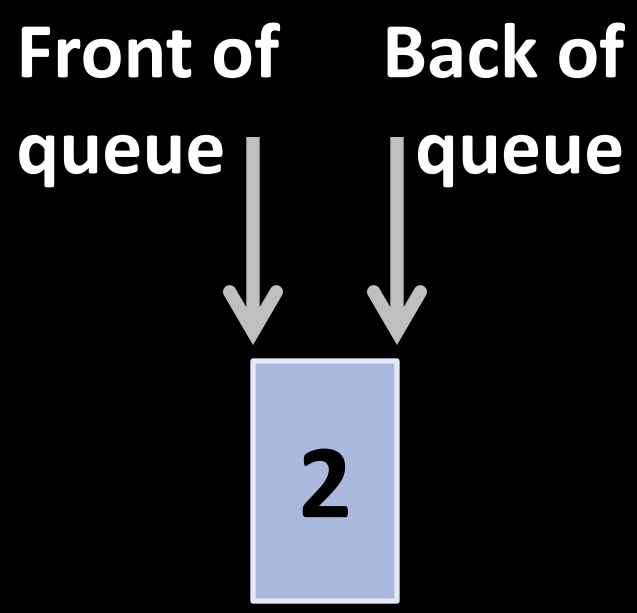
Queue Example

```
(start with empty queue)
enqueue(1)
enqueue(2)
```



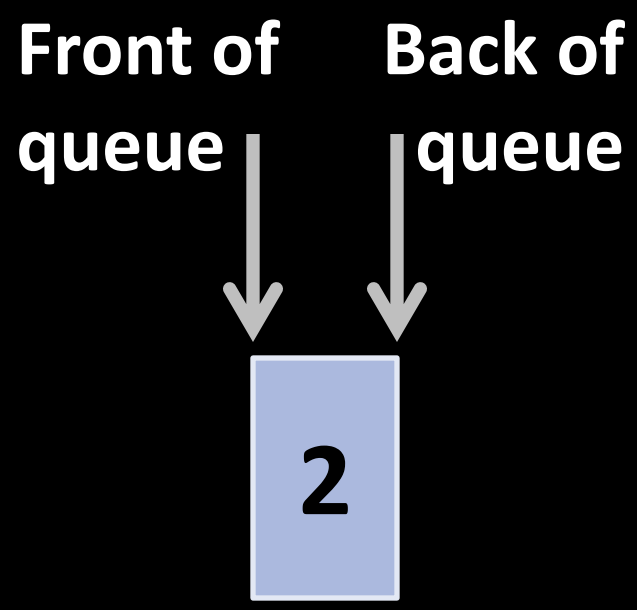
Queue Example

```
(start with empty queue)
enqueue(1)
enqueue(2)
dequeue() // 1
```



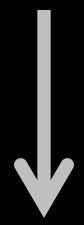
Queue Example

```
(start with empty queue)
enqueue(1)
enqueue(2)
dequeue() // 1
peek() // 2
```



Queue Example

```
(start with empty queue)
enqueue(1)
enqueue(2)
dequeue() // 1
peek() // 2
dequeue() // 2
```



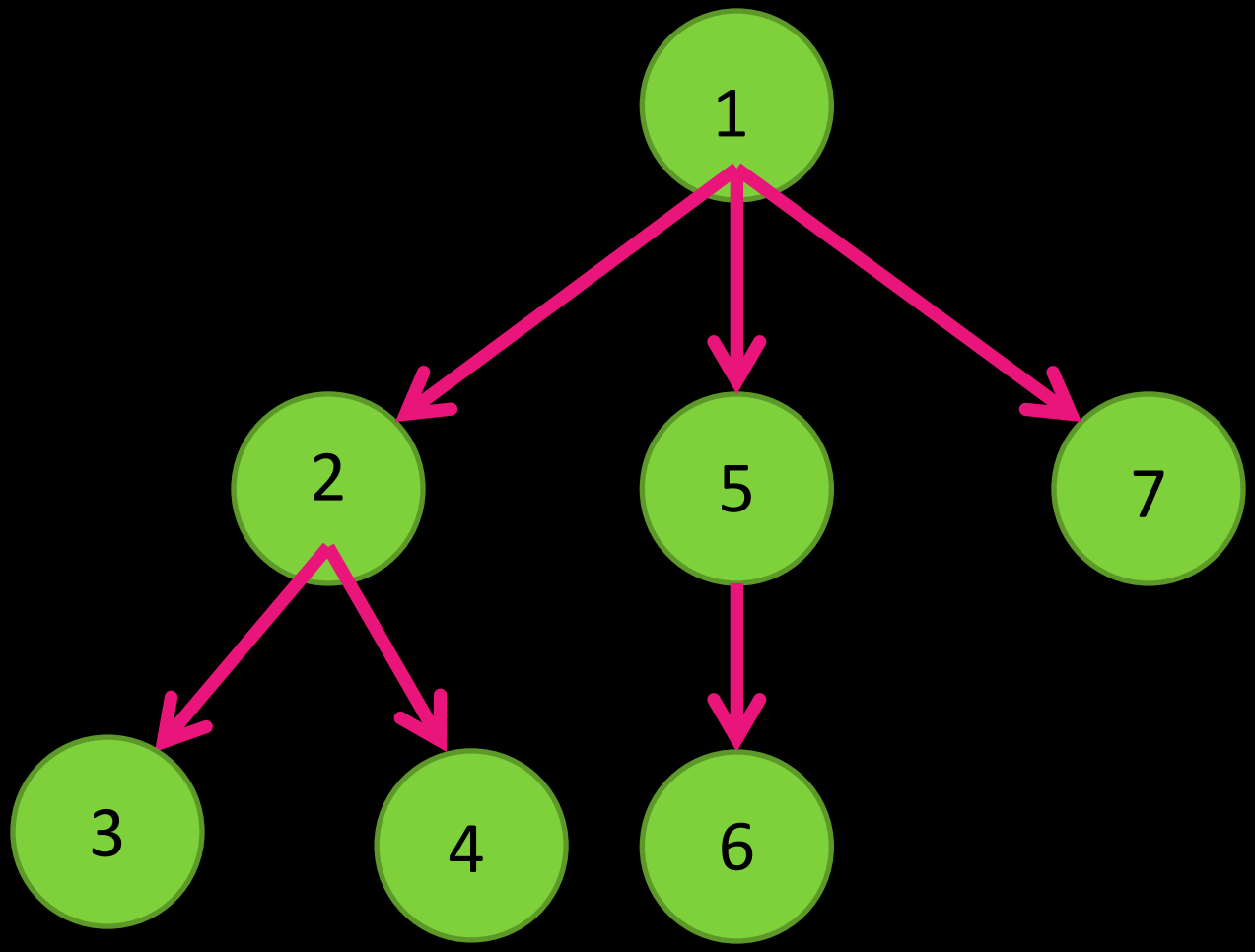
Front / back of queue

Tree Traversal

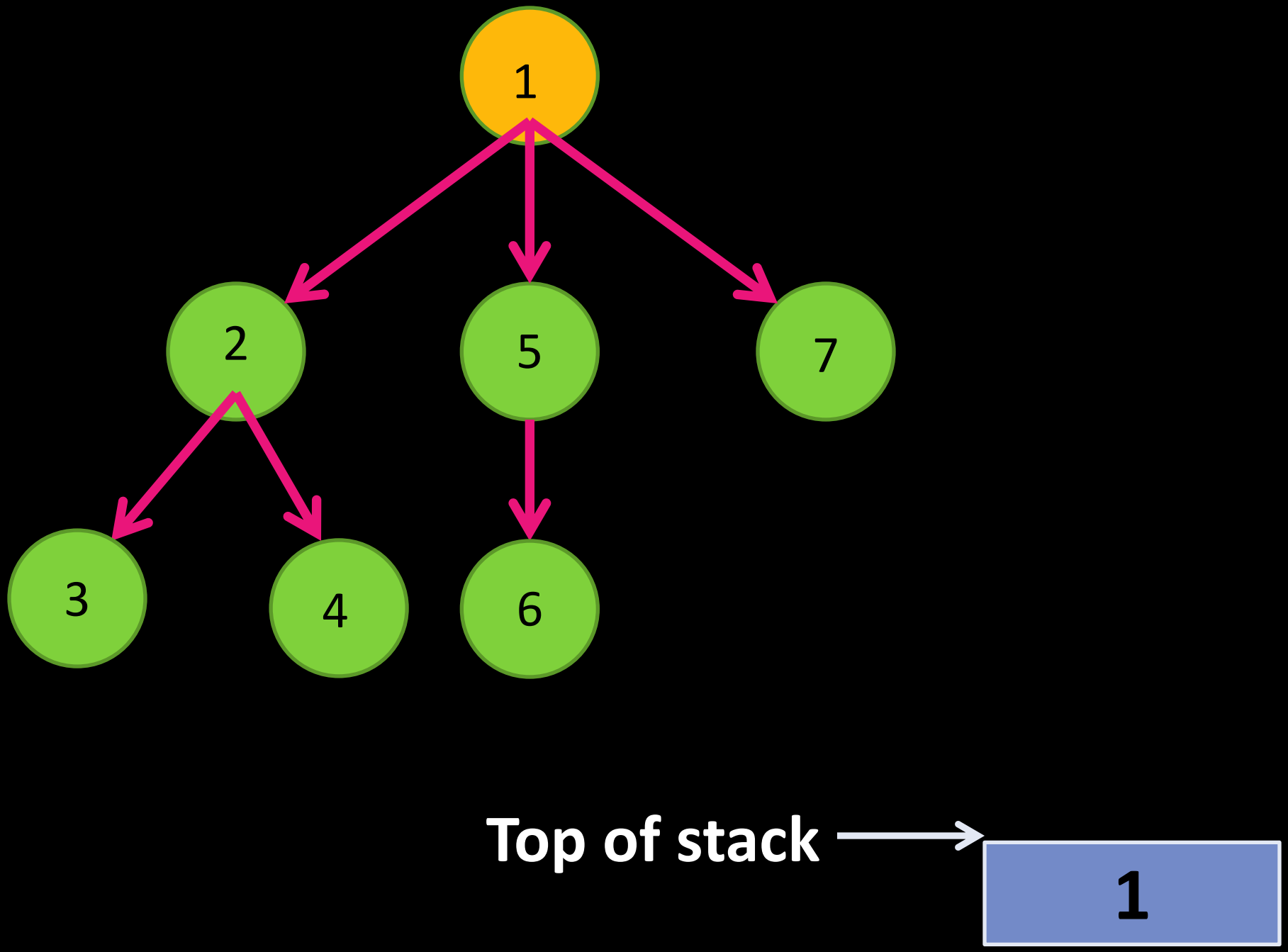
- Fringe: group of nodes to be visited next
- Algorithm:
  - Add root to fringe
  - While fringe is not empty:
    - Remove and process a node from fringe
    - Add its children to the fringe

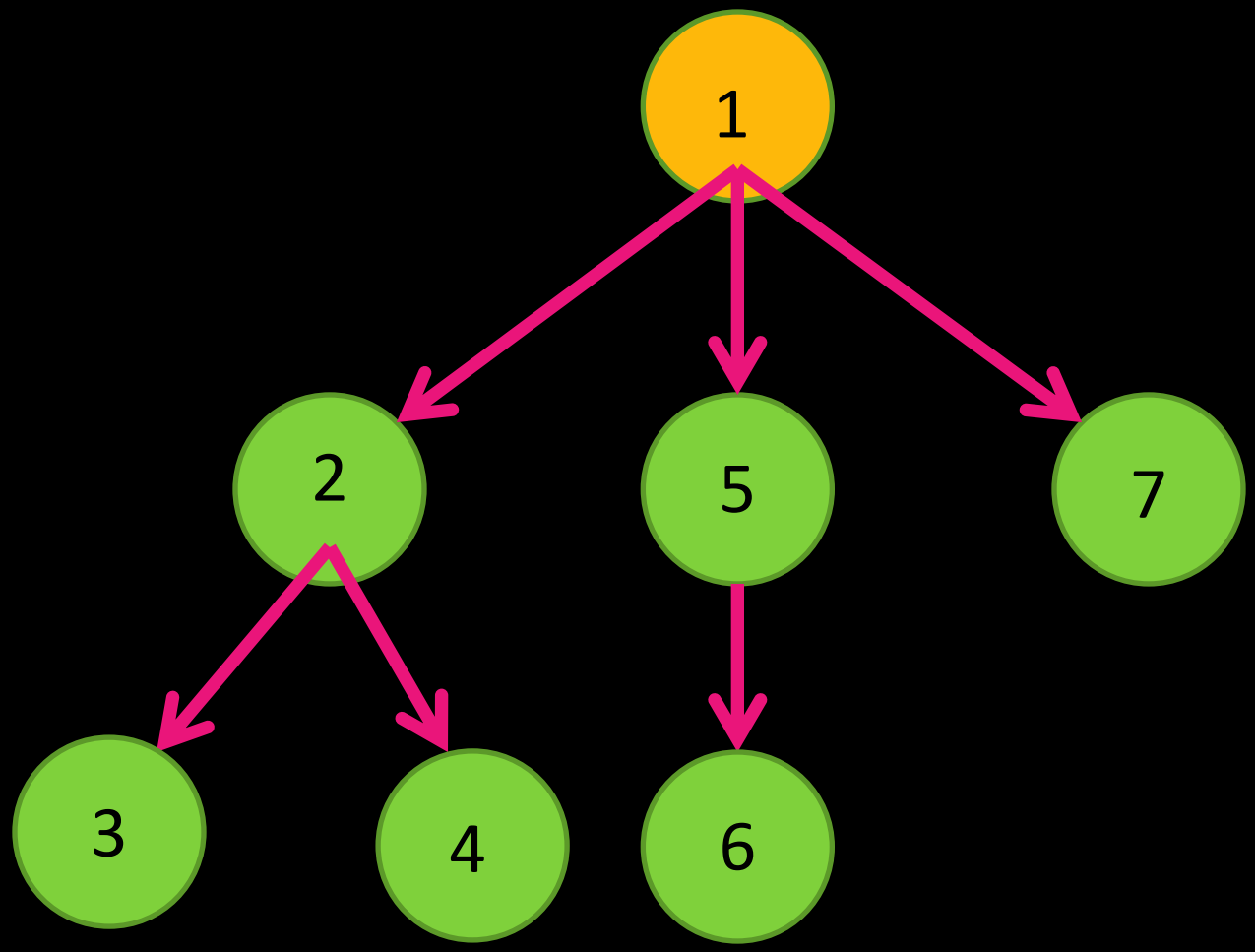
Tree Traversals: Depth First Search

- Node order:
  - Start at root
  - Go as deep as you can (choosing children arbitrarily)
  - Backtrack to unvisited nodes
- Use stack for fringe

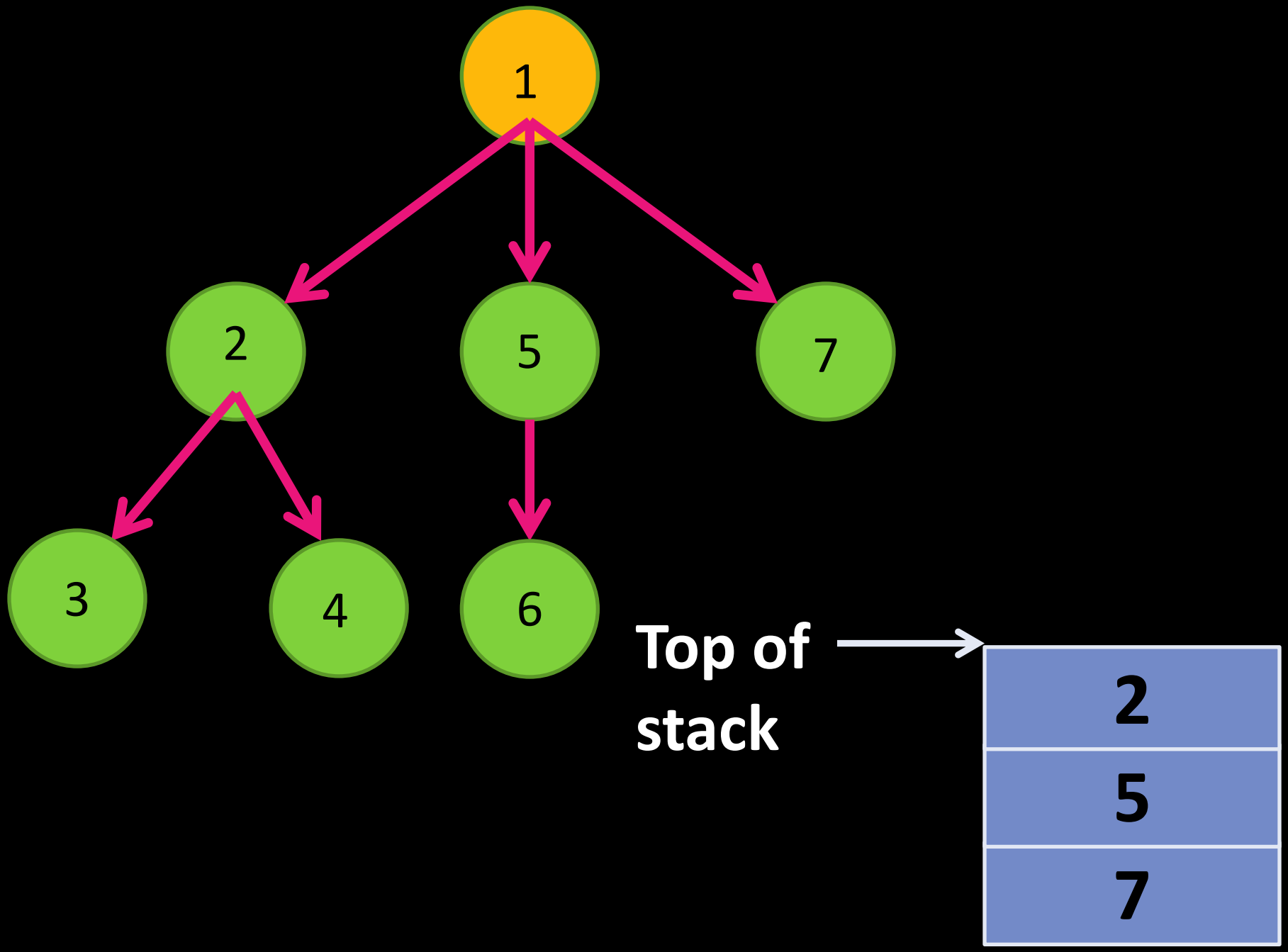


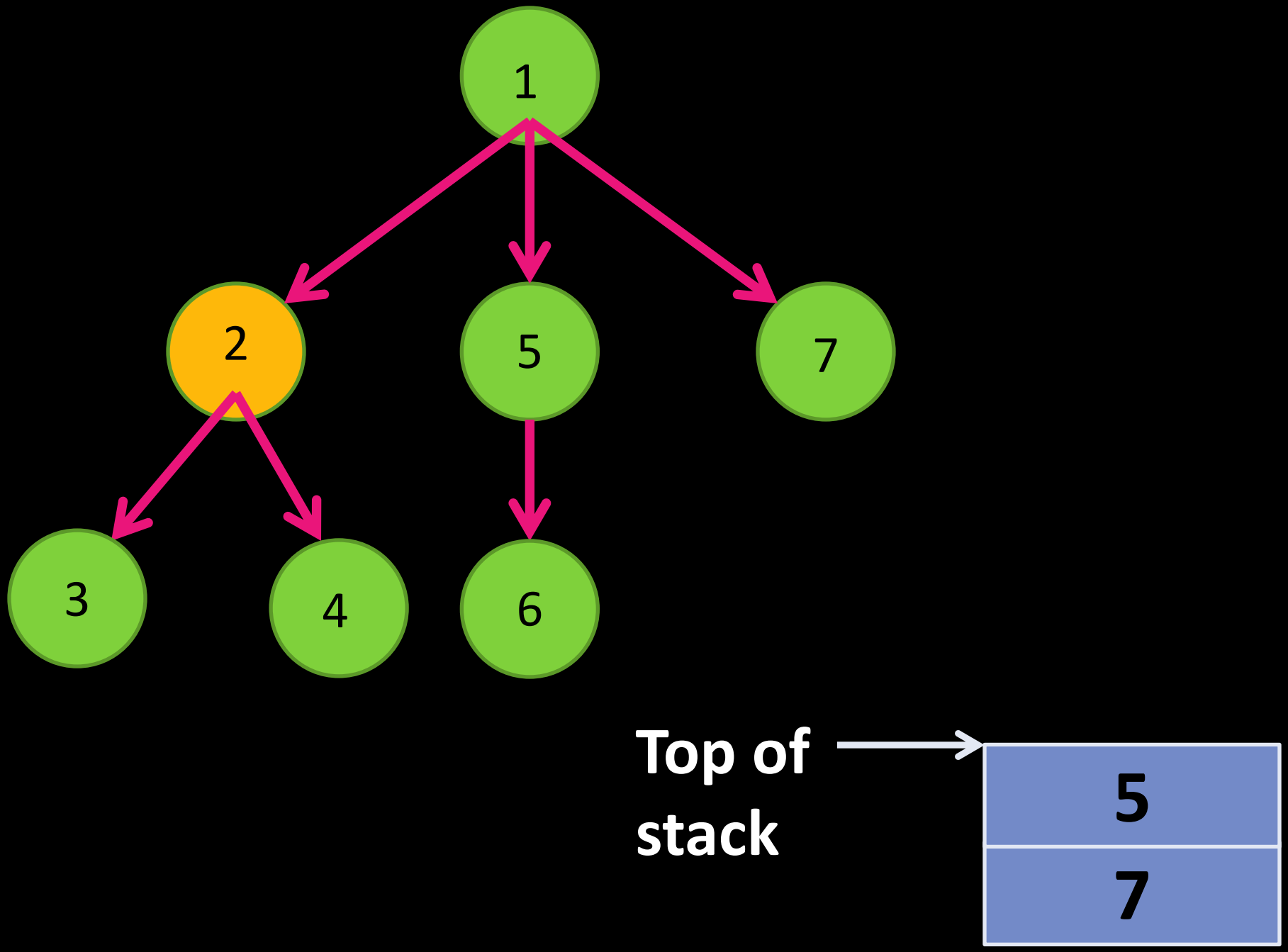
Top of stack ⟶

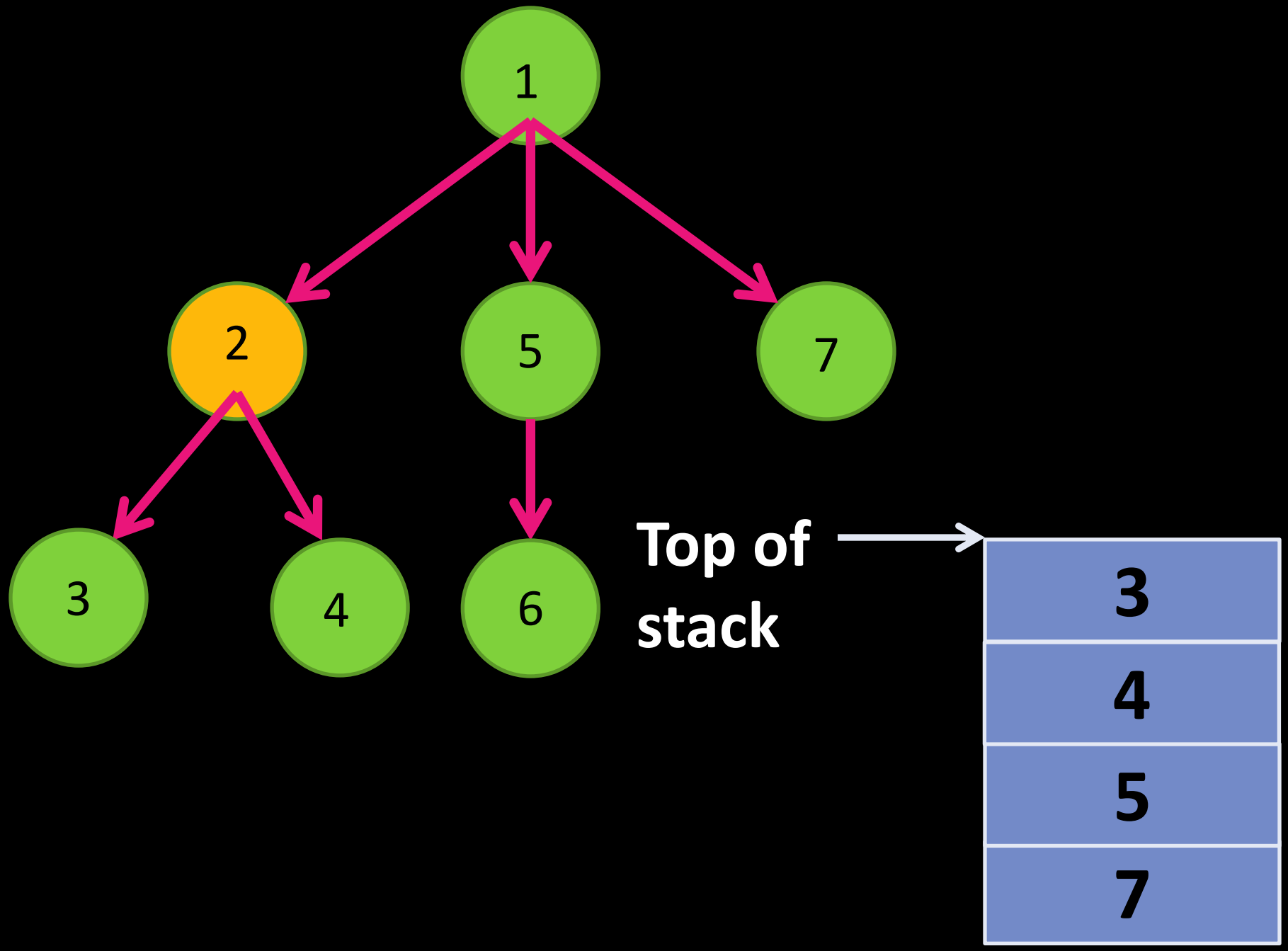

1
2
5
7
3
4
6
Top of stack
1

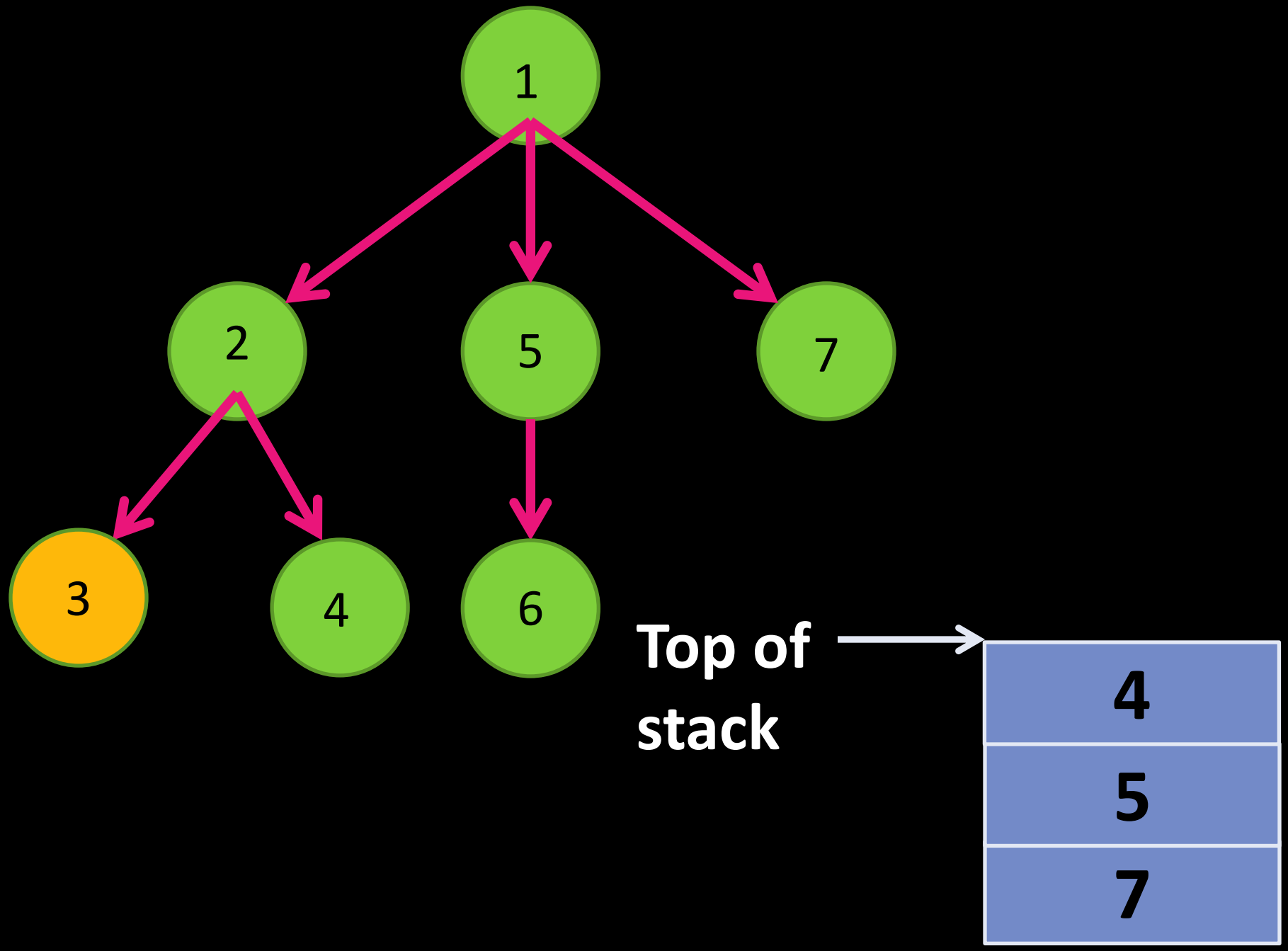


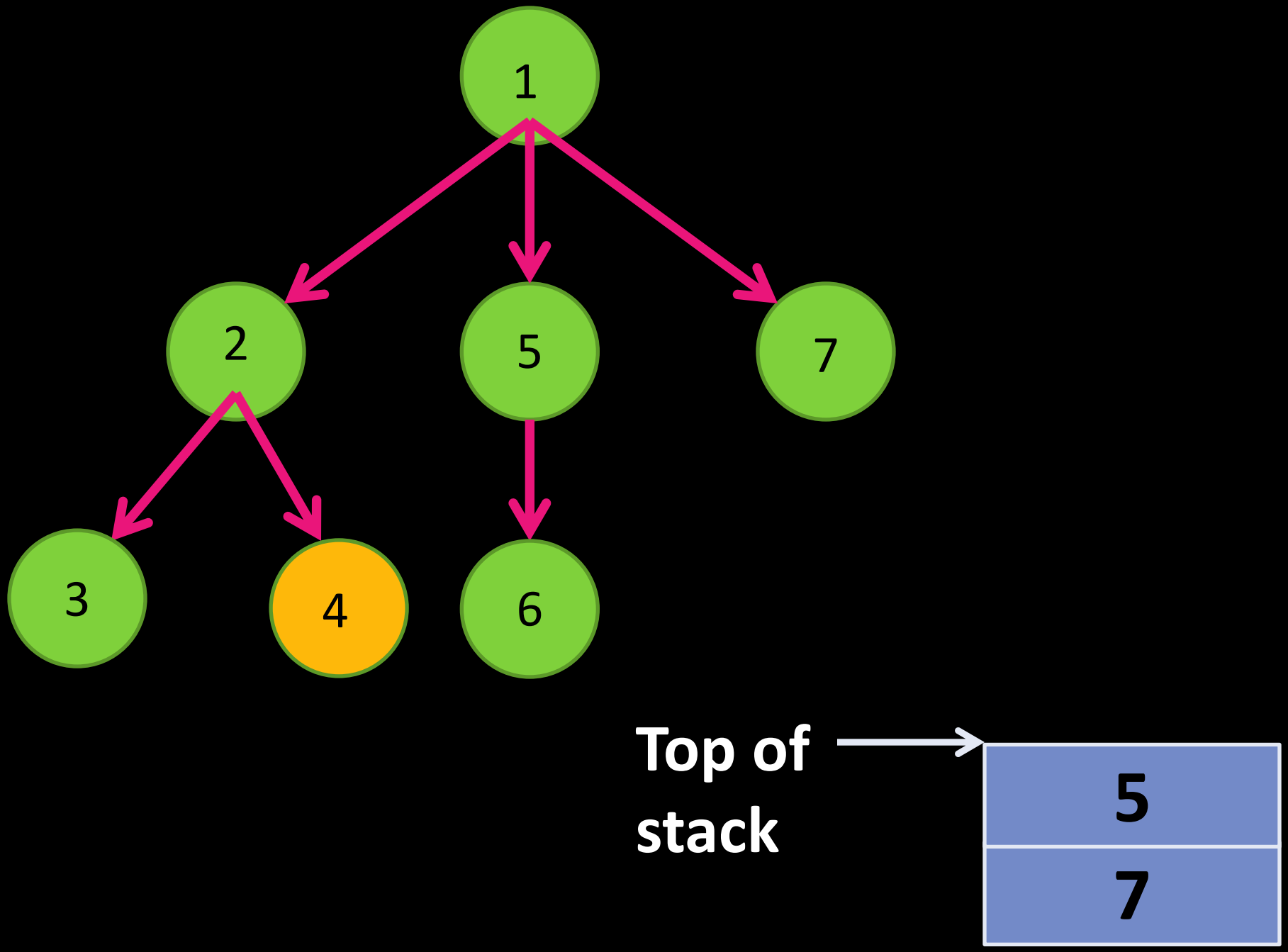
Top of stack ⟶

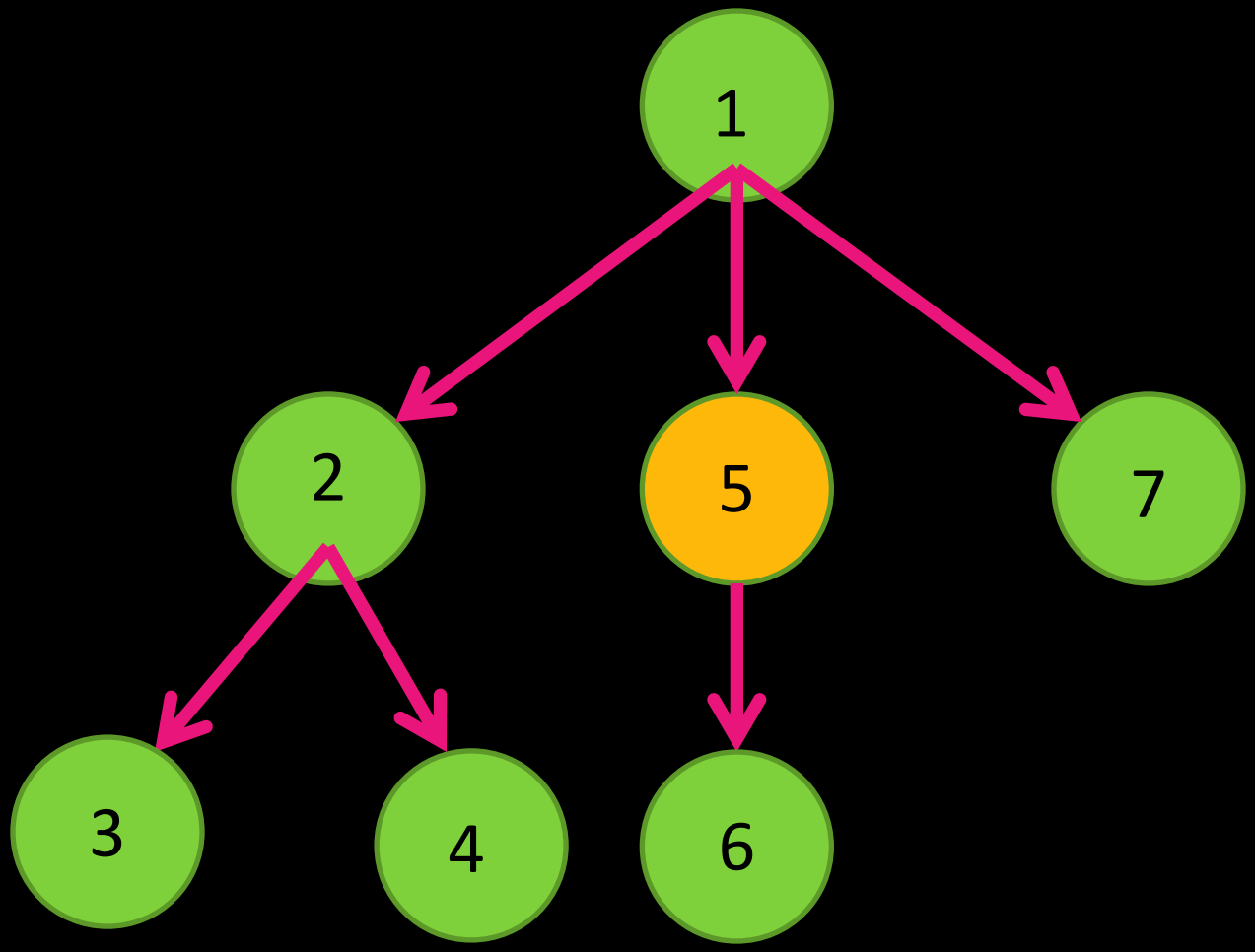

1
2
5
7
3
4
6
Top of stack
2
5
7


1
2
5
7
3
4
6
Top of stack
5
7

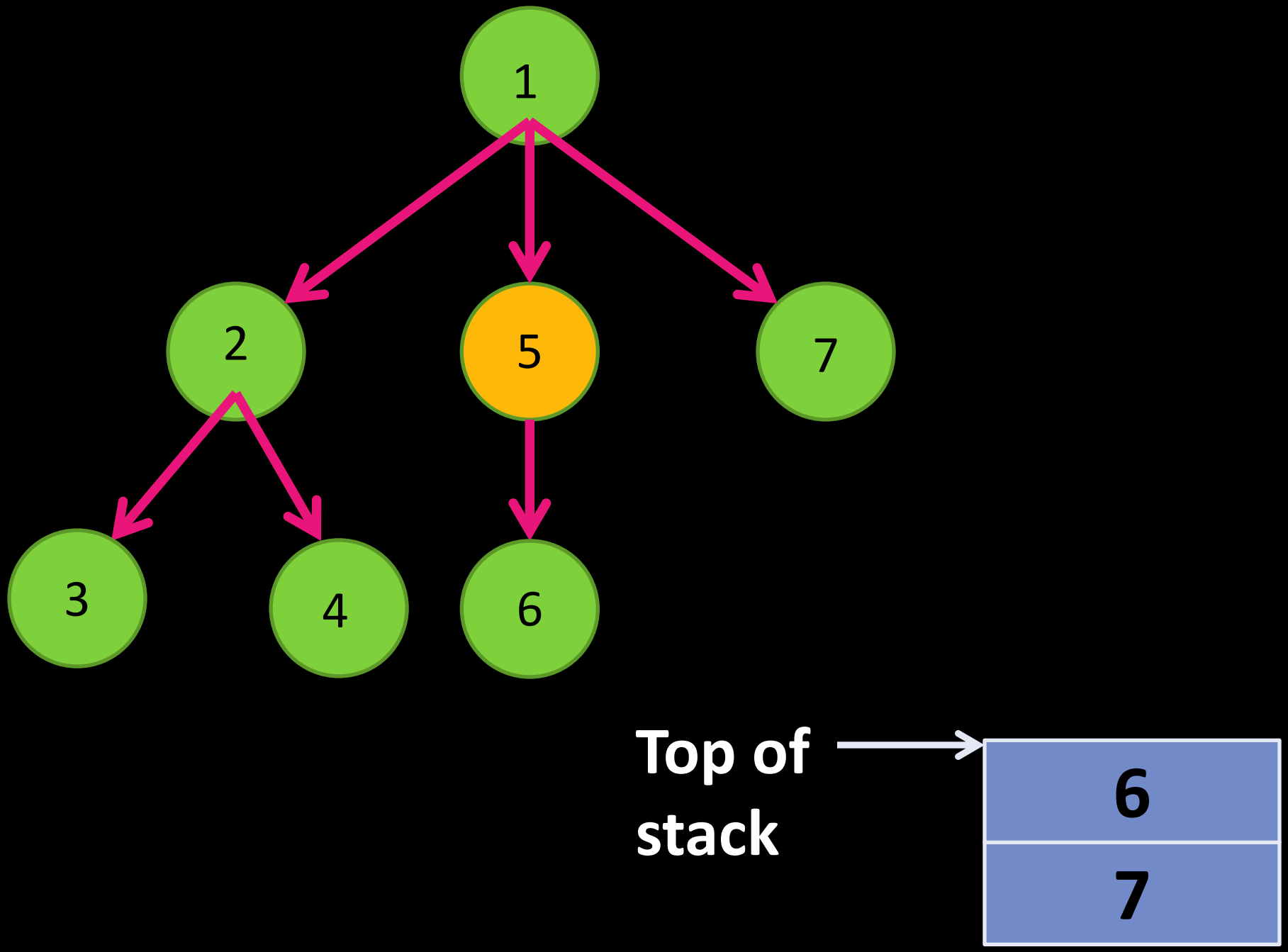

1
2
5
7
3
4
6
Top of stack
3
4
5
7

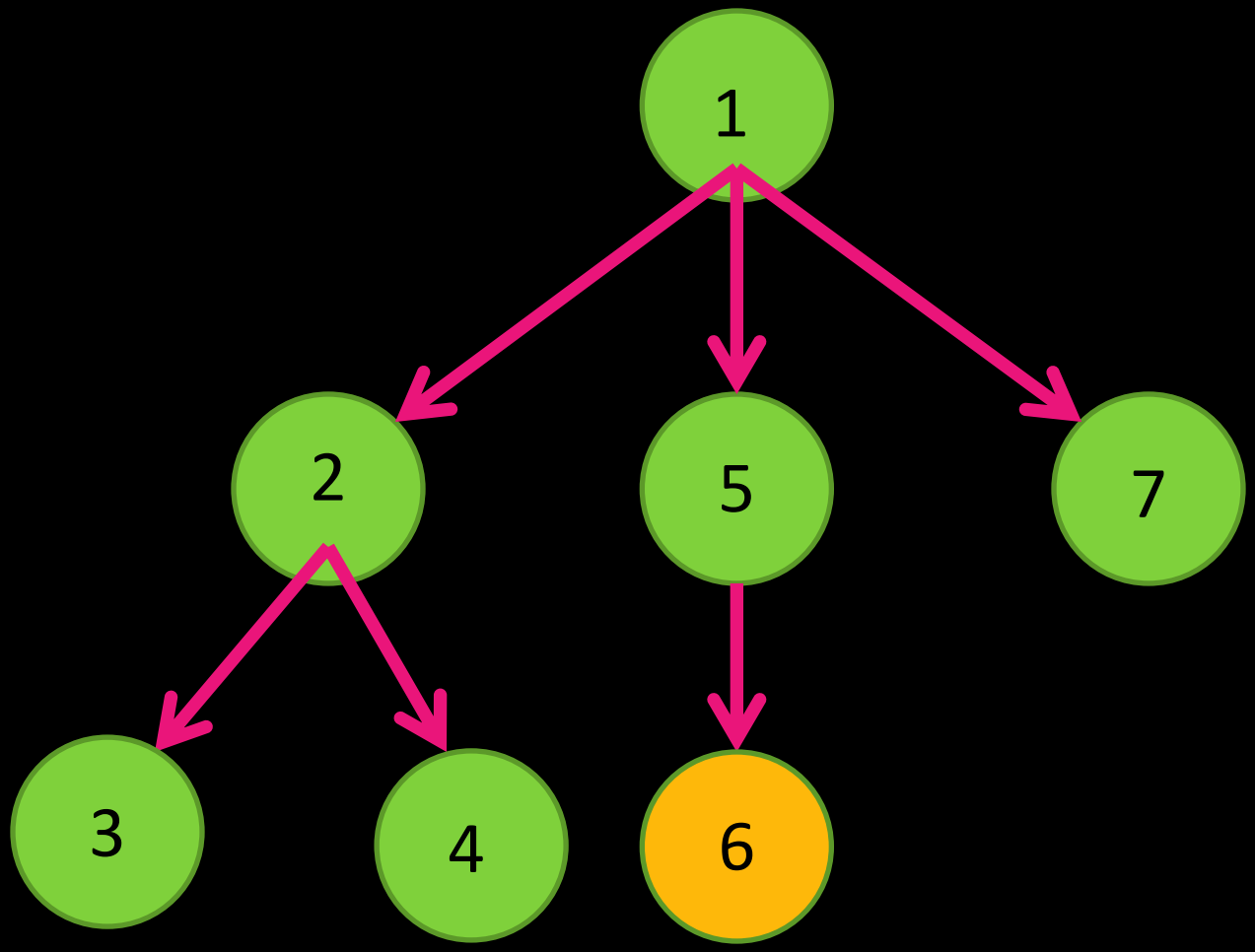

1
2
5
7
3
4
6
Top of stack
4
5
7


1
2
5
7
3
4
6
Top of stack
5
7

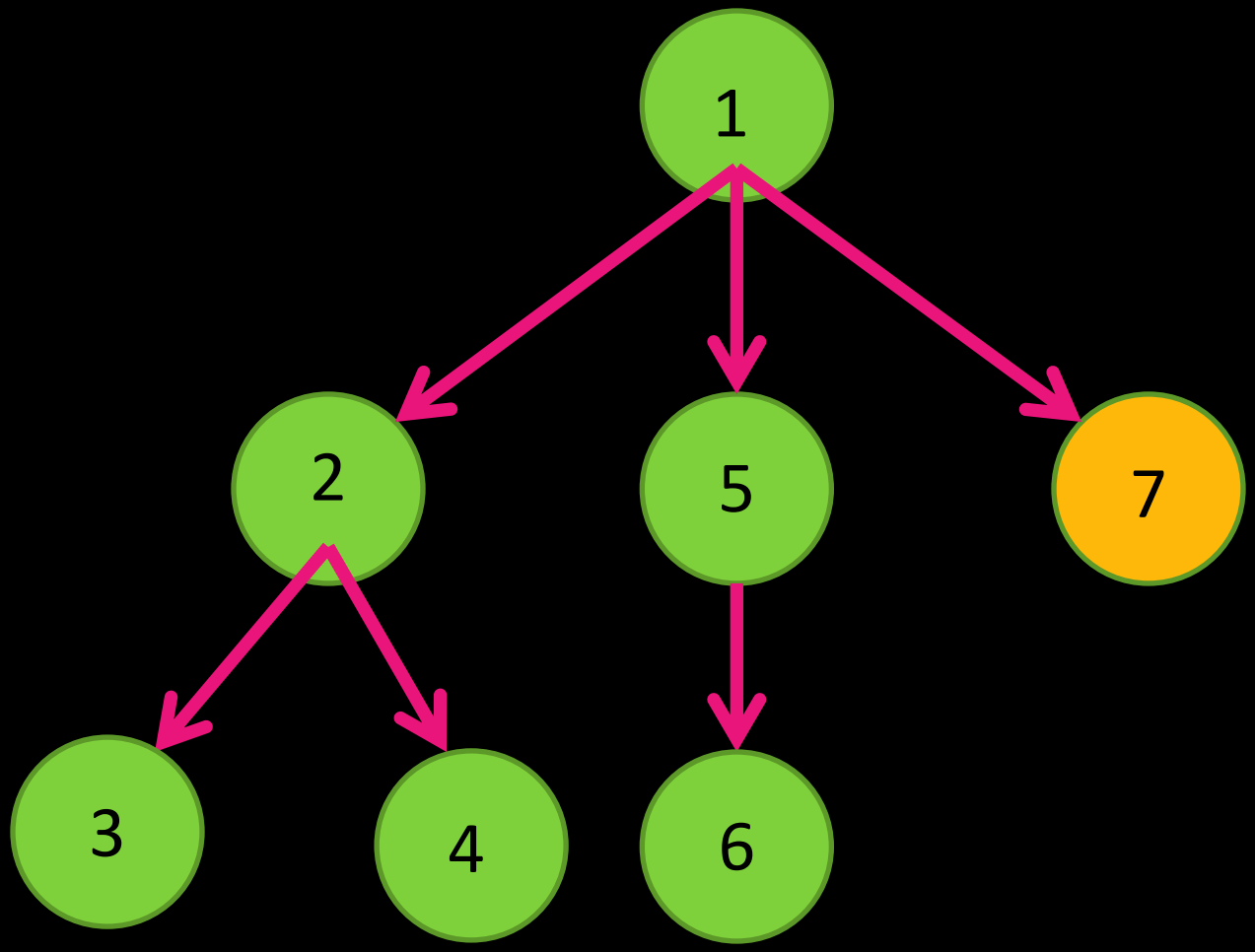


Top of stack

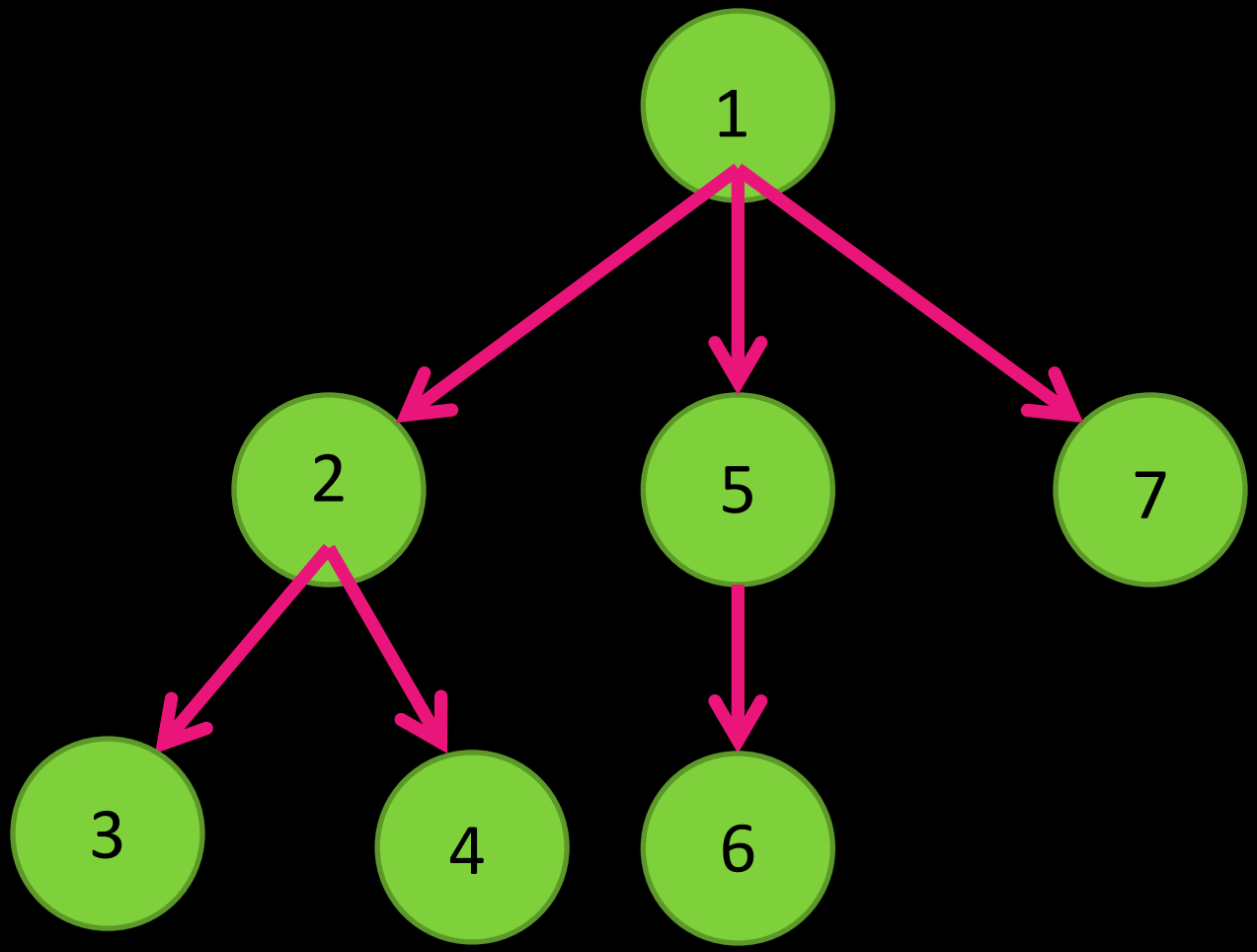

1
2
5
7
3
4
6
Top of stack
6
7



Top of stack → **7**



Top of stack →



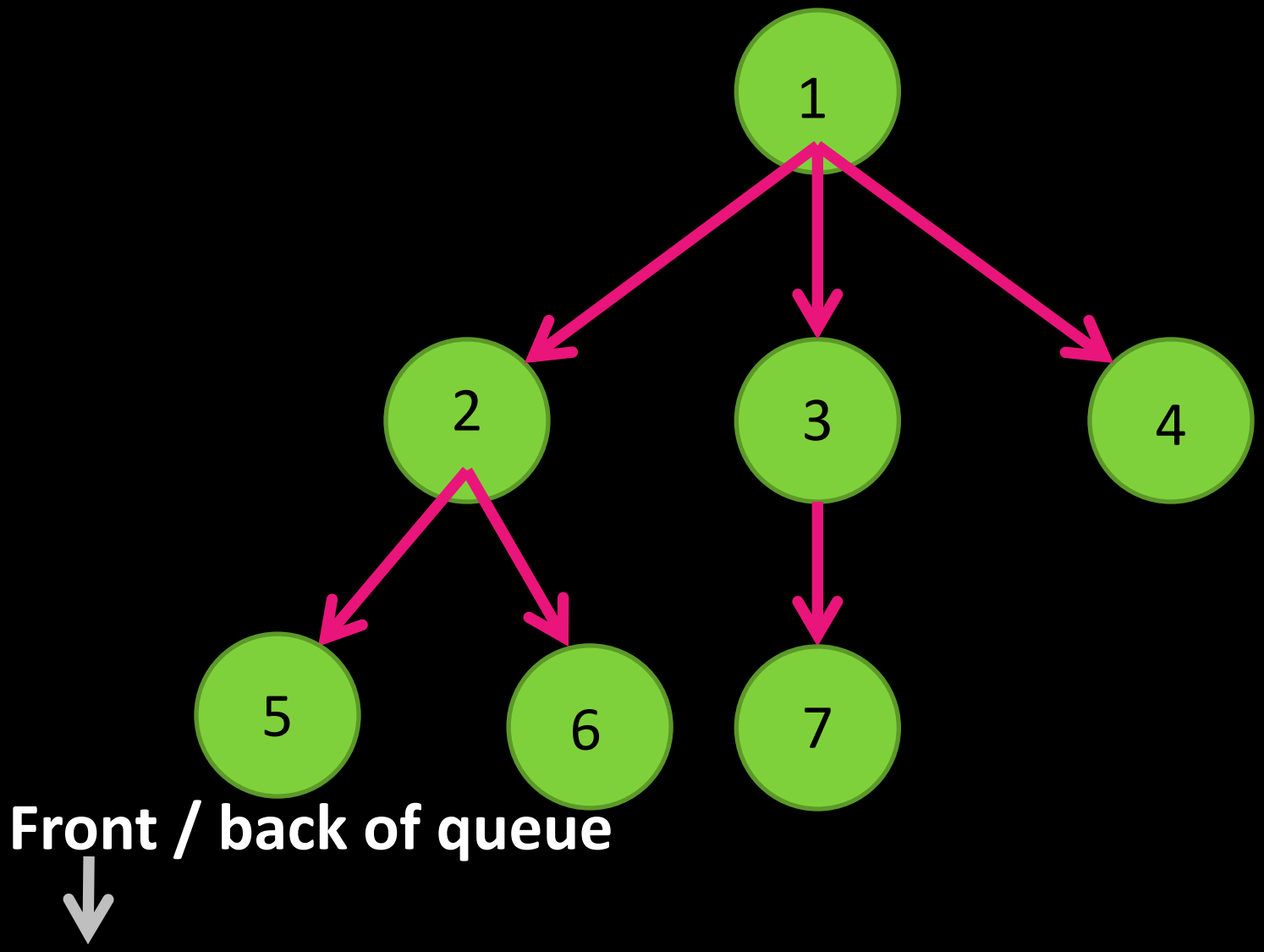
Top of stack →

Tree Traversal

- Fringe: group of nodes to be visited next
- Algorithm:
  - Add root to fringe
  - While fringe is not empty:
    - Remove and process a node from fringe
    - Add its children to the fringe

Tree Traversal: Breadth First Search

- Node order:
  - Start at root
  - Visit all nodes at next depth level
  - Repeat
- Use queue for fringe


1
2
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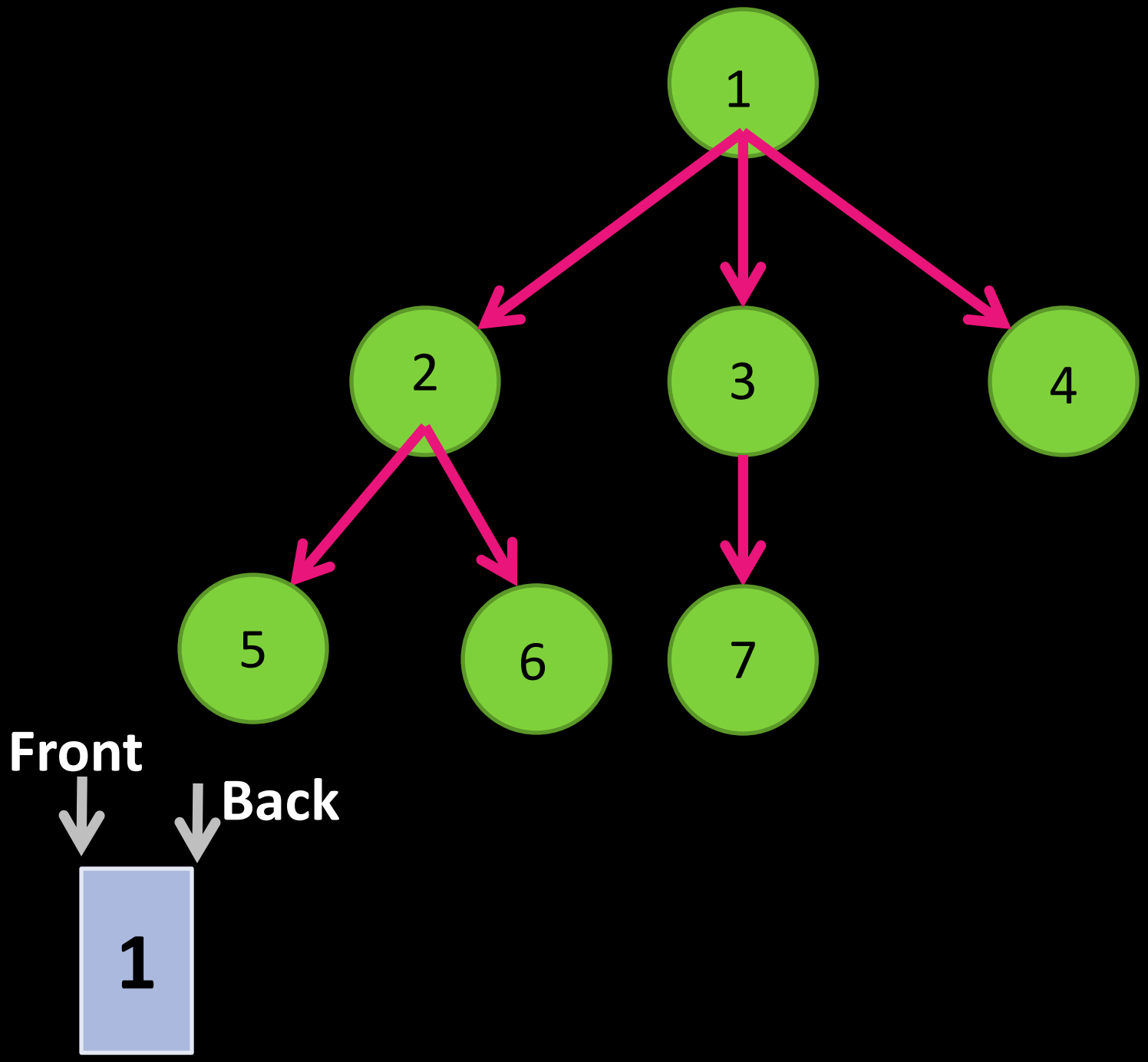
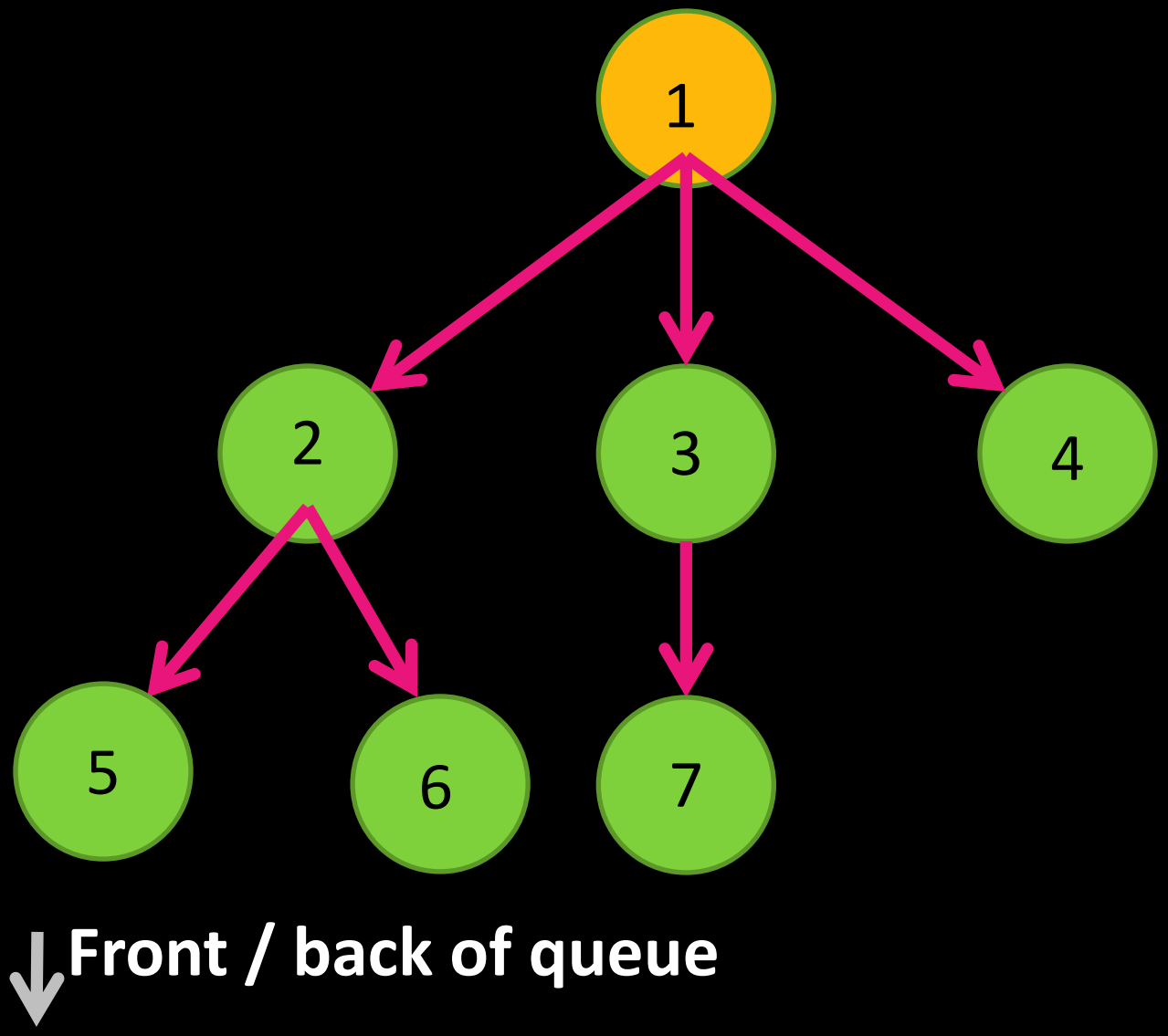
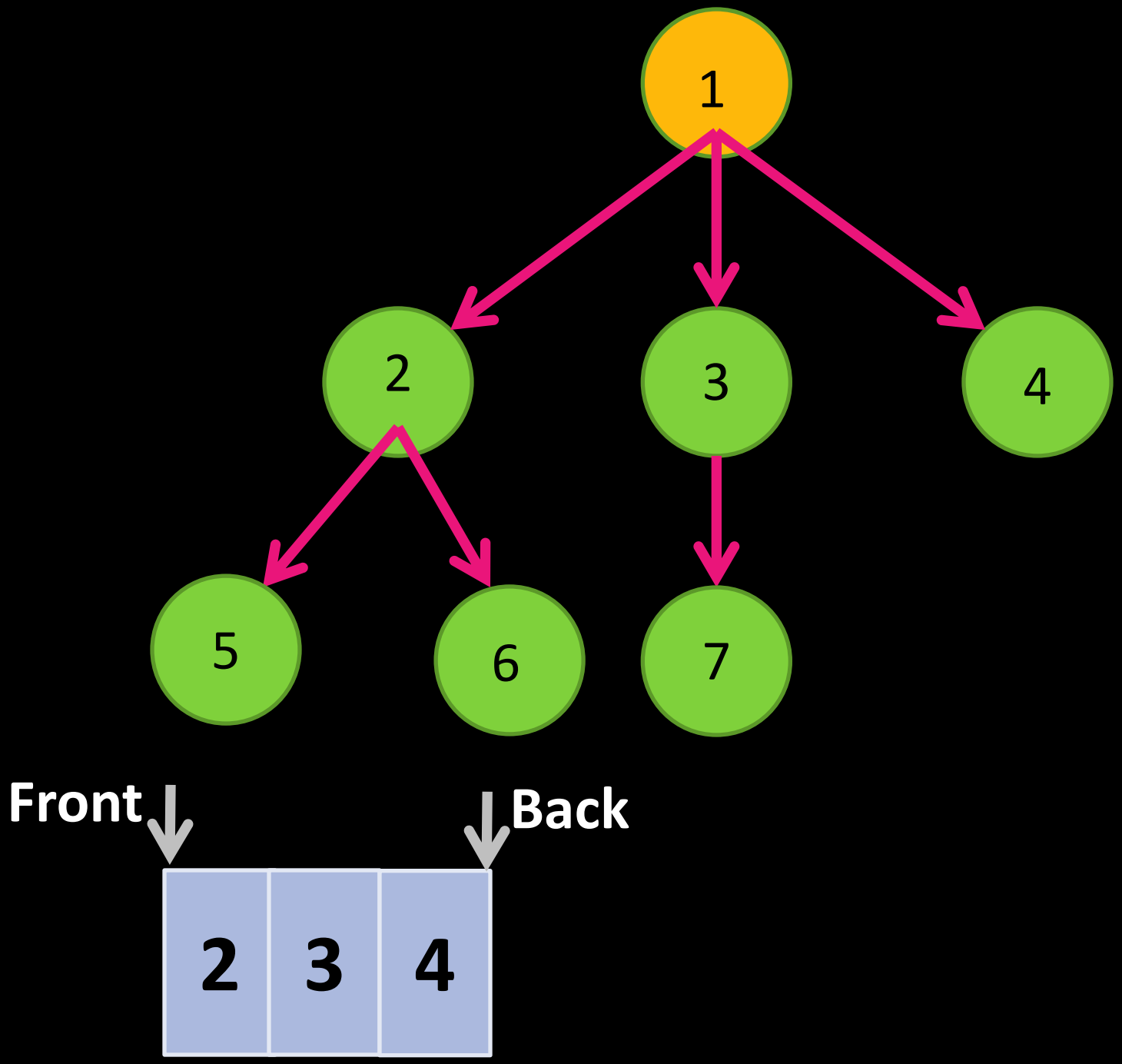
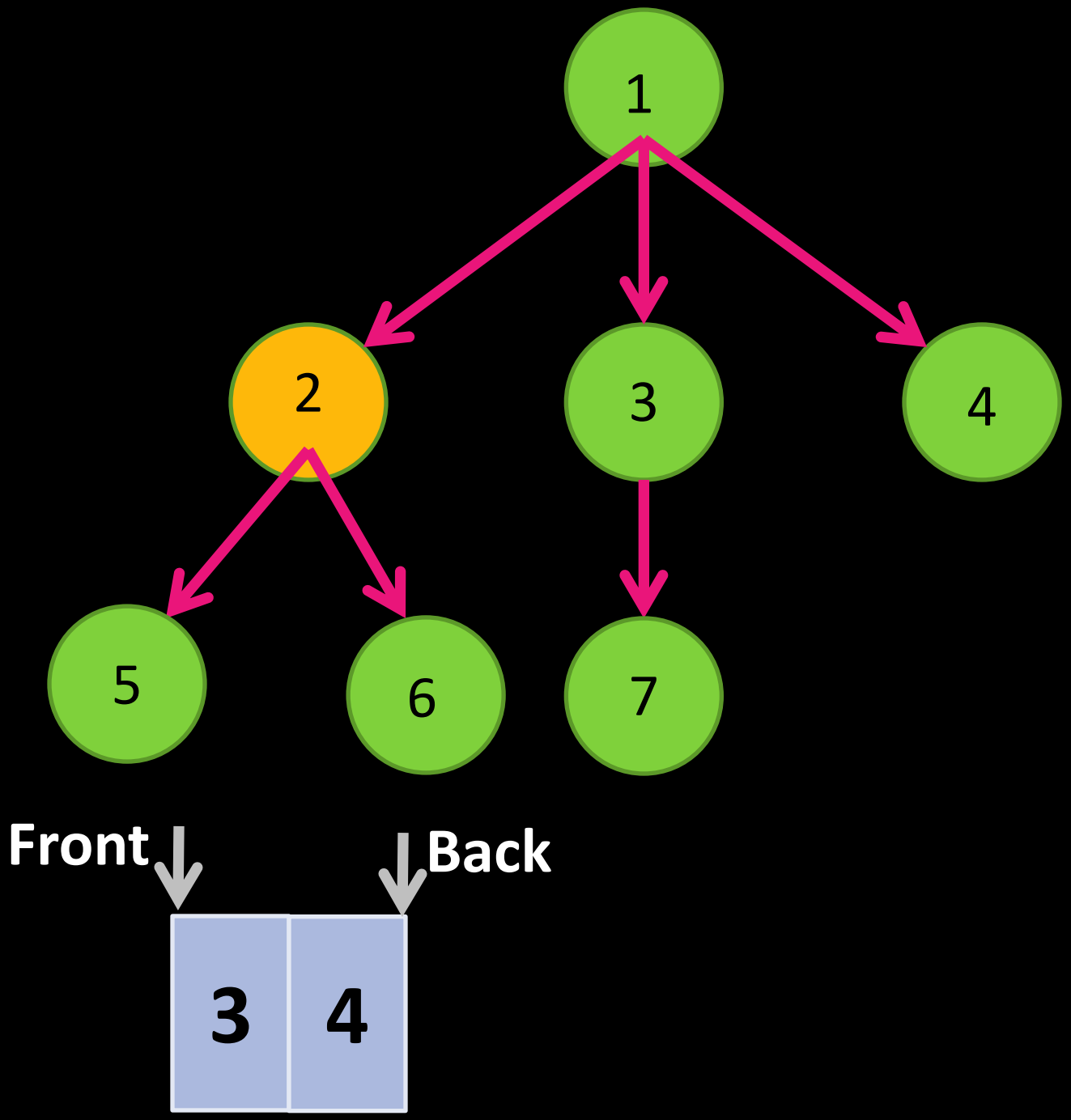
4
5
6
7
Front / back of queue


1
2
3
4
5
6
7
Front
Back
1


1
2
3
4
5
6
7
Front / back of queue


1
2
3
4
5
6
7
Front
Back
2
3
4


1
2
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Front
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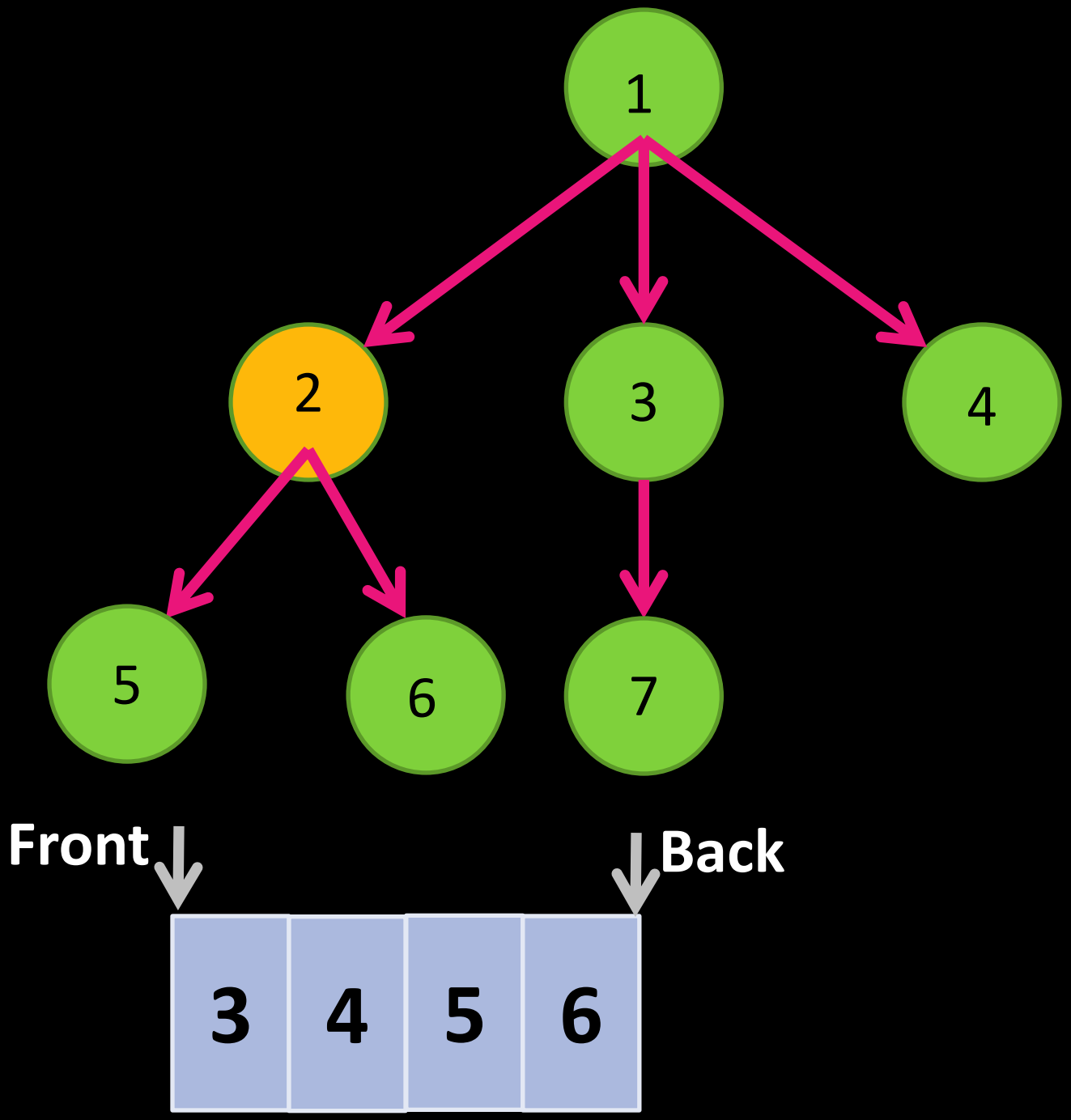
1
2
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Front
Back
3
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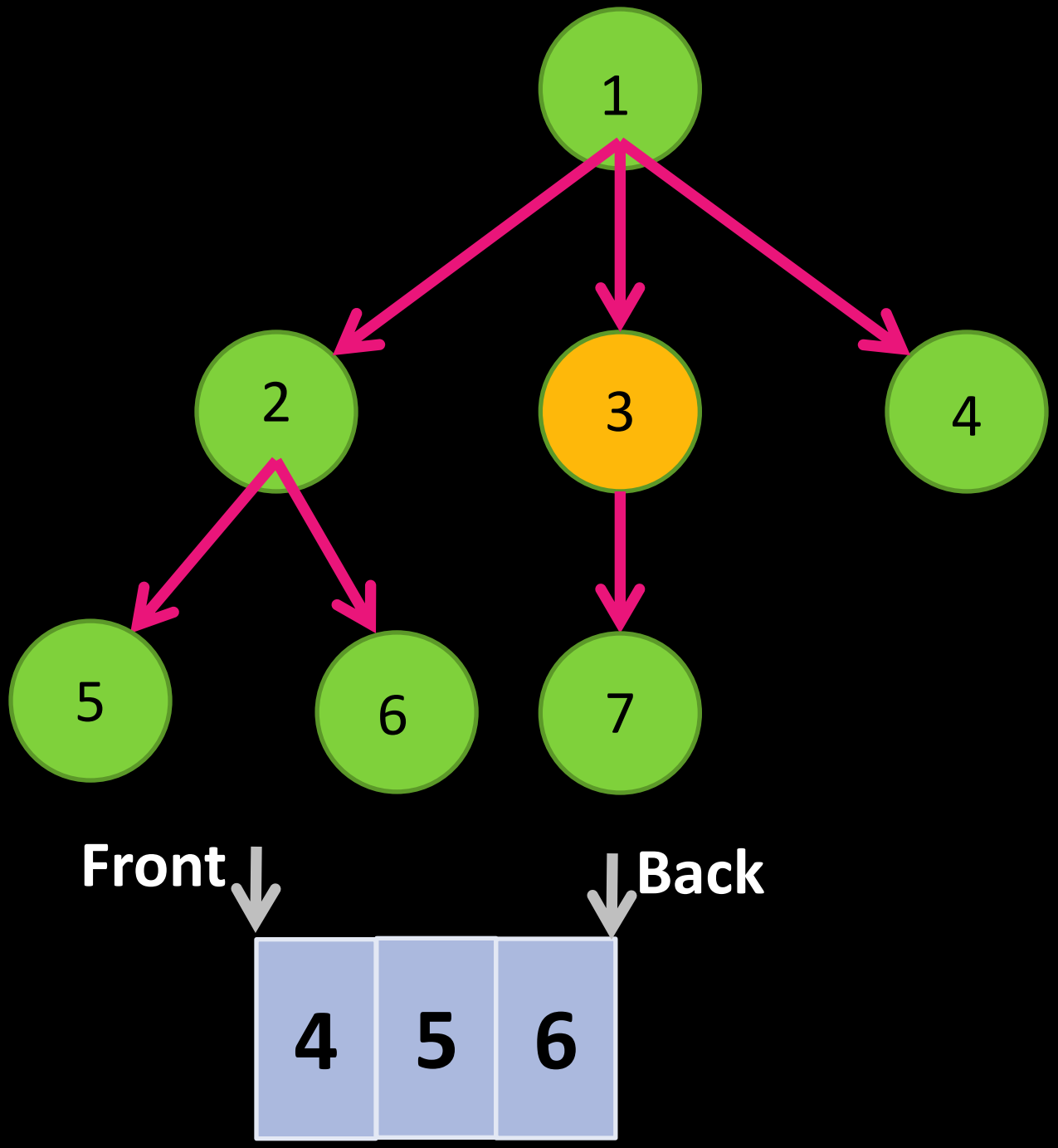
6


1
2
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6
7
Front
Back
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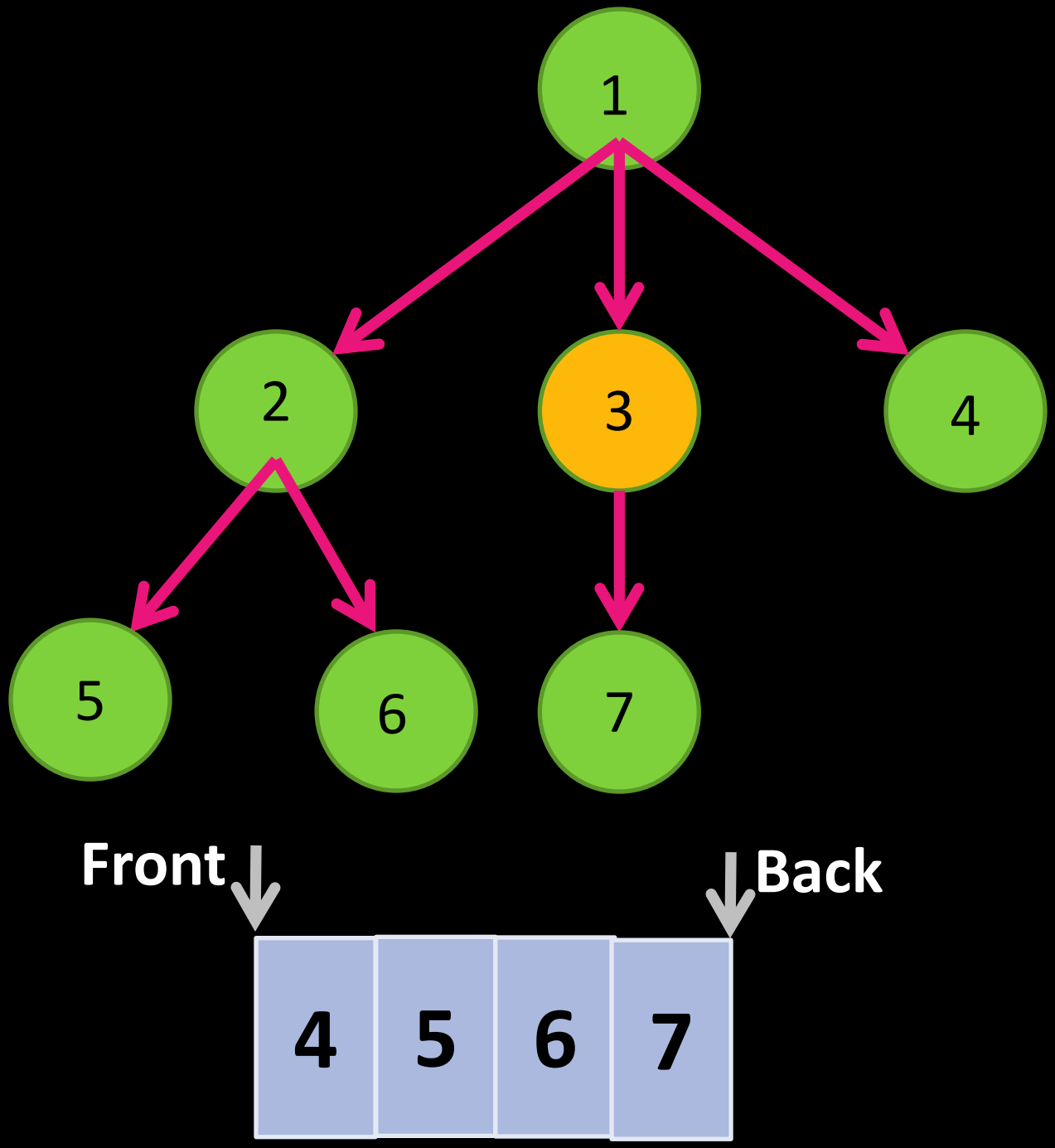
6


1
2
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4
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Front
Back
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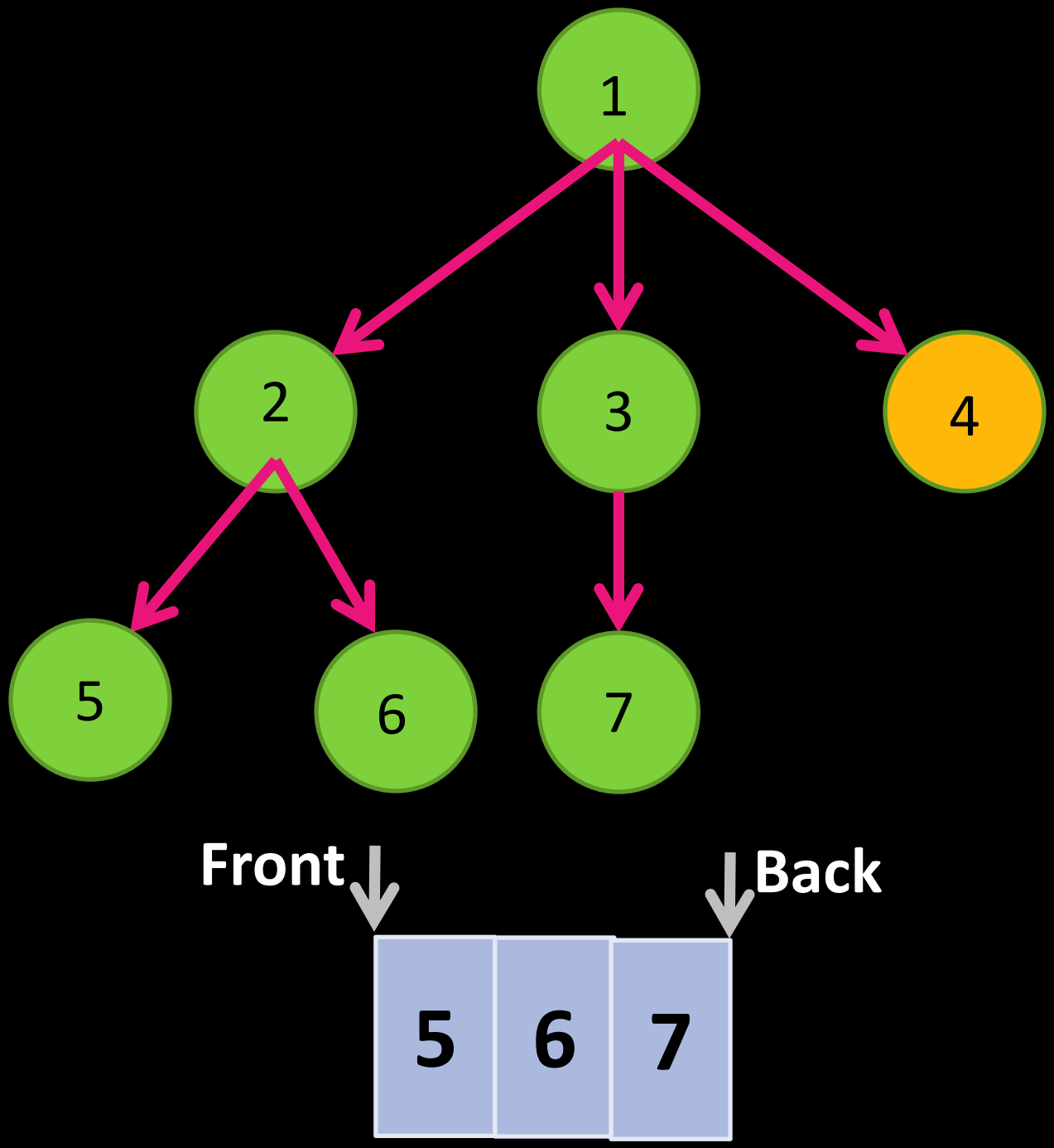
6
7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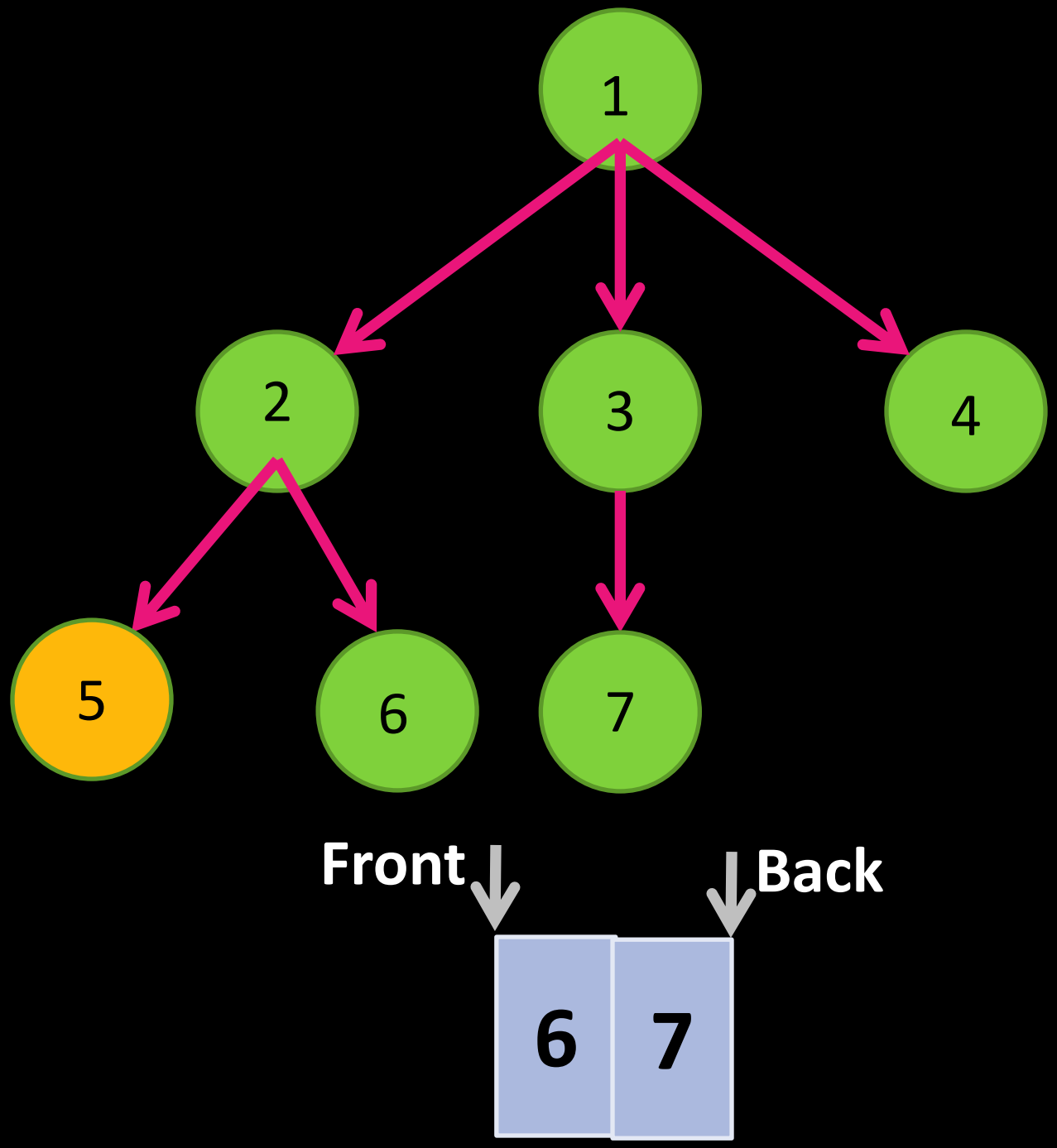
1
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6
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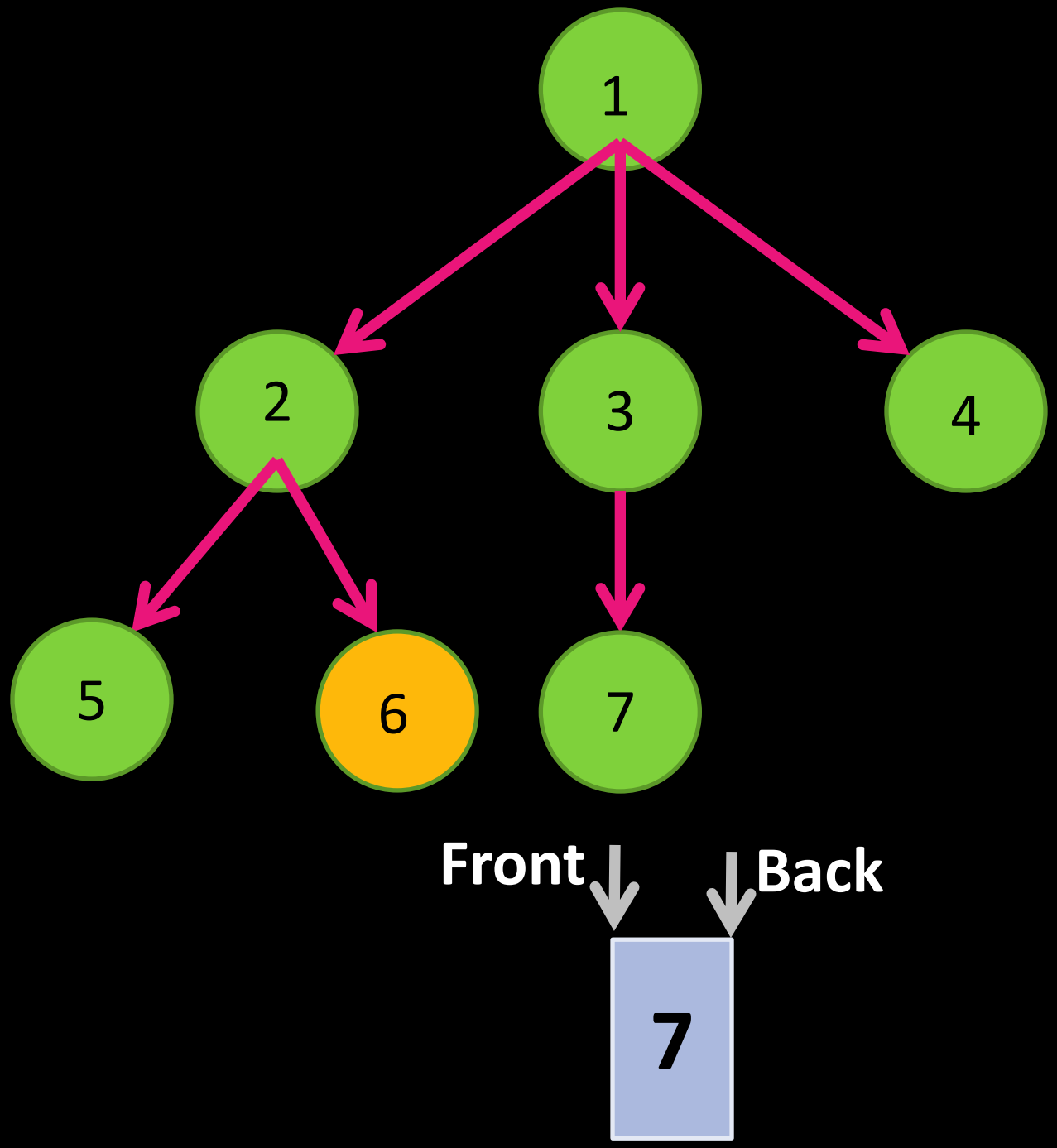
Front
Back
6
7


1
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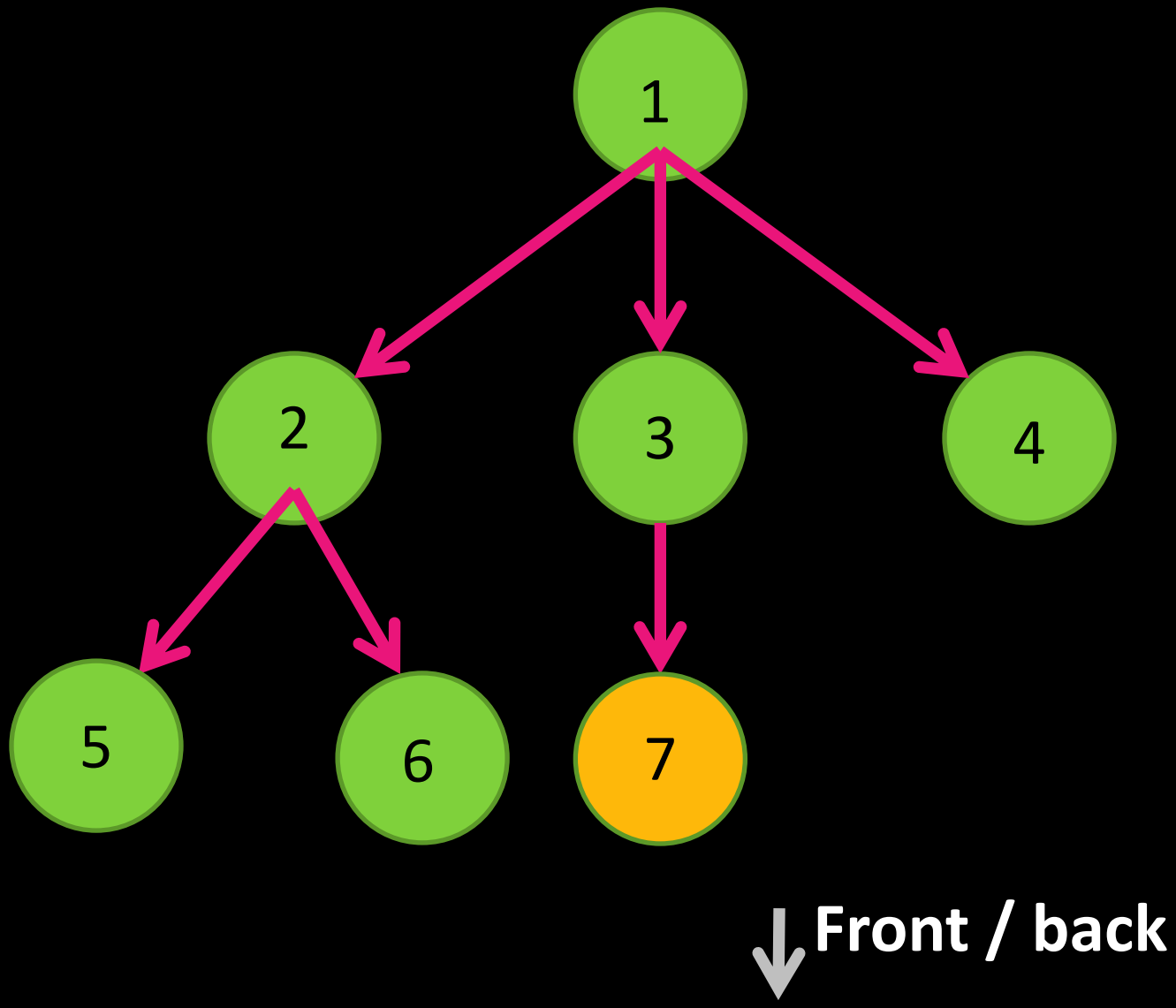
Front
Back
7

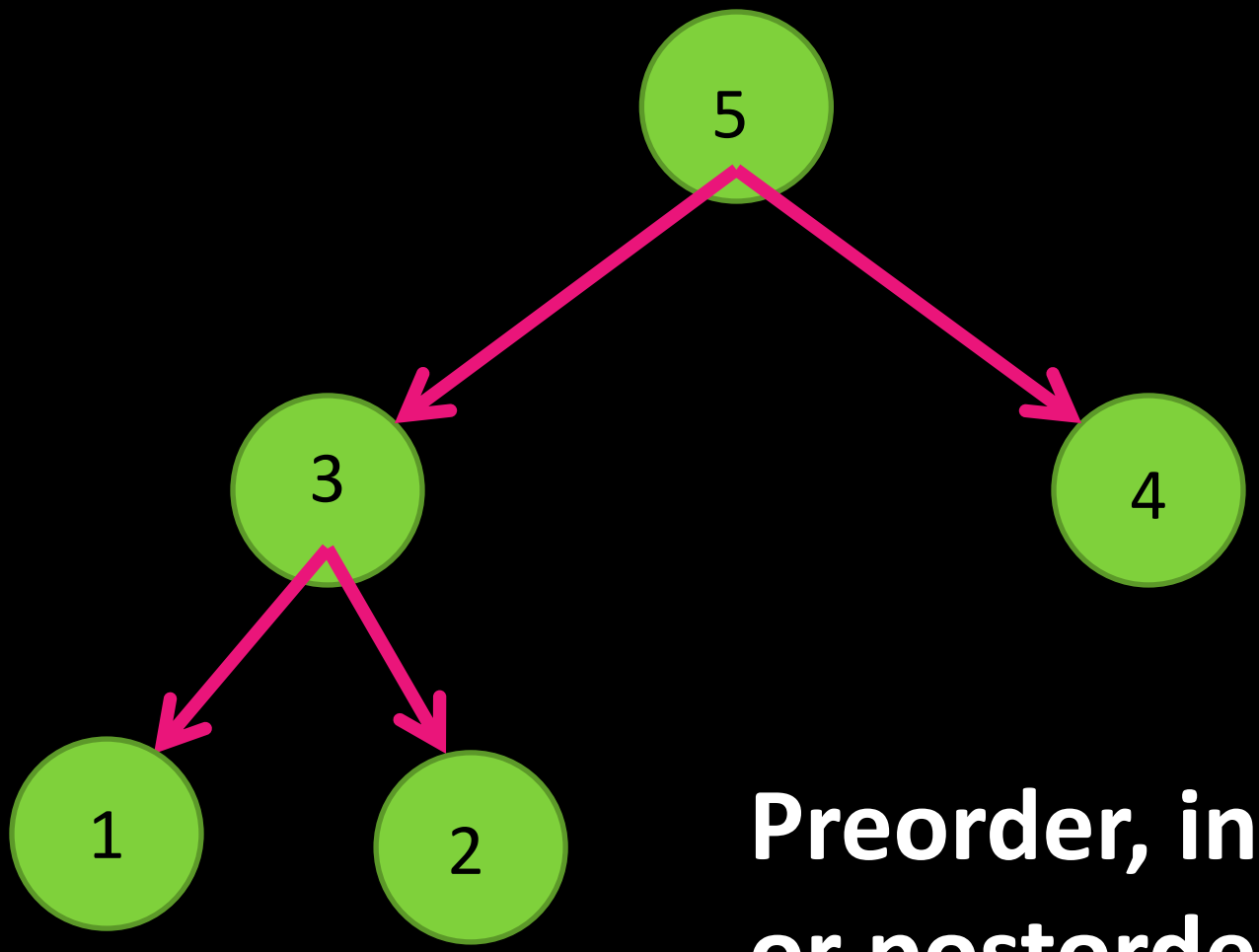

1
2
3
4
5
6
7
Front / back

Binary Trees

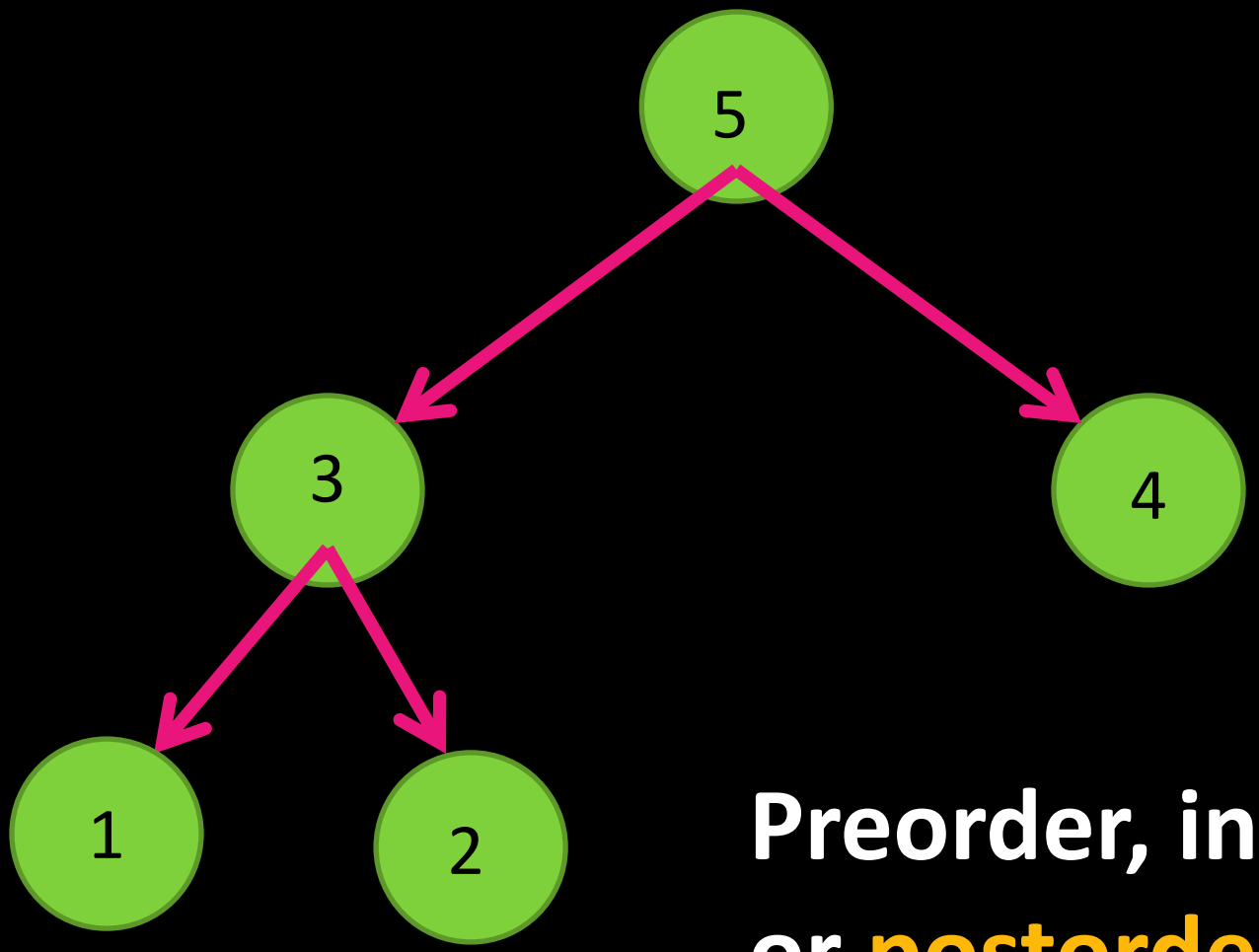
- Each node has up to 2 children
- Instead of ArrayList of children, nodes have myLeft and myRight instance variables

Binary Tree Traversals

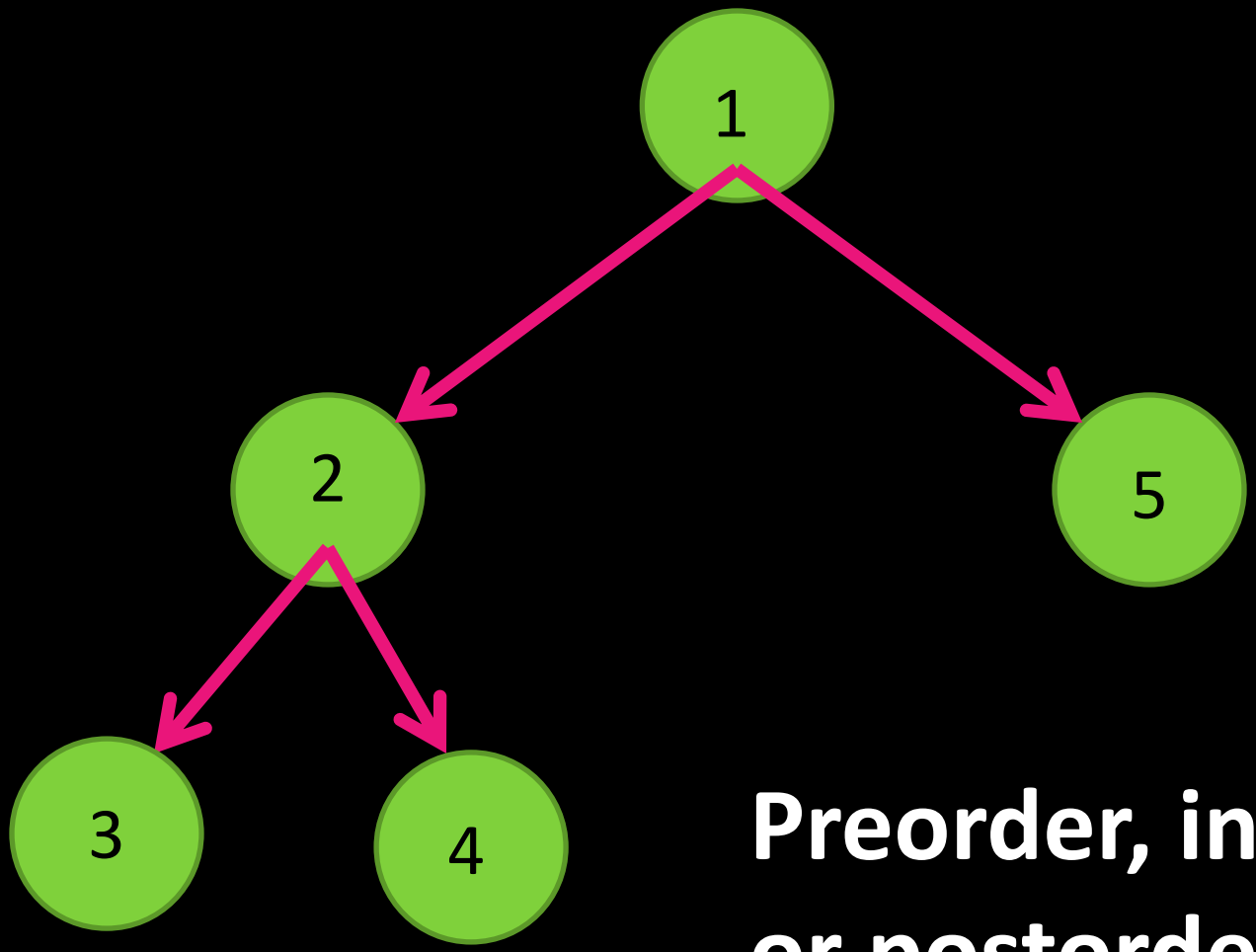
- Start at root
- A: Process left subtree
- B: Process current node’s element
- C: Process right subtree
- Preorder: BAC
- Inorder: ABC
- Postorder: ACB



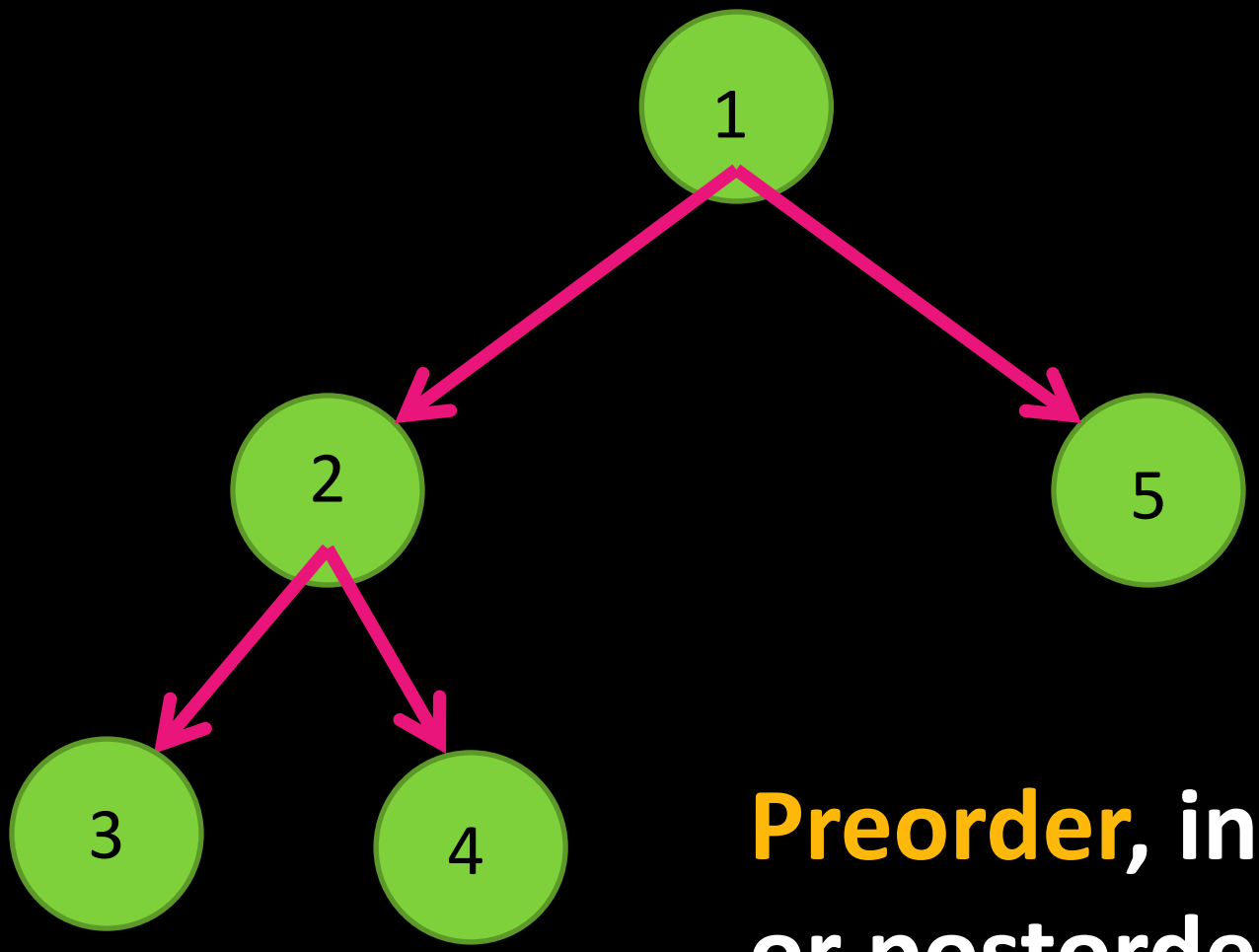
Preorder, inorder, or postorder?



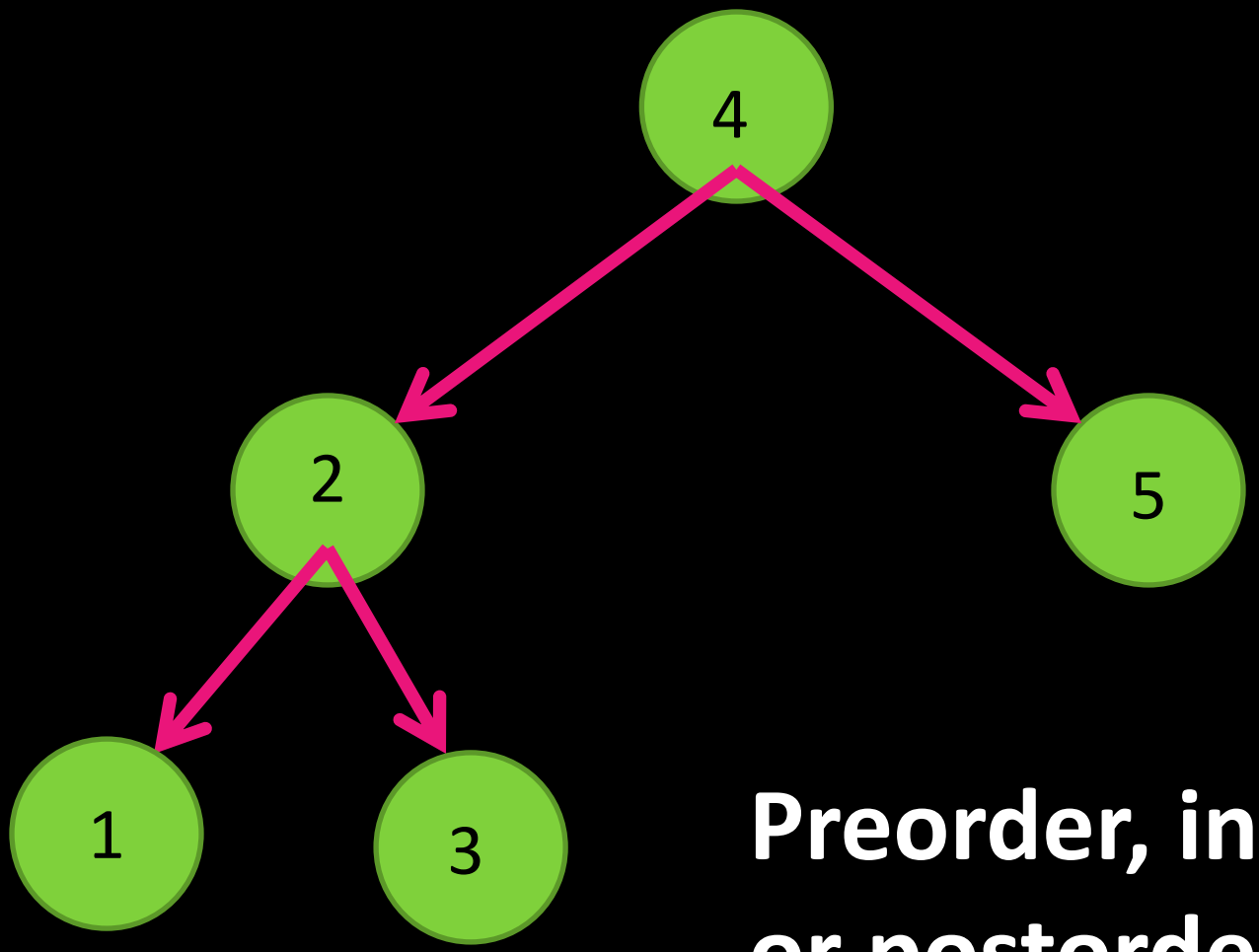
Preorder, inorder, or postorder?



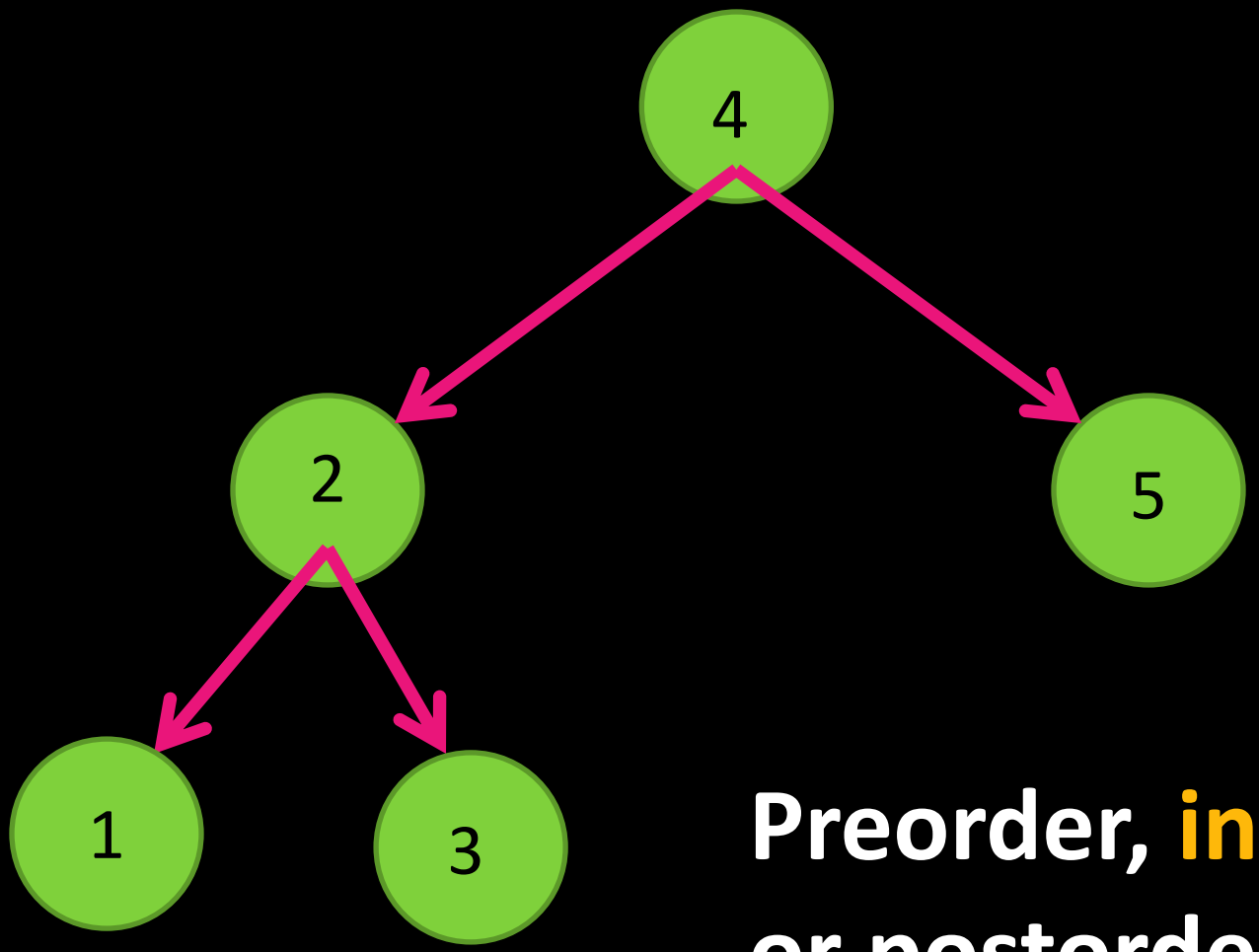
Preorder, inorder, or postorder?



Preorder, inorder, or postorder?



Preorder, inorder, or postorder?

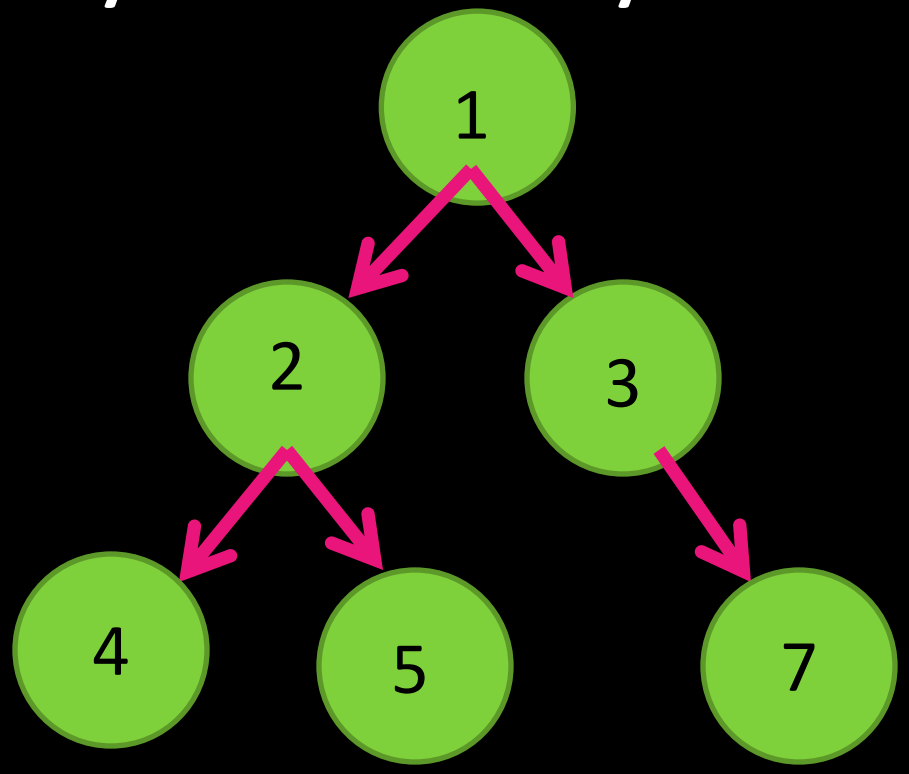


Preorder, inorder, or postorder?

Binary Tree Array Representation

- Root at index 1
- If node is at index i
  - **`myLeft`** at index $2i$
  - **`myRight`** at index $2i + 1$
- Nothing at index 0

Binary Tree Array Example

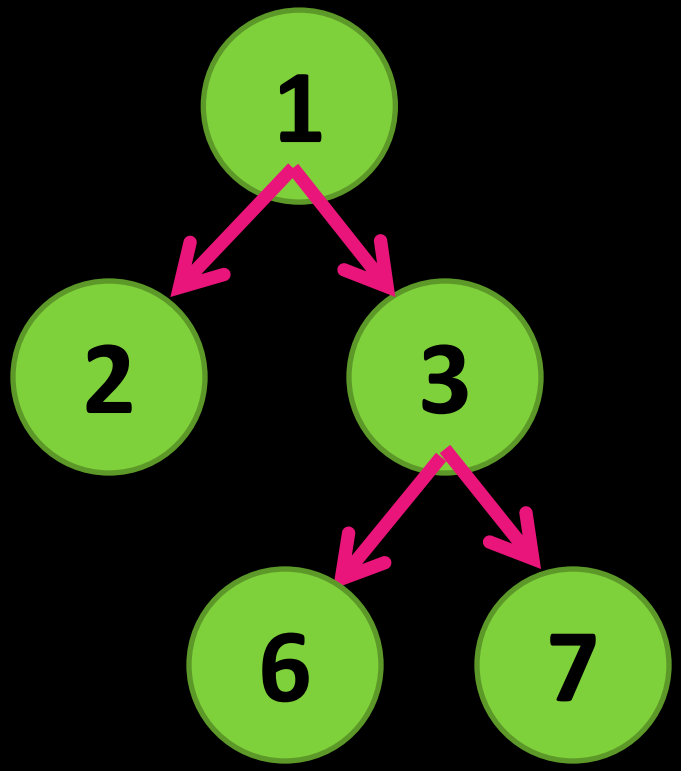


X	1	2	3	4	5	6	7

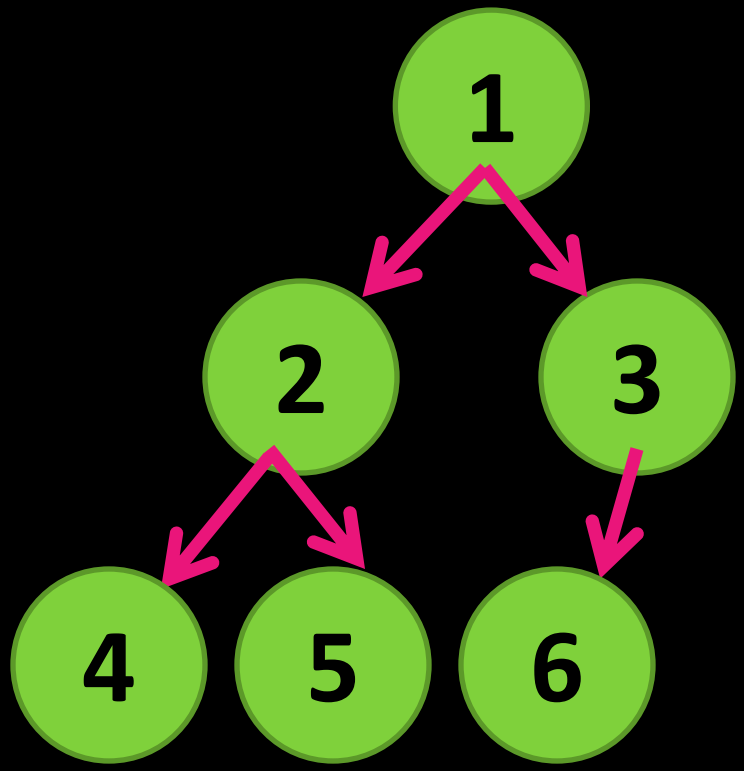
Types of Binary Trees

- Full: Every node has 2 children (except for leaves)
- Complete: Every level is as filled up as much as possible except possibly the bottom-most level, where all nodes are as left as possible
- Balanced: Height of left and right subtree of every node differ by at most 1

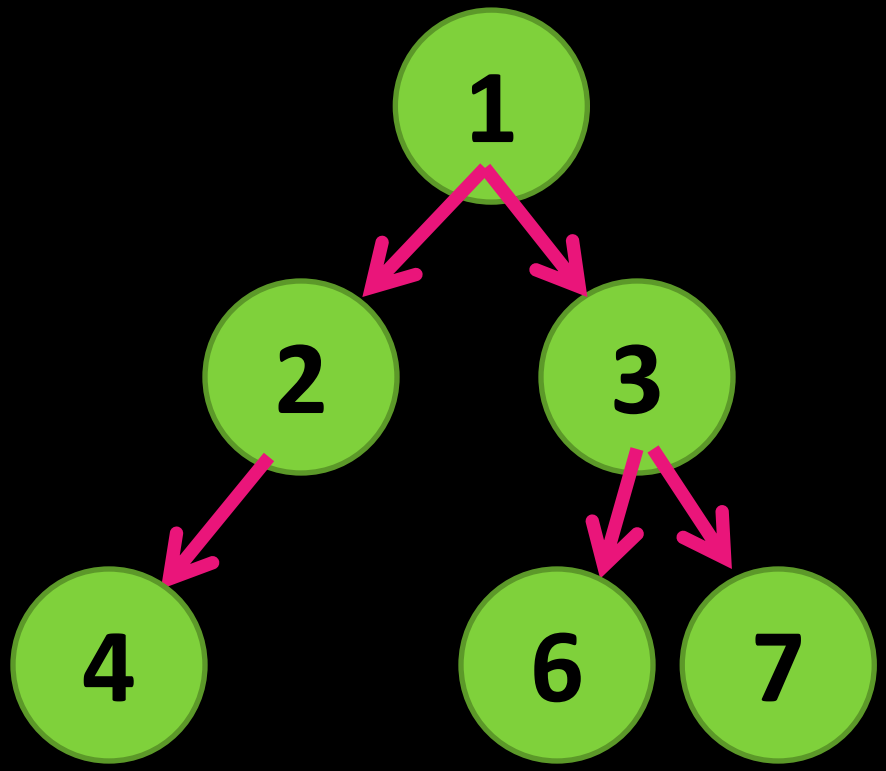
Full Binary Tree



Complete Binary Tree



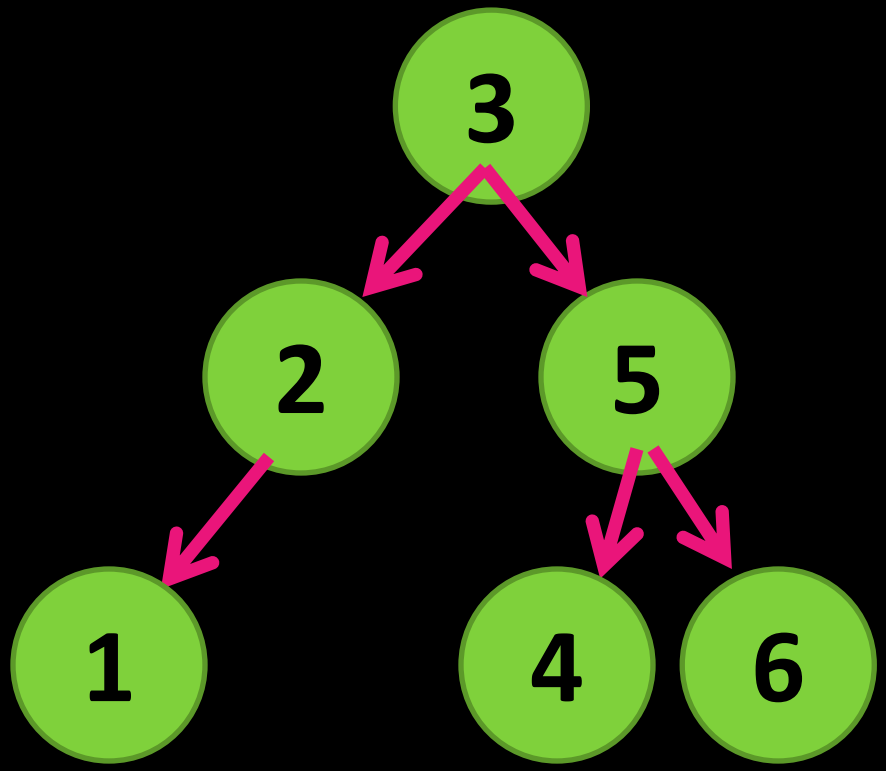
Balanced Binary Tree



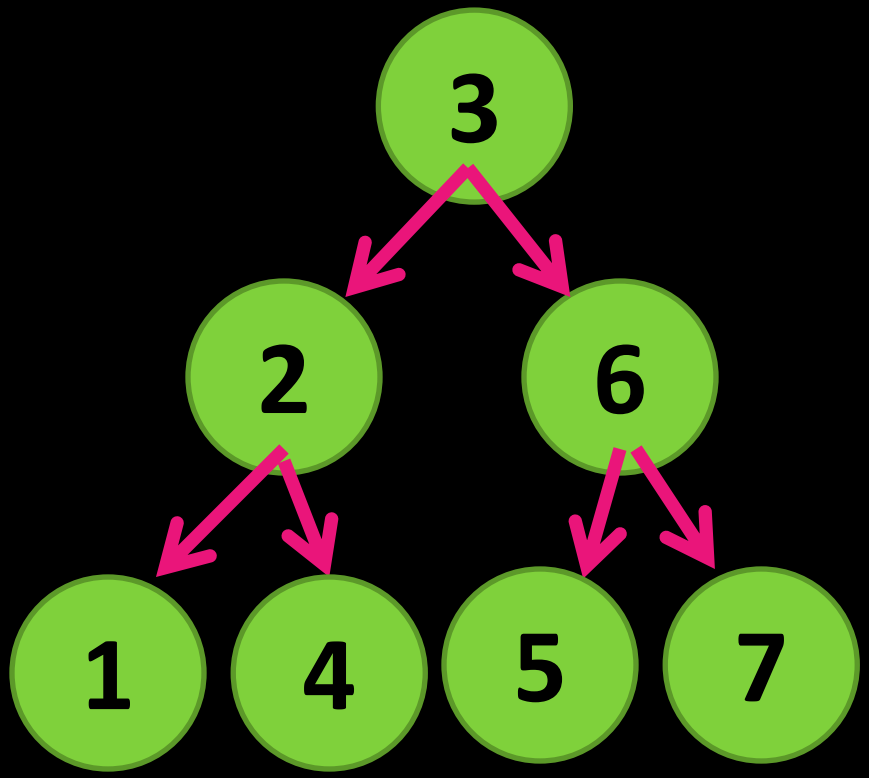
Binary Search Trees

- Invariants:
  - No nodes with duplicate keys
  - At each node, every node in the <u>left</u> subtree has key <u>less than</u> the current node's key
  - At each node, every node in the <u>right</u> subtree has key <u>greater</u> than the current node's key
  - Subtrees rooted at each node's children must also be binary search trees

BST Example



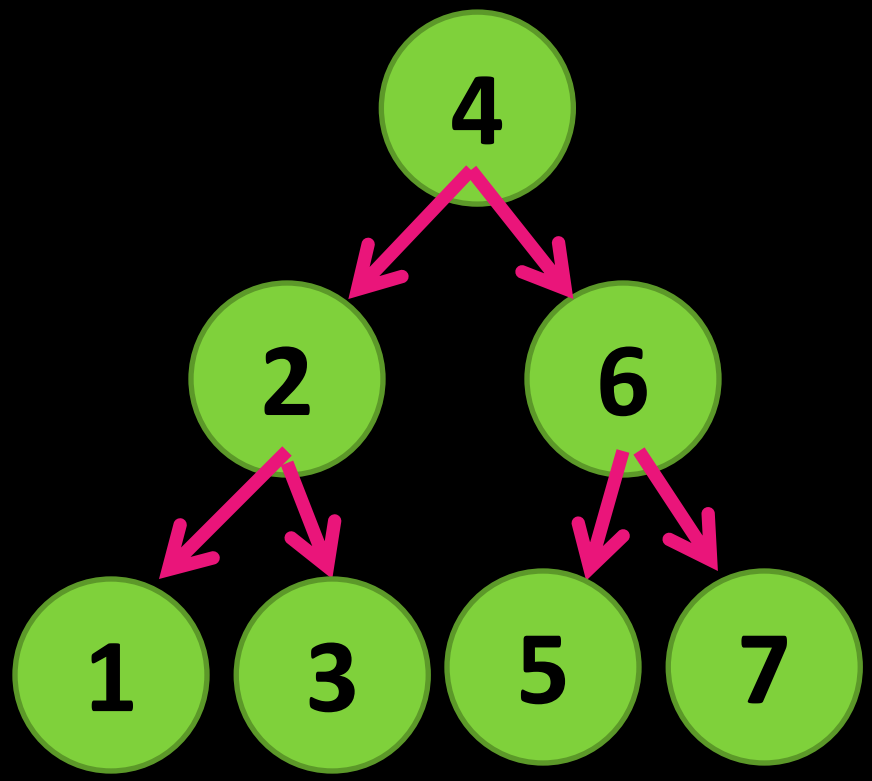
NOT A BST!



Binary <u>Search</u> Tree

- If reasonably well-balanced, searching for a key in a binary search tree takes $O(\log n)$ for a tree with n nodes

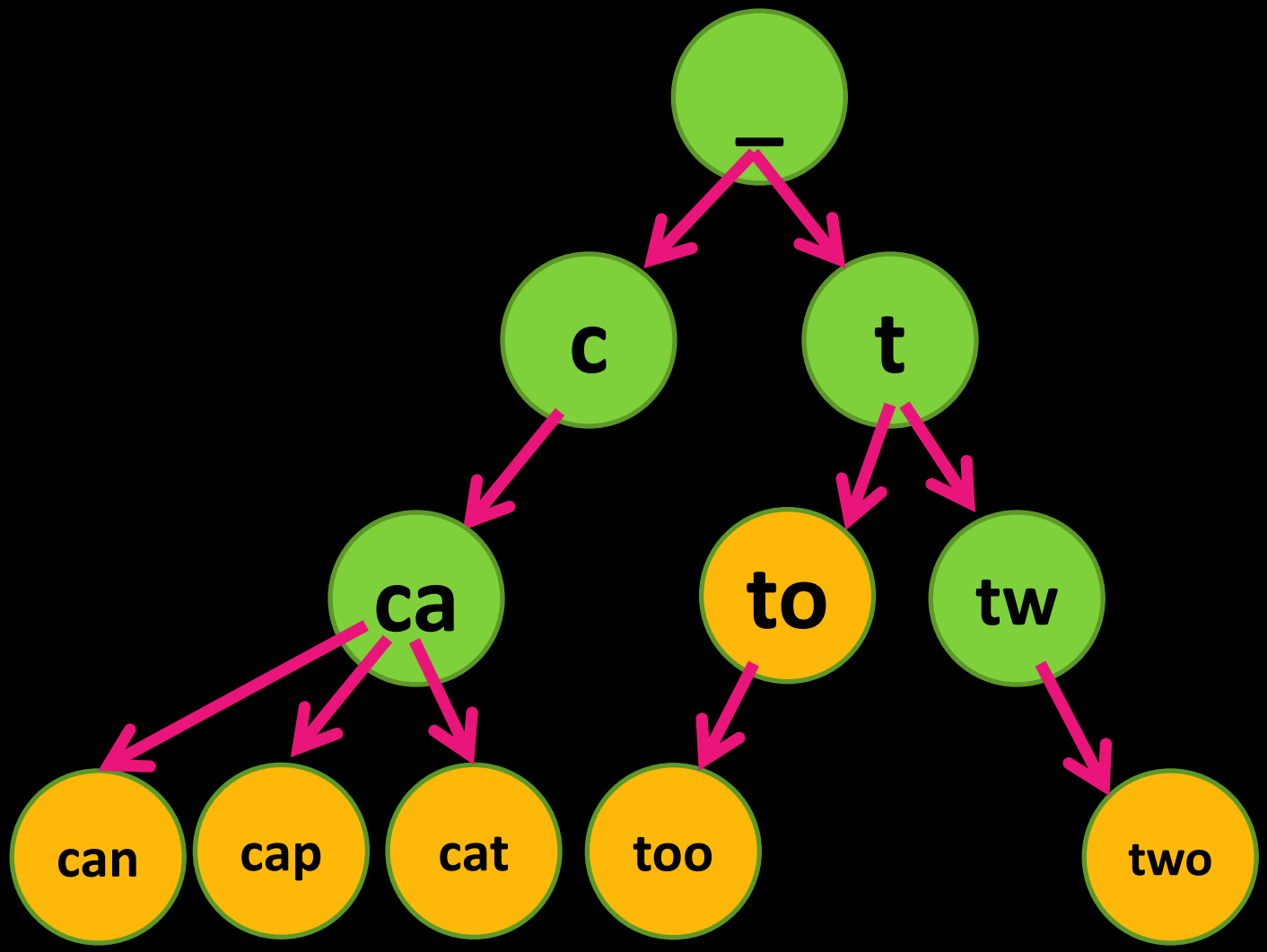
Inorder traversal of this BST?



Trie

- Each node's key is determined by keys of all nodes in path from root
- Useful as a dictionary for storing words!

Trie Example



See you tonight (maybe)