1 Thread-Level Parallelism

As powerful as data level parallelization is, it can be quite inflexible, as not all applications have data that can be vectorized. Multithreading, or running a single piece of software on multiple hardware threads, is much more powerful and versatile. OpenMP provides an easy interface for using multithreading within C programs. Some examples of OpenMP directives:

The `parallel` directive indicates that each thread should run a copy of the code within the block. If a for loop is put within the block, every thread will run every iteration of the for loop.

```c
#pragma omp parallel {
    ...  
}
```

The `parallel for` directive will split up iterations of a for loop over various threads. Every thread will run different iterations of the for loop. The following two code snippets are equivalent.

```c
#pragma omp parallel for
for (int i = 0; i < n; i++) {
    ...}

#pragma omp parallel {
    #pragma omp for
    for (int i = 0; i < n; i++) {
        ...}
}
```

There are two functions you can call that may be useful to you:

- `int omp_get_thread_num()` will return the number of the thread executing the code
- `int omp_get_num_threads()` will return the number of total hardware threads executing the code

For each question below, state and justify whether the program is sometimes incorrect, always incorrect, slower than serial, faster than serial, or none of the above. Assume the default number of threads is greater than 1. Assume no thread will complete before another thread starts executing. Assume `arr` is an `int[]` of length `n`.

(a) // Set element i of arr to i
```c
#pragma omp parallel
{
    for (int i = 0; i < n; i++)
        arr[i] = i;
}
```
Slower than serial: There is no `for` directive, so every thread executes this loop in its entirety. n threads running n loops at the same time will actually execute in the same time as 1 thread running 1 loop. Despite the possibility of false sharing, the values should all be correct at the end of the loop. Furthermore, the existence of parallel overhead due to the extra number of threads could slow down the execution time.

(b) // Set arr to be an array of Fibonacci numbers.
arr[0] = 0;
arr[1] = 1;
#pragma omp parallel for
for (int i = 2; i < n; i++)
    arr[i] = arr[i-1] + arr[i - 2];

Always incorrect (when n > 4): Loop has data dependencies, so the calculation of all threads but the first one will depend on data from the previous thread. Because we said “assume no thread will complete before another thread starts executing,” this code will always read incorrect values.

(c) // Set all elements in arr to 0;
int i;
#pragma omp parallel for
for (i = 0; i < n; i++)
    arr[i] = 0;

Faster than serial: The `for` directive actually automatically makes loop variables (such as the index) private, so this will work properly. The `for` directive splits up the iterations of the loop into continuous chunks for each thread, so there will be no data dependencies or false sharing.

What potential issue can arise from this code?

1 // Decrements element i of arr. n is a multiple of `omp_get_num_threads()`
2 #pragma omp parallel
3 {
4     int threadCount = omp_get_num_threads();
5     int myThread = omp_get_thread_num();
6     for (int i = 0; i < n; i++) {
7         if (i % threadCount == myThread) arr[i] -= 1;
8     }
9 }

False sharing arises because different threads can modify elements located in the same memory block simultaneously. This is a problem because some threads may have incorrect values in their cache block when they modify the value `arr[i]`, invalidating the cache block.
1.3 // Assume n holds the length of arr
2   double fast_product(double *arr, int n) {
3       double product = 1;
4       #pragma omp parallel for
5       for (int i = 0; i < n; i++) {
6           product *= arr[i];
7       }
8       return product;
9   }

(a) What is wrong with this code?

    The code has the shared variable product.

(b) Fix the code using #pragma omp critical

   double fast_product(double *arr, int n) {
       double product = 1;
       #pragma omp parallel for
       for (int i = 0; i < n; i++) {
           #pragma omp critical
           product *= arr[i];
       }
       return product;
   }

(c) Fix the code using #pragma omp reduction(operation: var).

   double fast_product(double *arr, int n) {
       double product = 1;
       #pragma omp parallel for reduction(*: product)
       for (int i = 0; i < n; i++) {
           product *= arr[i];
       }
       return product;
   }

2 Logic Gates

2.1 Label the following logic gates:

   NOT, AND, OR, XOR, NAND, NOR, XNOR

2.2 Convert the following to boolean expressions:

   (a) NAND
\[
\bar{A}B + \bar{A}B + AB
\]
(b) XOR
\[
\bar{A}B + AB
\]
(c) XNOR
\[
\bar{A}\bar{B} + AB
\]

Create an AND gate using only NAND gates.

How many different two-input logic gates can there be? How many \(n\)-input gates?

A truth table with \(n\) inputs has \(2^n\) rows. Each logic gate has a 0 or a 1 at each of these rows. Imagining a function as a \(2^n\)-bit number, we count \(2^{2^n}\) total functions, or 16 in the case of \(n = 2\).