

Direct-Mapped and Set Associative Caches

Instructor: Steven Ho

Great Idea #3: Principle of Locality/ Memory Hierarchy



- Why have caches?
 - Intermediate level between CPU and memory
 - In-between in size, cost, and speed
- Memory (hierarchy, organization, structures) set up to exploit *temporal* and *spatial locality*
 - Temporal: If accessed, will access again soon
 - Spatial: If accessed, will access others around it
- Caches hold a subset of memory (in *blocks*)
 - We are studying how they are designed for fast and efficient operation (lookup, access, storage)

- Fully Associative Caches:
 - Every block can go in any slot
 - Use random or LRU replacement policy when cache full
 - Memory address breakdown (on request)
 - Tag field is unique identifier (which block is currently in slot)
 - Offset field indexes into block (by bytes)
 - *Each* cache slot holds block data, tag, valid bit, and dirty bit (dirty bit is only for *write-back*)
 - The whole cache maintains LRU bits

- On memory access (read or write):
 - 1) Look at ALL cache slots in parallel
 - 2) If Valid bit is 0, then ignore (garbage)
 - 3) If Valid bit is 1 and Tag matches, then use that data
- On write, set Dirty bit if write-back

2⁶ = 64 B address space cache size (C) block size (K)

- Fully associative cache layout in our example
 - 6-bit address space, 16-byte cache with 4-byte blocks
 - How many blocks do we have? C/K = 4 blocks
 - LRU replacement (2 bits)
 - Offset 2 bits, Tag 4 bits LRU bits

$\mathbf{O}^{\mathbf{r}}$	ff	S	e	t	
<u> </u>	• •	-	<u> </u>	×.	

		V	Tag	00	01	10	11	LRU
Slot	0	Х	XXXX	0x??	0x??	0x??	0x??	XX
	1	Х	XXXX	0x??	0x??	0x??	0x??	XX
5101	2	Х	XXXX	0x??	0x??	0x??	0x??	XX
	3	Х	XXXX	0x??	0x??	0x??	0x??	XX

Yesterday's example was write through and looked like this

FA Cache Examples (3/4)

1) Consider the following *addresses* being requested: Starting with a cold cache: 0 2 2 0 16 20 8 4

000000 0 miss

1	0000 M[0]		M[1]	M[2]	M[3]	
0	0000	0x??	0x??	0x??	0x??	
0	0000	0x??	0x??	0x??	0x??	
0	0000	0x??	0x??	0x??	0x??	

000010 2 hit

1	L 0000 M[C		Л[0] M[1] (M[3]	
0	0000	0x??	0x??	0x??	0x??	
0	0000	0x??	0x??	0x??	0x??	
0	0000	0x??	0x??	0x??	0x??	

000010 2 hit

1	0000	M[0]	M[1]	M[2]	M[3]
0	0000	0x??	0x??	0x??	0x??
0	0000	0x??	0x??	0x??	0x??
0	0000	0x??	0x??	0x??	0x??

000000 0 hit

1	0000 M[0]		M[1]	M[2]	M[3]	
0	0000	0x??	0x??	0x??	0x??	
0	0000	0x??	0x??	0x??	0x??	
0	0000	0x??	0x??	0x??	0x??	

FA Cache Examples (3/4)

1) Consider the following addresses being requested: Starting with a cold cache: 0 2 2 0 16 20 8 4

010000 16 miss

1	0000	M[0]	M[1]	M[2]	M[3]
1	0100	M[16]	M[17]	M[18]	M[19]
0	0000	0x??	0x??	0x??	0x??
0	0000	0x??	0x??	0x??	0x??

001000

8 miss

1	0000	M[0]	M[1]	M[2]	M[3]
1	0100	M[16]	M[17]	M[18]	M[19]
1	0101	M[20]	M[21]	M[22]	M[23]
1	0010	M[8]	M[9]	M[10]	M[11]

010100 20 miss

1	0000	M[0]	M[1]	M[2]	M[3]
1	0100	M[16]	M[17]	M[18]	M[19]
1	0101	M[20]	M[21]	M[22]	M[23]
0	0000	0x??	0x??	0x??	0x??

000100

4 miss

1	0000	M[0]	M[1]	M[2]	M[3]
1	0100	M[16]	M[17]	M[18]	M[19]
1	0101	M[20]	M[21]	M[22]	M[23]
1	0010	M[8]	M[9]	M[10]	M[11]

• 8 requests, 5 misses – ordering matters!

FA Cache Examples (4/4)

3) Original sequence, but double block size to 8B Starting with a cold cache: 8 20 0 2 2 4 16 0

0	1	000	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]
miss	0	000	0x??	0x??	0x??	0x??	0x??	0x??	0x??	0x??
000010										
2	1	000	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]
hit	0	000	0x??	0x??	0x??	0x??	0x??	0x??	0x??	0x??
000100										
4	1	000	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]
hit	0	000	0x??	0x??	0x??	0x??	0x??	0x??	0x??	0x??
001000										
8	1	000	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]
miss	1	001	M[8]	M[9]	M[10]	M[11]	M[12]	M[13]	M[14]	M[15]

FA Cache Examples (4/4)

3) Original sequence, but double block size

Starting with a cold cache: 0 2 4 8 20 16 0

0100				_			_		
20	1 000	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]
miss	1 001	M[8]	M[9]	M[10]	M[11]	M[12]	M[13]	M[14]	M[15]

010000										
16	1	010	M[16]	M[17]	M[18]	M[19]	M[20]	M[21]	M[22]	M[23]
hit	1	001	M[8]	M[9]	M[10]	M[11]	M[12]	M[13]	M[14]	M[15]

000000

0	1	010	M[16]	M[17]	M[18]	M[19]	M[20]	M[21]	M[22]	M[23]
miss	1	001	M[8]	M[9]	M[10]	M[11]	M[12]	M[13]	M[14]	M[15]

000010

2	1	010	M[16]	M[17]	M[18]	M[19]	M[20]	M[21]	M[22]	M[23]
hit	1	000	M[0]	M[1]	M[2]	M[3]	M[4]	M[5]	M[6]	M[7]

8 requests, 4 misses – cache parameters matter!

2

Question:

Starting with the same cold cache as the first 3 examples, which of the sequences below will result in the final state of the cache shown here:

0	1	0000	M[0]	M[1]	M[2]	M[3]	
1	1	0011	M[12]	M[13]	M[14]	M[15]	LRU
2	1	0001	M[4]	M[5]	M[6]	M[7]	10
3	1	0100	M[16]	M[17]	M[18]	M[19]	

(A)	0	2	12	4	16	8	0	6
(B)	0	8	4	16	0	12	6	2
(C)	6	12	4	8	2	16	0	0
(D)	2	8		4	6	16	12	



Question:

Starting with the same cold cache as the first 3 examples, which of the sequences below will result in the final state of the cache shown here:

0	1	0000	M[0]	M[1]	M[2]	M[3]	
1	1	0011	M[12]	M[13]	M[14]	M[15]	LRU
2	1	0001	M[4]	M[5]	M[6]	M[7]	10
3	1	0100	M[16]	M[17]	M[18]	M[19]	





Question:

Starting with the same cold cache as the first 3 examples, which of the sequences below will result in the final state of the cache shown here:

0	1	0000	M[0]	M[1]	M[2]	M[3]		
1	1	0011	M[12]	M[13]	M[14]	M[15]	Γ	LRU
2	1	0001	M[4]	M[5]	M[6]	M[7]		10
3	1	0100	M[16]	M[17]	M[18]	M[19]		

(A)	0	2	12	4	16	8	0	6
(B)	0	8	4	16	0	12	6	2
(C)	6	12	4	8	2	16	0	0
(D)	2	8	0	4	6	16	12	0



Memory Accesses

• The picture so far:



Handling Write Hits

- Write hits (D\$)
 - 1) Write-Through Policy: Always write data to cache and to memory (*through* cache)
 - Forces cache and memory to always be consistent
 - Slow! (every memory access is long)
 - Include a *Write Buffer* that updates memory in parallel with processor

Assume present in all schemes when writing to memory

Handling Write Hits

- Write hits (D\$)
 - 2) Write-Back Policy: Write data <u>only to cache</u>, then update memory when block is removed
 - Allows cache and memory to be inconsistent
 - Multiple writes collected in cache; single write to memory per block
 - Dirty bit: Extra bit per cache row that is set if block was written to (is "dirty") and needs to be written back

Handling Cache Misses

- Miss penalty grows as block size does
- Read misses (I\$ and D\$)
 - Stall execution, fetch block from memory, put in cache, send requested data to processor, resume
- Write misses (D\$)
 - Always have to update block from memory
 - We have to make a choice:
 - Carry the updated block into cache or not?

Write Allocate

- *Write Allocate* policy: when we bring the block into the cache after a write miss
- No Write Allocate policy: only change main memory after a write miss
 - Write allocate almost always paired with write-back
 - Eg: Accessing same address many times -> cache it
 - No write allocate typically paired with write-through
 - Eg: Infrequent/random writes -> don't bother caching it

Updated Cache Picture

- Fully associative, write through
 - Same as our simplified examples from before
- Fully associative, write back

		V	D	Tag	00	01	10	11	
	0	Х	Х	XXXX	0x??	0x??	0x??	0x??	
Slot	1	Х	Х	XXXX	0x??	0x??	0x??	0x??	Lŀ
	2	Х	Х	XXXX	0x??	0x??	0x??	0x??	**
	3	Х	Х	XXXX	0x??	0x??	0x??	0x??	

• Write miss procedure (write allocate or not) only affects **behavior**, not design

How do we use this thing?

- Nothing changes from the programmer's perspective
 - Still just issuing lw and sw instructions
- The rest is handled in hardware:
 - Checking the cache
 - Extracting the data using the offset
- Why should a programmer care?
 - Understanding cache parameters = faster programs

Agenda

- Review of yesterday
- Administrativia
- Direct-Mapped Caches
- Set Associative Caches
- Cache Performance

Administrivia

- HW3/4 Due today
- HW5 Released, due next Monday (7/23)
- Project 3 Due Friday (7/20)
 - Parties tonight @Soda 405/411 and Friday @Woz
 (4-6pm for both)
 - If you ask for help please diagnose problem spots
- Guerilla Session on Wed. 4-6pm @Soda 405
- Midterm 2 is coming up! Next Wed. in lecture
 - Covering up to Performance
 - Review Session Sunday 2-4pm @GPB 100

Direct-Mapped Caches (1/3)

- Each memory block is mapped to *exactly one* slot in the cache (*direct-mapped*)
 - Every block has only one "home"
 - Use hash function to determine which slot
- Comparison with fully associative
 - Check just one slot for a block (faster!)
 - No replacement policy necessary
 - Access pattern may leave empty slots in cache

Direct-Mapped Caches (2/3)

- Offset field remains the same as before
- Recall: blocks consist of adjacent bytes
 - Do we want adjacent blocks to map to same slot?
 - Index field: Apply hash function to block address to determine *which slot* the block goes in
 - (*block* address) modulo (# of *blocks* in the cache)
- Tag field maintains same function (identifier), but is now shorter

TIO Address Breakdown

• Memory address fields:



- Meaning of the field sizes:
 - O bits \leftrightarrow 2⁰ bytes/block = 2⁰⁻² words/block
 - | bits \leftrightarrow 2[|] slots in cache = cache size / block size

Direct-Mapped Caches (3/3)

- What's actually in the cache?
 - Block of data ($8 \times K = 8 \times 2^{\circ}$ bits)
 - Tag field of address as identifier (T bits)
 - Valid bit (1 bit)
 - Dirty bit (1 bit if write-back)
 - No replacement management bits!
- Total bits in cache = # slots × (8×K + T + 1 + 1) = 2^{I} × (8× 2^{O} + T + 1 + 1) bits

DM Cache Example (1/5)

- Cache parameters:
 - Direct-mapped, address space of 64B, block size of 4B, cache size of 16B, write-through
- TIO Breakdown: Memory Addresses: XX XX XX- $O = \log_2(4) = 2$ Block address
 - Cache size / block size = 16/4 = 4, so $I = log_2(4) = 2$
 - $-A = \log_2(64) = 6$ bits, so T = 6 2 2 = 2
- Bits in cache = $2^2 \times (8 \times 2^2 + 2 + 1) = 140$ bits

DM Cache Example (2/5)

- Cache parameters:
 - Direct-mapped, address space of 64B, block size of 4B, cache size of 16B, write-through

Offcot

Offset – 2 bits, Index – 2 bits, Tag – 2 bits

				Uliset							
		V	Tag	00	01	10	11				
Index	00	Х	XX	0x??	0x??	0x??	0x??				
	01	Х	XX	0x??	0x??	0x??	0x??				
	10	Х	XX	0x??	0x??	0x??	0x??				
	11	Х	XX	0x??	0x??	0x??	0x??				

• 35 bits per index/slot, 140 bits to implement

DM Cache Example (3/5)



Which blocks map to each row of the cache? (see colors)

On a memory request: (let's say 001011_{two})

1) Take Index field (10)

2) Check if Valid bit is true in that row of cache

3) If valid, then check if Tag matches

DM Cache Example (4/5)

• Consider the sequence of memory address accesses Starting with a cold cache: 0 2 4 8 20 16 0 2

000000 0 miss

00	1	00	M[0]	M[1]	M[2]	M[3]
01	0	00	0x??	0x??	0x??	0x??
10	0	00	0x??	0x??	0x??	0x??
11	0	00	0x??	0x??	0x??	0x??

000100 4 miss

00	1	00	M[0]	M[1]	M[2]	M[3]
01	1	00	M[4]	M[5]	M[6]	M[7]
10	0	00	0x??	0x??	0x??	0x??
11	0	00	0x??	0x??	0x??	0x??

000010 2 hit

00	1	00	M[0]	M[1]	M[2]	M[3]
01	0	00	0x??	0x??	0x??	0x??
10	0	00	0x??	0x??	0x??	0x??
11	0	00	0x??	0x??	0x??	0x??

001000 8 miss

00	1	00	M[0]	M[1]	M[2]	M[3]
01	1	00	M[4]	M[5]	M[6]	M[7]
10	1	00	M[8]	M[9]	M[10]	M[11]
11	0	00	0x??	0x??	0x??	0x??

DM Cache Example (5/5)

• Consider the sequence of memory address accesses Starting with a cold cache: 0 2 4 8 20 16 0 2

010100 20 miss

00	1	00	M[0]	M[1]	M[2]	M[3]
01	1	00	M[4]	M[5]	M[0]	M[7]
10	1	00	M[8]	M[9]	M[10]	M[11]
11	0	00	0x??	0x??	0x??	0x??

000000 0 miss

00	1	01	M[16]	M[17]	M[18]	M[19]
01	1	01	M[20]	M[21]	M[22]	M[23]
10	1	00	M[8]	M[9]	M[10]	M[11]
11	0	00	0x??	0x??	0x??	0x??

010000 16 miss

00	1	90	M[0]	M[1]	M[2]	M[3]
01	1	01	M[20]	M[21]	M[22]	M[23]
10	1	00	M[8]	M[9]	M[10]	M[11]
11	0	00	0x??	0x??	0x??	0x??

000010 2 hit

00	1	00	M[0]	M[1]	M[2]	M[3]
01	1	01	M[20]	M[21]	M[22]	M[23]
10	1	00	M[8]	M[9]	M[10]	M[11]
11	0	00	0x??	0x??	0x??	0x??

8 requests, 6 misses – last slot was never used!

Worst-Case for Direct-Mapped

- Cold DM \$ that holds four 1-word blocks
- Consider the memory accesses: 0, 16, 0, 16,...



Comparison So Far

- Fully associative
 - Block can go into any slot
 - Must check ALL cache slots on request ("slow")
 - TO breakdown (i.e. I = 0 bits)
 - "Worst case" still fills cache (more efficient)
- Direct-mapped
 - Block goes into one specific slot (set by Index field)
 - Only check ONE cache slot on request ("fast")
 - TIO breakdown
 - "Worst case" may only use 1 slot (less efficient)

Meet the Staff

	Sukrit	Suvansh
Favorite Villain	The Lannisters	Logisim [De]Evolution
What would you protest	Prerequisite enforcement	CS Design requirement
What are you passionate about?	Musicc	American football
What you'd want to be famous for?	Arora's Algorithm	Facial Hair

Agenda

- Review of yesterday
- Administrivia
- Direct-Mapped Caches
- Set Associative Caches
- Cache Performance

Set Associative Caches

- Compromise!
 - More flexible than DM, more structured than FA
- N-way set-associative: Divide \$ into sets, each of which consists of N slots
 - Memory block maps to a set determined by Index field and is placed in any of the N slots of that set
 - Call N the *associativity*
 - New hash function:
 (block address) modulo (# sets in the cache)
 - Replacement policy applies to every set

Effect of Associativity on TIO (1/2)

- Here we assume a cache of fixed size (C)
- Offset: # of bytes in a block (same as before)
- Index: Instead of pointing to a *slot*, now points to a *set*, so I = log₂(C÷K÷N)
 - Fully associative (1 set): 0 Index bits!
 - Direct-mapped (N = 1): max Index bits
 - Set associative: somewhere in-between
- Tag: Remaining identifier bits (T = A − I − O)

Effect of Associativity on TIO (2/2)

 For a fixed-size cache, each increase by a factor of two in associativity doubles the number of blocks per set (i.e. the number of slots) and halves the number of sets – decreasing the size of the Index by 1 bit and increasing the size of the Tag by 1 bit



Example: Eight-Block Cache Configs







Eight-way set associative (fully associative)

Tag	Data														

- Total size of \$ = # sets × associativity
- For fixed \$ size, associativity ↑ means # sets ↓ and slots per set ↑
- With 8 blocks, an 8-way set associative \$ is same as a fully associative \$

Block Placement Schemes

Place memory block 12 in a cache that holds 8 blocks



- Fully associative: Can go in any of the slots (all 1 set)
- **Direct-mapped:** Can only go in slot (12 mod 8) = 4
- 2-way set associative: Can go in either slot of set (12 mod 4) = 0

SA Cache Example (1/5)

- Cache parameters:
 - 2-way set associative, 6-bit addresses, 1-word blocks, 4-word cache, write-through
- How many sets?
 - $C \div K \div N = 4 \div 1 \div 2 = 2 \text{ sets}$
- TIO Breakdown:

 $-0 = \log_2(4) = 2$, $| = \log_2(2) = 1$, T = 6 - 1 - 2 = 3

Memory Addresses: XXX X XX



SA Cache Example (2/5)

- Cache parameters:
 - 2-way set associative, 6-bit addresses, 1-word blocks, 4-word cache, write-through

Offset – 2 bits, Index – 1 bit, Tag – 3 bits
 Offset

			V	Tag	00	01	10	11	_
	0	0	Х	XXX	0x??	0x??	0x??	0x??	LRU
Indov	U	1	Х	XXX	0x??	0x??	0x??	0x??	Х
muex	4	0	Х	XXX	0x??	0x??	0x??	0x??	LRU
	T	1	Х	XXX	0x??	0x??	0x??	0x??	Х

 37 bits per slot, 37*2 = 74 bits per set, 2*74 = 148 bits to implement

SA Cache Example (3/5)



Each block maps into one set (either slot) (see colors)

On a memory request: (let's say 001011_{two})

1) Take Index field (0)

2) For EACH slot in set, check valid bit, then compare Tag

SA Cache Example (4/5)

• Consider the sequence of memory address accesses Starting with a cold cache: 0 2 4 8 20 16 0 2

000000 0 miss

0	0	1	000	M[0]	M[1]	M[2]	M[3]
U	1	0	000	0x??	0x??	0x??	0x??
1	0	0	000	0x??	0x??	0x??	0x??
T	1	0	000	0x??	0x??	0x??	0x??

000100 4 miss

0	0	1	000	M[0]	M[1]	M[2]	M[3]
U	1	0	000	0x??	0x??	0x??	0x??
1	0	1	000	M[4]	M[5]	M[6]	M[7]
T	1	0	000	0x??	0x??	0x??	0x??

2 hit 000 M[0] M[1] [M[2]] M[3] 0 0 0 000 0x?? 0x?? 0x?? 0x?? 1 0 000 0x?? 0x?? 0x?? 0x?? 0

1	0	000	0x??	0x??	0x??	0x??

001000 8 miss

000010

0	1	000	M[0]	M[1]	M[2]	M[3]
1	1	001	M[8]	M[9]	M[10]	M[11]
10	1	000	M[4]	M[5]	M[6]	M[7]
[⊥] 1	0	000	0x??	0x??	0x??	0x??

SA Cache Example (5/5)

Consider the sequence of memory address accesses
 Starting with a cold cache:
 O
 A
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 A
 B
 B
 A
 B
 B
 B
 Consider the sequence of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Constraint of memory address accesses
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold cache:
 Constraint of memory address accesses
 Starting with a cold c

010100 20 miss

0	1	000	M[0]	M[1]	M[2]	M[3]
1	1	001	M[8]	M[9]	M[10]	M[11]
0	1	000	M[4]	M[5]	M[6]	M[7]

1 1 010 M[20] M[21] M[22] M[23]

000000 0 miss

0	0	1	010	M[16]	M[17]	M[18]	M[19]
U	1	1	001	M[8]	M[9]	M[10]	M[11]
1	0	1	000	M[4]	M[5]	M[6]	M[7]
T	1	1	010	M[20]	M[21]	M[22]	M[23]

0 1 000 M[0] M[1] M[2] M[3]

Λ	U	┺					
U	1	1	001	M[8]	M[9]	M[10]	M[11]
1	0	1	000	M[4]	M[5]	M[6]	M[7]
T	1	1	010	M[20]	M[21]	M[22]	M[23]

000010 2 hit

0)[1	010	M[16]	M[17]	M[18]	M[19]
1	-	1	000	M[0]	M[1]	M[2]	M[3]
10)	1	000	M[4]	M[5]	M[6]	M[7]
[⊥] 1	- [1	010	M[20]	M[21]	M[22]	M[23]

• 8 requests, 6 misses

Worst Case for Set Associative

 Worst case for DM was repeating pattern of 2 into same cache slot (HR = 0/n)

- Set associative for N > 1: HR = (n-2)/n

- Worst case for N-way SA with LRU?
 - Repeating pattern of at least N+1 that maps into same set
 0, 8, 16, 0, 8,
 - Back to HR = 0:

000	M[863]]9]
000	M[0-3]]



Question: What is the TIO breakdown for the following cache?

- 32-bit address space
- 32 KiB 4-way set associative cache
- 8 word blocks

A = 32, C = 32 KiB = 2¹⁵ B, N = 4, K = 8 words = 32 B



 $O = \log_{2}(K) = 5 \text{ bits}$ $C/K = 2^{10} \text{ slots}$ $C/K/N = 2^{8} \text{ sets}$ $I = \log_{2}(C/K/N) = 8 \text{ bits}$ T = A - I - O = 19 bits

Summary

- Set associativity determines flexibility of block placement
 - Fully associative: blocks can go anywhere
 - Direct-mapped: blocks go in one specific location
 - N-way: cache split into sets, each of which have n slots to place memory blocks