

# University of California, Berkeley – College of Engineering

Department of Electrical Engineering and Computer Sciences

Spring 2008

Instructor: Dr. Dan Garcia

2008-03-09



After the exam, indicate on the line above where you fall in the emotion spectrum between “sad” & “smiley”...

<i>Last Name</i>	<b>Answer Key</b>
<i>First Name</i>	
<i>Student ID Number</i>	
<i>Login</i>	cs61c-
<i>Login First Letter (please circle)</i>	a b c d e f g h i j k l m
<i>Login Second Letter (please circle)</i>	a b c d e f g h i j k l m n o p q r s t u v w x y z
<i>The name of your LAB TA (please circle)</i>	Ben Brian Casey David Keaton Matt Omar
<i>Name of the person to your Left</i>	
<i>Name of the person to your Right</i>	
<i>All the work is my own. I had no prior knowledge of the exam contents nor will I share the contents with others in CS61C who have not taken it yet. (please sign)</i>	

## a) Instructions (Read Me!)

- Don't Panic!
- This booklet contains 6 numbered pages including the cover page. Put all answers on these pages; don't hand in any stray pieces of paper.
- Please turn off all pagers, cell phones & beepers. Remove all hats & headphones. Place your backpacks, laptops and jackets at the front. Sit in every other seat. Nothing may be placed in the “no fly zone” spare seat/desk between students.
- Question 0 (1 point) involves filling in the front of this page and putting your name & login on every front sheet of paper.
- You have 180 minutes to complete this exam. The exam is closed book, no computers, PDAs or calculators. You may use one page (US Letter, front and back) of notes and the green sheet.
- There may be partial credit for incomplete answers; write as much of the solution as you can. We will deduct points if your solution is far more complicated than necessary. When we provide a blank, please fit your answer within the space provided. You have 3 hours...relax.

Question	0	1	2	3	4	5	Total
Minutes	1	36	36	36	36	36	180
Points	1	14	15	15	15	15	75
Score	1	14	15	15	15	15	75





**Question 3: You won't mind this question one bit! (15 pts, 36 min)**

We wish to implement a **bit** array, where we can read and write a particular *bit*. Normally for read/write array access, we would just use bracket notation (e.g., `x=A[5]`; `A[5]=y`), but since a bit is *smaller* than the smallest datatype in C, we have to design our own `GetBit()` and `SetBit()` functions. We'll use the following typedefs to make our job easier:

```
typedef uint8_t  bit_t;      // If it's a single bit, value is in least significant bit.
typedef uint32_t index_t;   // The index into a bit_t array to select which bit is used
```

E.g., imagine a 16-bit bit array: `bit_t A[2]`; `A[1]=0x82`; `A[0]=0x1F`; Internally, A would look like this:

	8				2				1				F			
<b>Array A:</b>	1	0	0	0	0	0	1	0	0	0	0	1	1	1	1	1
<b>Bit index:</b>	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

`GetBit(A,0)` would return 1, as would `GetBit(A,1)`, `GetBit(A,2)`, `GetBit(A,3)`, and `GetBit(A,4)`.  
`GetBit(A,5)` would return 0, as would `GetBit(A,6)`, `GetBit(A,7)`, and `GetBit(A,8)`. Etc.

- a) How much space would the largest **usable** bit array take up? 512 MiB  
 "Usable" means we could read and write every bit in the array.  
 Express your answer in IEC format. E.g., 128 KiB, 32TiB, etc. \_\_\_\_\_

- b) Write `setBit` in C. *You may not need to use all the lines.*

```
void SetBit(bit_t A[], index_t n, bit_t b) { // b is either 0 or 1
    A[n/8] = ( A[n/8] & ~(1 << (n%8)) ) | (b << (n%8)); // The one liner. Or ...

    _____
    uint32_t byte_index = n/8; // (1) Find out which byte we want
    uint8_t bit_index = n%8; // (2) Find where within that byte is the bit

    _____

    A[byte_index] &= ~(1 << bit_index); // (3) Reset that bit in the byte

    _____

    A[byte_index] |= b << bit_index; // (4) Assign that bit in the byte

    _____
}
```

- c) Write `GetBit(bit_t A[], index_t n)` in MAL; `$v0` should be 1 if the bit is on, and 0 if it's off.  
 Hint: it might help if you start from the `srlv` and work backwards.

```
GetBit:    srl    $t0, $a1, 3           # $t0 = byte_index = n/8
           _____ #
           addu   $t1, $a0, $t0       # $t1 = A + byte_index
           _____ #
           lbu    $t2, 0($t1)         # $t2 = byte = *(A+byte_index) = A[byte_index]
           _____ #
           andi   $t3, $a1, 7         # $t3 = bit_index = n%8
           _____ #
           srlv   $v0,$t2,$t3         # $v0 = byte >> bit_index (slide bit to lsb slot)
           srlv   $v0,$t2,$t3         # "srlv rd,rt,rs" means (in C): rd = rt >> rs
           andi   $v0,$v0,1           # $v0 &= 1 (mask out the lsb bit)
           _____ #

           jr     $ra                 # $v0 better be either a 0 or 1
```

**Question 4: Did somebody say “Free Lunch”?! (15 pts, 36 min)**

Consider two competing 5-bit floating point formats. Each contains the same fields (sign, exponent, significand) and follows the same general rules as the 32-bit IEEE standard (denorms, biased exponent, non-numeric values, etc.), but allocates its bits differently.

Implementation “LEFT”: 

S	EE	FF
---	----	----

  
*scratch space (show all work here)*

Implementation “RIGHT”: 

S	EEE	F
---	-----	---

  
*scratch space (show all work here)*

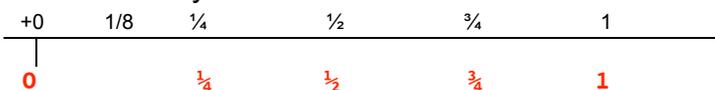
Exponent Bias:  $2^{\#Es-1}-1 = 2^{2-1}-1 = 1$   
 Denorm implicit exponent:  $-(Bias-1) = 0$   
 Number of NaNs: 6

Exponent Bias:  $2^{\#Es-1}-1 = 2^{3-1}-1 = 3$   
 Denorm implicit exponent:  $-(Bias-1) = -2$   
 Number of NaNs: 2

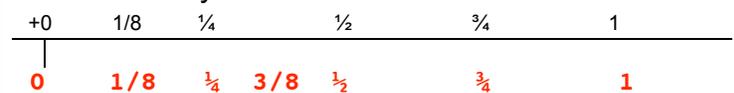
What	Number	Bit Pattern
<i>Smallest non-zero pos denorm</i>	$2^0 \times 0.01 = \frac{1}{4}$	<b>0b00001=0x1</b>
<i>Largest non-infinite pos value</i>	$2^1 \times 1.11 = 3.5$	<b>0b01011=0xB</b>
<i>Negative Infinity</i>	$-\infty$	<b>0b11100=0x1C</b>

What	Number	Bit Pattern
<i>Smallest non-zero pos denorm</i>	$2^{-2} \times 0.1 = 1/8$	<b>0b00001=0x1</b>
<i>Largest non-infinite pos value</i>	$2^3 \times 1.1 = 12$	<b>0b01101=0xD</b>
<i>Negative Infinity</i>	$-\infty$	<b>0b11110=0x1E</b>

Mark every representable number in the range [+0,1] as a vertical line on the number line below. We’ve already done it for +0.



Mark every representable number in the range [+0,1] as a vertical line on the number line below. We’ve already done it for +0.



Which implementation is able to represent more *integers*, LEFT or (RIGHT) ? (circle one)

**Question 5: Three's a Crowd... (15 pts, 36 min)**

Breaking news! We have just developed hardware that has 3-states: {false=0, true=1, and maybe=2}! Now we can store all our numbers in base 3. The race is on to develop a good encoding scheme for integer values.

Decimal	Ternary
5	12 <sub>three</sub>
26	222 <sub>three</sub>
27	1000 <sub>three</sub>

- a) To warm up, first do some simple conversions between decimal and unsigned ternary. We've done one for you.
- b) Suppose we have N ternary digits (*tets*, for short). What is the largest unsigned integer that can be stored?

$3^N - 1$

---

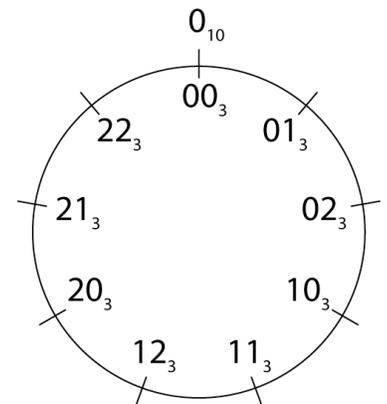
Ok, now that we've got unsigned numbers nailed down, let's tackle the negatives. We'll look to binary representations for inspiration.

- c) Name two disadvantages of a *sign and magnitude* approach in ternary. Suppose a leading 0 means positive, and a leading 1 means negative, similar to what we did in the binary days. **There are only two signs, but three digits (we waste 1/3 of our numbers)**

**Wierd gap behavior -> now we jump to 2's mystery zone from 11...11 + 1.**

---

- d) Maybe *three's complement* will be more promising. To make sure we understand what that means, let's begin with a very small example – say a 2-tet number. Fill in the following number ring of tet-patterns with the values we'd like them to represent (just as in two's complement, we want all zeros to be zero, and want a balanced number of positive and negative values). **Going clockwise from 00<sub>three</sub> : 0, 1, 2, 3, 4, -4, -3, -2, -1**



- e) Recall that for an N-bit *two's complement* number, the bit-pattern of the largest positive number looks like 011...11. For an N-tet *three's complement* number, what does the tet-pattern of the largest positive number look like?

                    11...11                    

- f) Provide (in pseudocode) an algorithm for *negating* an N-tet three's complement number.

**Flip all tets across 1. (i.e., 0 -> 2, 1-> 1, 2 -> 0)  
Add 1. (same as 2's complement!)**