State

1. Fill out the timing diagram for the circuit below:

```
      +---+      +---+      +---+
   IN-|D Q|-s0-|D Q|-s1-|D Q|--Out
      +-^-+      +-^-+      +-^-+
        |          |          |
   CLK--+-------+-------+

   clk  __|‾|_|‾|_|‾|_|‾|_|‾|_|‾|_
   in   _____|‾|__|‾‾‾‾‾‾|_____
   s0   ▓▓▓|_____|‾‾‾‾‾|____
   s1   ▓▓▓▓▓▓|_____|‾‾‾‾|____
   out  ▓▓▓▓▓▓▓▓▓▓|_____|‾|__
```

2. Fill out the timing diagram for the circuit below:

```
      +---+      +---+
   A--|D Q|-R1-|D Q|-R2--
      +-^-+      +-^-+
        |          |
   CLK--+---|>o--+

   clk  __|‾|__|‾|__|‾|__|‾|__
   !clk ‾‾|_|‾‾|_|‾‾|_|‾‾|_|‾‾
   A    _|‾‾|____|‾‾‾‾‾|_____
   R1   ▓▓|_____|‾|_
   R2   ▓▓▓▓▓|_____|_
```
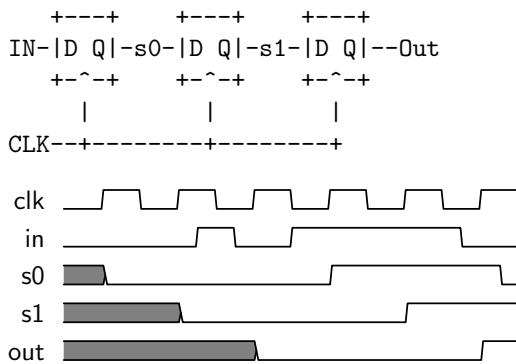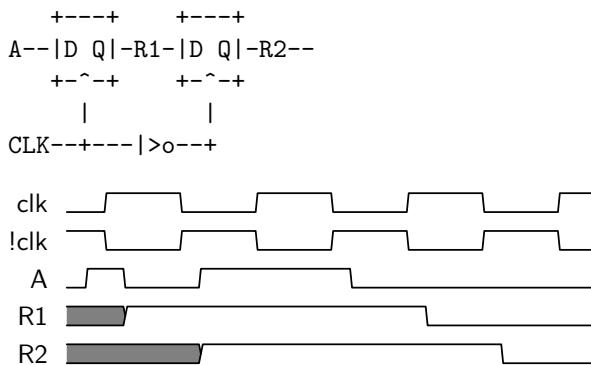
Logic Gates

1. Label the following logic gates:
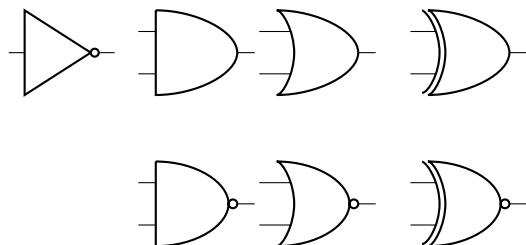
> **Solution:** not, and, or, xor, nand, nor, xnor

2. Convert the following to boolean expressions:
   (a) NAND

Solution: $\bar{A}\bar{B} + \bar{A}B + A\bar{B}$

(b) XOR

Solution: $\bar{A}B + A\bar{B}$

(c) XNOR

Solution: $\bar{A}\bar{B} + AB$

3. Create an AND gate using only NAND gates.

Solution:

A ─┐
   │ >o─┐ >o─ Output
B ─┘   │
   ─────┘
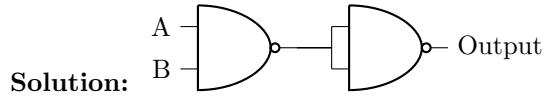
4. How many different two-input logic gates can there be? How many n-input logic gates?

> **Solution:** A truth table with n inputs has 2^n rows. Each logic gate has a 0 or a 1 at each of these rows. Imagining a function as a 2^n-bit number, we count 2^{2^n} total functions, or 16 in the case of n = 2.

Boolean Logic

$1 + A = 1$ $\qquad A + \bar{A} = 1$ $\qquad A + AB = A$ $\qquad (A + B)(A + C) = A + BC$
$0B = 0$ $\qquad\qquad B\bar{B} = 0$ $\qquad\quad A + \bar{A}B = A + B$
DeMorgan's Law: $\quad \overline{AB} = \bar{A} + \bar{B} \quad \overline{A + B} = \bar{A}\bar{B}$

1. Minimize the following boolean expressions:

   (a) Standard: $(A + B)(A + \bar{B})C$

   > **Solution:**
   > $$(AA + A\bar{B} + AB + B\bar{B})C = (A + A(\bar{B} + B))C = AC \tag{1}$$

   (b) Grouping & Extra Terms: $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + A\bar{B}C$

   > **Solution:**
   > $$\bar{A}\bar{C}(\bar{B} + B) + A\bar{C}(B + \bar{B}) + AC(B + \bar{B}) \;=\; \bar{A}\bar{C} + A\bar{C} + AC \tag{2}$$
   > $$= \bar{A}\bar{C} + A\bar{C} + A\bar{C} + AC \tag{3}$$
   > $$= (\bar{A} + A)\bar{C} + A(\bar{C} + C) \tag{4}$$
   > $$= A + \bar{C} \tag{5}$$

   (c) DeMorgan's: $\overline{A(\bar{B}\bar{C} + BC)}$

   > **Solution:**
   > $$\overline{A(\bar{B}\bar{C} + BC)} \;=\; \bar{A} + \overline{\bar{B}\bar{C} + BC} \tag{6}$$
   > $$= \bar{A} + \overline{\bar{B}\bar{C}}\,\overline{BC} \tag{7}$$
   > $$= \bar{A} + (B + C)(\bar{B} + \bar{C}) \tag{8}$$
   > $$= \bar{A} + B\bar{C} + \bar{B}C \tag{9}$$